

## PART I

### CHAPTER I

## VALUATION PRINCIPLES

### INTRODUCTION

1. The fact that life assurance has developed on the level annual premium basis has rendered the question of maintaining adequate reserves one of the utmost importance. The acceptance of a level annual premium for a risk which is generally continuously increasing with time requires the retention of the excess amounts paid in the early years against the arrival of the time when the level premium by itself is insufficient to meet the current risk. This phenomenon distinguishes life assurance from most other forms of insurance where the creation of reserves springs rather from the desire to improve the security offered than from any theoretical requirements.

2. Although the level premium basis is in undisputed possession of the field, it is an interesting fact that attempts have been made in the past to supersede it by a system whereby each year's risk was paid for as it arose. This system, known as Assessmentism, produced premiums which increased as the age advanced and, as the healthy lives could withdraw at any time without loss, heavy selection could be, and indeed was, exercised against the company. There was no necessity for holding reserves apart from an allowance for current unexpired risk. Group life assurance is undertaken at the present time in this country on the basis of assessmentism, but since the insurance unit is the group and not the individual, the objections mentioned lose much of their force.

### NECESSITY FOR VALUATION

3. Granting that the holding of reserves may be an essential, it may be asked: in what circumstances is it necessary to ascertain the amount of these reserves by independent calculation? If the premiums are adequate and if the business progresses as anticipated in the bases on which the premiums are founded, the necessary

reserve will obviously be the balance of funds in hand. If, however, the experience of the working of the fund differs from that contemplated in the premium bases, the funds in hand may be more or less than are required. It is in these circumstances (and they are the rule and not the exception) that the determination of what reserve should be in hand becomes necessary.

4. The purposes for which it may be necessary to value the liabilities are:

- (a) to determine whether a fund is solvent,
- (b) to fix how much of the fund is surplus and available for distribution as bonus.

It should be noted that in the first case the implication is that the funds are probably less than are necessary, while in the second case a surplus over requirements is anticipated. The purpose for which a valuation is being made should colour the choice of bases and method to be used. Thus, where valuation with a view to a declaration of bonus is being made, it would be prudent to set a full and an unquestionably adequate value on the liabilities because the declaration of a bonus is equivalent to the conversion of part of the fund, at present free, into a definite liability. If later it is found that through an under-estimation of the value of the liabilities too much surplus was so converted, the damage has been done and cannot easily be put right. Where, however, a test of solvency is required, bases and methods should conform as closely as practicable to the anticipated experience of the fund.

#### VALUATION FORMULAE AVAILABLE

5. The liability under a life assurance contract may be determined in a number of ways, *e.g.*

- (a) by a prospective process,
- (b) by a retrospective process,
- (c) by means of prepared tables of policy values,
- (d) by an accumulative process from the liability at a previous valuation as suggested by Searle (*J.I.A.* Vol. xxx, p. 493).

Valuations in the United Kingdom are invariably conducted by means of the prospective formula on account of the necessity for

stating the values of the sums assured and bonuses separately from the values of the premiums in the Board of Trade returns (Assurance Companies Act 1909, Schedule IV). None of the other formulae gives this information directly and must perforce be considered as useful merely for special cases.

Apart from the adaptability of the prospective formula referred to above, there are other considerations which weigh in its favour. The valuation of bonus declared from time to time is facilitated and the determination of the cost of a given rate of bonus is made a simple task.

The general form of the prospective formula used is

$$V = A - \pi a.$$

Its reduction to the form  $V = I - (\pi + d) a$  would greatly reduce the arithmetical work but in this form it suffers from the same limitations as the other methods mentioned.

#### VALUATION BASES—GENERAL REMARKS

6. The prospective value of a policy being the difference between the present value of the liabilities thereunder and the present value of the future premiums, it becomes important to consider these liabilities and premiums and the bases upon which their present value may be determined.

The liabilities consist of the sum assured and existing vested bonuses, future expenses of running the business and provision for future bonuses and sometimes for deferred bonuses.

The assets consist of the future office premiums payable under the policy.

As will be seen in a later chapter the choice of bases for discounting these assets and liabilities is involved with the question of maintaining a certain rate of bonus in the future, and great care must be exercised at this stage if confusion of mind is to be avoided.

Before proceeding to study the papers which develop the subject of valuation methods and assumptions it would be well to consider the question divested of the complications of bonuses.

7. What should be the liability under a whole life assurance without profits at age  $(x + n)$  under which an office premium of  $P'_x$  is payable?

When the policy was effected the estimated rates of mortality, interest and expense which it was thought would prevail during its currency, coupled with a margin for contingencies, resulted in a premium of  $P'_x$ .

