

300 PROBLEMS IN SPECIAL AND GENERAL RELATIVITY

Einstein's theories of special relativity and general relativity form a core part of today's undergraduate (or master's-level) physics curriculum. This is a supplementary problem book or student's manual, consisting of 150 problems in each of special and general relativity i.e., in total 300 problems. The problems have been collected, developed, tested, and refined by the authors over the past two decades from homework and exams given at KTH Royal Institute of Technology, Stockholm, Sweden, starting in the late 1990s. They are a mixture of short-form and multipart extended problems, with hints provided where appropriate. Complete solutions are elaborated for every problem, in a different section of the book; some solutions include brief discussions on their physical or historical significance. The extensive and fully worked out solutions are the main feature of the book and have been revised several times by the authors. Designed as a companion text to complement a main relativity textbook, it does not assume access to any specific textbook. This is a helpful resource for advanced students, for self-study, as a source of problems for university teaching assistants, or as an inspiration for instructors and examiners constructing problems for their lectures, homework, or exams.

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Mattias Blennow , Tommy Ohlsson

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With Complete Solutions

MATTIAS BLENNOW

KTH Royal Institute of Technology

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To our wives, Ana and Linda

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Preface

This book is a supplementary book in the form of a “problem book” or “student’s manual” in special and general relativity consisting of a total of 300 problems (150 problems each in special and general relativity) with complete and elaborate solutions. It is intended as a companion text to a main textbook, but does not assume any particular textbook. It may be used for self-study act as a source of problems for classes, or as inspiration for teachers and examiners looking to construct new problems for lectures, homework, or exams.

The problems have been collected over the course of about two decades from homework and exams given at KTH Royal Institute of Technology, Stockholm, Sweden, starting in the late 1990s. The extensive and fully worked-out solutions are the main feature of the book and have been revised several times by the authors.

The book is divided into the following chapters:

“Notation, Concepts, and Conventions in Relativity Theory”;

1. Problems in “Special Relativity Theory”;
2. Problems in “General Relativity Theory”; and
3. “Solutions to Problems” in both special and general relativity,

where the first, unnumbered chapter introduces and sets the stage for both special and general relativity and is intended to be a brief review. The structure of the book is to first present the problems belonging to each main chapter (i.e., Chapters 1 and 2), which are further split into sections in order to obtain a better overview. The solutions are then presented in Chapter 3 (i.e., they do not follow immediately after the problem formulations). The main purpose of this is to suppress the urge for the reader to look at the solution to a problem before making a proper attempt. Some of the problems and solutions are illustrated by figures.

The target audience of the book is students and teachers of special and/or general relativity courses at the master’s level that may benefit from it in the way described

above. It will generally be too advanced for the relativity covered by the typical introductory modern physics courses at the bachelor's level, and most likely not advanced enough for an in-depth study at the PhD level.

Finally, we would like to acknowledge our colleagues Jouko Mickelsson, Håkan Snellman, Edwin Langmann, and Teresia Månsson, who have given important contributions to some of the problem statements included in this book. We would also like to thank our editor, Vince Higgs, at Cambridge University Press for a smooth and constructive process with the publication of this book, Torbjörn Bäck for supporting us in developing this book, and Marcus Pernow for proofreading earlier versions of the problem statements and solutions in special relativity. In addition, the KTH Royal Institute of Technology in Stockholm, Sweden, and the University of Iceland in Reykjavik, Iceland, are acknowledged for their hospitality and financial support.