

## INDEX OF SYMBOLS

*The numbers denote pages.* $>, <, \geq, \leq$  4, 8, 27 $=, \equiv$  23

0 10

 $\infty$  11 $\sqrt{\quad}$  24 $!, \perp$  35 $\searrow, \nearrow$  43 $\rightarrow$  44 $(x|y)$  18 $|x|$  44,  $|z|$  283 $\operatorname{am} z$  283 $e$  51 $\varpi, \pi$  183 $i$  282 $f(x)$  135 $f(x) \rightarrow L$  198 $\lim_{n \rightarrow \infty}$  44 $\lim_{x \rightarrow a} f(x)$  198 $\lim_{x \rightarrow \infty} f(x)$  207 $x \rightarrow \infty$  $\lim_{h \searrow 0} f(h)$  232 $Dy, D_x y, f'(x), \frac{dy}{dx}$ , etc. 143, 216 $\int_a^b f(x) dx$  149 $\int f(x) dx$  251 $[F(x)]_a^b$  251 $a^m$  85 $\log_a b$  94 $\cos x, \sin x$ , etc. 204 $C(x), S(x)$  176 $\cosh x, \sinh x, \tanh x$ , etc. 189, 191 $\operatorname{arc} \cos x$  (or  $\cos^{-1} x$ ), etc. 205, 227 $\overline{C}(x), \overline{S}(x)$  186 $T(x), \overline{T}(x)$  189 $\operatorname{arg} \cosh x$  (or  $\cosh^{-1} x$ ), etc. 191The symbols “ $\neq$ ,” “ $\nrightarrow$ ,” etc. denote the negation of “ $=$ ,” “ $\rightarrow$ ,” etc.

## GENERAL INDEX

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