At a subsequent valuation it would not be right to make these same assumptions blindly. If conditions have materially changed, effect must be given to these changes. The initial assumptions were after all merely estimates which may prove to be inaccurate and at a valuation the opportunity is offered whereby steps may be taken to mitigate the effect of these involuntary errors of judgment. If the opportunity is not taken and if the changes indicated are of an adverse character to the office, the evil day is only postponed and the inevitable trouble will occur with much greater force. This question has been stressed advisedly as there may be a temptation to leave matters alone and trust to the swing of the pendulum to restore the original situation. Such a procedure, besides being contrary to the traditions of life assurance, is unsound in theory.

#### VALUATION BASES—MORTALITY

8. In making a choice of suitable bases, the three underlying factors mentioned will be considered in turn.

In the first place, the rate of mortality should be that which is likely to be experienced during the future duration of the contract. In the present state of knowledge this means the rate of mortality disclosed by some applicable past experience, due effect being given, strictly speaking, to the question of selection, but an attempt has been made in the tables based on the Offices' Annuitant Experience, 1900–20, definitely to forecast the mortality of the future. There are considerable difficulties in the way of making a valuation which takes account of the duration of the policies since entry (usually called a "select" valuation) and in Chapter v the matter will be referred to in more detail. At this stage it is sufficient to say that strict theory requires a "select" valuation or at least one of the approximations that may be made thereto.

## VALUATION BASES—INTEREST

9. Secondly, the rate of interest should be the net rate which may reasonably be anticipated over the future duration of the contracts. As the unexpired term of the contracts may vary from one to one hundred years, it might be thought that different or perhaps decreasing rates of interest depending on the possible future durations should be used. To give effect to this is practically impossible and the rate of interest chosen will be an average rate suitable to the composition of the business. It is customary to base this estimate of the future course of interest rates on conditions and tendencies at the date of valuation. If the valuation is being made with a view to the distribution of a bonus, when, as mentioned before, adequacy of reserves is imperative, the rate of interest would generally be fixed somewhat lower than the rate ruling at the date of valuation. Due allowance would, however, be made for the future trend of interest rates if this were well marked, considerable weight being given to the generally admitted economic dictum that over a long period interest rates may be expected to fall.

It is pertinent to enquire to what extent the interest yield on the existing fund at the date of valuation should influence the choice of the valuation rate of interest. So far as the reserves in hand are concerned, if they are invested in securities suitably married to the liabilities so that reinvestment on a lower interest basis is improbable, it is true they will produce the rate of interest yielded by the fund. But there is still the problem of the investment of the future premiums and interest in excess of the amount required yearly to pay claims and expenses. These excesses can only be invested at the interest rates current when they arise, and the present interest yield on the fund (which may be artificial on account of the method of treatment of the book values of the securities) is not a good guide to the determination of the probable rate of accumulation of future net income.

The influence of the present yield on the fund on the valuation rate of interest will therefore depend on whether the fund is in a stationary or decreasing state (as would be found in the later life of closed funds) or whether it is increasing.

It should be stated that the foregoing remarks ignore the influence of the bonus system on the choice of the interest rate. It will be seen later that many of the theoretical niceties are submerged by the artifice of lowering the valuation rate of interest for the purpose of creating a reserve for maintaining bonus rates in the future. At the present stage, however, it is desirable that the principles governing the question should be set out free from the confusion which may result from extraneous matters.

#### VALUATION BASES—EXPENSES

10. Lastly, the rate of expense should bear direct relation to the expenses that the office incurs in conducting its business. As these expenses are largely under the control of the office, the rate at the date of valuation would in normal circumstances be appropriate. Allowance is made by taking credit for only a portion of future office premiums, the balance, or loading, being reserved to form the fund for meeting future expenses.

The amount of loading thrown off the office premiums must, if the valuation is to be a proper one, be an adequate quantity. The determination of this loading is a matter of some difficulty and, moreover, one which lends itself to abuse in unscrupulous hands. If the valuation bases of interest and mortality happen to be the same as those on which the office premiums were calculated, it would seem reasonable in a valuation to reserve the actual loading contained in the office premium. In other words, the future premiums taken credit for would be the net premiums underlying the office premium scale and the valuation would become what is called a net premium valuation. It is, however, assumed here that the expenses will be constant each year and will correspond to the level expense loadings in the office premiums. This is not the case under modern conditions where the expenses of the first year are much heavier than those of subsequent years. Moreover, it is improbable that the valuation would be conducted on the same bases as those on which the premium scale was founded, even if they be known definitely.

One expedient would be to throw off a percentage from the office premiums equal to that which the total expenses bear to the total

premiums (*i.e.* the expense ratio of the office). It will be observed that this allowance is too great in the case of business which has been some time on the books, as the renewal expenses of such business, expressed as a percentage of the premiums, will be substantially less than the expense ratio of the office which is derived from a consideration of all the expenses including those arising from new business. On the other hand, if the bare future renewal expenses of the existing business were reserved, no provision would be made (at least directly) for the cost of future replacements of business which it is necessary to contemplate in a continuing concern. The use of a percentage comparable to the office's expense ratio secures that the reserve for future expenses shall be based on an average figure which reflects both the cost of obtaining new business and of running existing business.

