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978-1-316-50973-9 - An Introductory Course of Mathematical Analysis

Charles Walmsley

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An Introductory Course of
MATHEMATICAL ANALYSIS

BY

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WITH A PREFACE BY

W. H. YOUNG, Sc.D., F.R.S.

CAMBRIDGE
AT THE UNIVERSITY PRESS
1926

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UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781316509739

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First published 1926

First paperback edition 2015

A catalogue record for this publication is available from the British Library

ISBN 978-1-316-50973-9 Paperback

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PREFACE

By

Dr W. H. YOUNG, F.R.S.

THIS book had its origin in, and is one of, the tangible results of my reorganisation of the Mathematical Department at Aberystwyth in the University of Wales, during my four years tenure of office there. The problem I had to solve was a difficult one. It was one of the crying needs of the Departments of Physics and Chemistry that their students should by the end of their first year have become acquainted with the concepts and processes of Differentiation and Integration. A considerable portion of these students, however, were, on their arrival at the College, entirely ignorant of Trigonometry, the knowledge of which is presupposed in treatises, even elementary, on the Calculus. In drawing up a Syllabus for the non-geometrical portion of the First Year Course, I was thus led to arrange that the notion of a trigonometrical function appeared at a comparatively late stage and in an analytical form; a procedure, for the rest, in accordance with what has been felt as desirable by more than one distinguished mathematician, in view of the very restricted range of the geometrical definition *per se* of a trigonometrical function. Moreover the well-known interest felt by the Celtic race for abstract ideas, as well as the circumstance that the First Year Class necessarily contained students who had in view the possibility of taking Honours in Mathematics at a later stage, justified a rather more theoretical treatment than has been usual in this country, not only in the above connection but throughout the course.

The Syllabus, as printed in the Programme of the University College of Wales for the year 1922—1923, is as follows:

“FIRST YEAR COURSE. *Elementary Algebraical Analysis*:—The fundamental laws of algebra; the real number; equalities and equations; inequalities and inequations; indices; nature and use of logarithms; simple infinite series; rational functions; formation of simple transcendental functions by means of power series; the idea of a limit; the idea of continuity; differentiation of rational functions and of the simpler functions represented by power series; the

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idea of a definite integral; its intuitive properties; identification of the geometrical and analytical definitions of the trigonometrical functions; simple applications of the notions of differentiation and integration.”

The prominence given to *inequalities*, and more especially to *inequations*, in this Syllabus is in accordance with Continental practice. Their comparative neglect in English teaching has seriously handicapped in the past the researcher, and has interfered with the efficiency of the ordinary mathematical student. *Logarithms* were intended to be introduced almost simultaneously with *indices*, reappearing, of course, under the head of *simple transcendental functions represented by power series*. In the remainder of the course the *idea of limit* is fundamental, the method of approach being understood to be that by *monotone sequences** and making free use of the notions of *upper and lower bounds*, on which I have laid great stress and of which I have made systematic use in my papers.

The task of actually lecturing, under my direction, on the new Syllabus, I entrusted to one of my assistants, Mr Walmsley. In view of the somewhat revolutionary character of the changes introduced into the curriculum and with the object of securing that the matter communicated to the students was what was intended, partly also because I thought a great want would thereby be supplied, I suggested to Mr Walmsley that the Course should be written out and subsequently form the basis of a joint book by the two of us. Unfortunately, only the first few manuscript pages were submitted to me, so that the present volume appears under his name alone and on his sole responsibility.

I have only to add that the success that attended the reform at Aberystwyth warrants a confident hope that the present volume, attempting as it does to embody the main features of the Schedule, will find its place, and that an important one, among the books habitually used at school and college.

* Mr Walmsley has not followed my usage of restricting the use of the word *sequence* to such *successions* as have an unique limit; but where the succession is monotone the two notions, of course, coincide.

W. H. YOUNG.

COLLONGE, LA CONVERSION,

July, 1926.

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AUTHOR'S PREFACE

I HAVE to express my indebtedness to various works of the following authors: G. H. Hardy, T. J. P. A. Bromwich, E. W. Hobson, W. H. Young, C. J. de la Vallée Poussin, and E. Goursat.

I have to thank Mr Arthur Berry, Vice-Provost of King's College, Cambridge, for many valuable improvements, and Mr H. W. Unthank, of the University College, Exeter, for reading the proofs.

C. W.

EYAM, DERBYSHIRE,
August, 1926.

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