WHAT IS A QUANTUM FIELD THEORY?

Quantum field theory (QFT) is one of the great achievements of physics, of profound interest to mathematicians. Most pedagogical texts on QFT are geared toward budding professional physicists, however, whereas mathematical accounts are abstract and difficult to relate to the physics. This book bridges the gap. While the treatment is rigorous whenever possible, the accent is not on formality but on explaining what the physicists do and why, using precise mathematical language. In particular, it covers in detail the mysterious procedure of renormalization. Written for readers with a mathematical background but no previous knowledge of physics and largely self-contained, it presents both basic physical ideas from special relativity and quantum mechanics and advanced mathematical concepts in complete detail. It will be of interest to mathematicians wanting to learn about QFT and, with nearly 300 exercises, also to physics students seeking greater rigor than they typically find in their courses.

MICHEL TALAGRAND is the recipient of the Loève Prize (1995), the Fermat Prize (1997), and the Shaw Prize (2019). He was a plenary speaker at the International Congress of Mathematicians and is currently a member of the *Académie des sciences* (Paris). He has written several books in probability theory and well over 200 research papers.

"This book accomplishes the impossible task: It explains to a mathematician, in a language that a mathematician can understand, what is meant by a quantum field theory from a physicist's point of view. The author is completely and brutally honest in his goal to truly explain the physics rather than filtering out only the mathematics, but is at the same time as mathematically lucid as one can be with this topic. It is a great book by a great mathematician."

- Sourav Chatterjee, *Stanford University*

"Talagrand has done an admirable job of making the difficult subject of quantum field theory as concrete and understandable as possible. The book progresses slowly and carefully but still covers an enormous amount of material, culminating in a detailed treatment of renormalization. Although no one can make the subject truly easy, Talagrand has made every effort to assist the reader on a rewarding journey though the world of quantum fields."

- Brian Hall, *University of Notre Dame*

"A presentation of the fundamental ideas of quantum field theory in a manner that is both accessible and mathematically accurate seems like an impossible dream. Well, not anymore! This book goes from basic notions to advanced topics with patience and care. It is an absolute delight to anyone looking for a friendly introduction to the beauty of QFT and its mysteries."

- Shahar Mendelson, *Australian National University*

"I have been motivated to try and learn about quantum field theories for some time but struggled to find a presentation in a language that I as a mathematician could understand. This book was perfect for me: I was able to make progress without any initial preparation and felt very comfortable and reassured by the style of exposition."

- Ellen Powell, *Durham University*

"In addition to its success as a physical theory, quantum field theory has been a continuous source of inspiration for mathematics. However, mathematicians trying to understand quantum field theory must contend with the fact that some of the most important computations in the theory have no rigorous justification. This has been a considerable obstacle to communication between mathematicians and physicists. It is why, despite many fruitful interactions, only very few people would claim to be well versed in both disciplines at the highest level.

There have been many attempts to bridge this gap, each emphasizing different aspects of quantum field theory. Treatments aimed at a mathematical audience often deploy sophisticated mathematics. Michel Talagrand takes a decidedly elementary approach to answering the question in the title of his book, assuming little more than basic analysis. In addition to learning what quantum field theory is, the reader will encounter in this book beautiful mathematics that is hard to find anywhere else in such clear pedagogical form, notably the discussion of representations of the Poincaré group and the BPHZ Theorem. The book is especially timely given the recent resurgence of ideas from quantum field theory in probability and partial differential equations. It is sure to remain a reference for many decades."

- Philippe Sosoe, *Cornell University*

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A First Introduction for Mathematicians

MICHEL TALAGRAND

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If all mathematics were to disappear, physics would be set back exactly one week.

Richard Feynman

Physics should be made as simple as possible, but not simpler.

Albert Einstein

The career of a young theoretical physicist consists of treating the harmonic oscillator in ever-increasing levels of abstraction.

Sydney Coleman

Contents