11. It should be remarked that the function called the expense ratio of an office is not always entirely satisfactory or reliable as a guide. It may be unduly heavy if the office has been expanding rapidly, owing to the greater proportion of the more expensive new business entering into its composition, or it may be artificially low on account of the presence of a great quantity of business at high rates of premium. Its use, as suggested here, can only be defended on the grounds that no unusual circumstances are present and that it is fitting to make provision for the future on the assumption that the policy of the office will remain substantially the same as it is at the date of valuation. It should be evident that the determination of the proper percentage to reserve is a matter of some difficulty and gives ample scope for ingenuity and, unfortunately for the reputation of this method of valuation, plenty of opportunity for unsound manipulation.

#### BONUS RESERVE METHOD

12. The method of valuation described in the foregoing paragraphs, and known as the Gross Premium method, may be said to be the obvious and logical way of valuing non-participating policies, as it seeks to take account of the actual facts and to give due effect to them. To extend the principles to participating policies it is merely

necessary to remember that the office premiums include a bonus loading in addition to provision for expenses and that if credit is to be taken for the office premiums less a percentage for expenses, then a *pro forma* liability of the present value of the future bonus that it is expected to declare must be set up. In other words, to the gross premium value of the policy should be added the present value of the future bonus on the same bases of interest and mortality. To the extent that some small margin of safety must in prudence be retained on the rates of interest and mortality used, a small profit may be expected from these sources, and it therefore arises that the full rate of anticipated bonus need not be treated as a specific liability nor need so full a percentage be thrown off the office premiums for expenses. This method is known as the Bonus Reserve method of valuation. It has many ardent advocates but also some hostile critics. It has been strongly attacked because it treats future bonus as a liability, in complete contradiction of the true nature of bonus, but on the other hand it has been stoutly defended for the way in which it keeps the true facts in view, in contrast to its chief competitor for favour, the Net Premium method, which keeps none of the facts in view although making indirect provision for future bonus by the use of an artificially low rate of interest, as will be discussed in a later chapter.

#### NET PREMIUM METHOD—GENERAL REMARKS

**13.** The outstanding characteristic of the net premium method of valuation is the fact that credit is taken, not for a percentage of the actual premium to be received under the terms of a policy, but for the net or pure premium according to the bases of interest and mortality used in the valuation. If it should happen that the valuation bases coincided with the office premium bases, then the effect of the net premium method would be to reserve the expense loadings in the office premiums, and this represented generally the state of affairs when, nearly one hundred years ago, the method began to gain in popularity.

To appreciate the evolution of the net premium method, the influence the advent of bonuses had upon it and the reasons why it has completely ousted the more reasonable bonus reserve method,



reference must be made to Sir Gerald Ryan's historical survey contained in *J.I.A.* Vol. xxxviii, pp. 70–75, 80–87, 88–96. Following these extracts the whole of Warner's paper in *J.I.A.* Vol. xxxvii, p. 57, including the discussion, should be studied. This paper, which is the standard work on the method, deals fully with the problem (previously referred to) introduced by the fact that while expense loadings are paid by policy-holders by level annual sums, the actual expenses are heaped up at the beginning of a contract's existence. Further reference is made to other methods of avoiding the consequences of this feature in paragraph 16 *et seq.*

Fuller information on the bonus reserve method of valuation is contained in Coutts's paper (*J.I.A.* Vol. xlii, p. 161) and the discussion thereon, and papers by McNair Jones (*J.S.S.* Vol. iii, p. 36) and Elderton (*J.I.A.* Vol. lxii, p. 62). These papers introduce considerations which are related to the problem of distribution of surplus and are referred to again in a later chapter, and at this stage they should be studied for what they tell of the methods of valuation. To divorce valuation from distribution of surplus is impracticable in the sense that the valuation is the instrument by which the amount of cash surplus available for distribution is determined, but at this stage, in studying papers, attention should be fixed more on methods than on consequences.

14. In reading Sir Gerald Ryan's survey it is interesting to note the reasons he gives for the supersession of the method of valuation based on the Northampton Table used by the Equitable Society in its early days. Particularly suggestive is the statement that the second line of attack against this method "sought to demolish the idea that a result scientifically sound can be produced by false premises". The success of the attack was complete, but when the same weapon is used in turn against the net premium method its failure is as complete as was its previous success. Sir Gerald ventured to prophesy that the next few years after the time of writing (1902) would see some vital changes in the methods of valuation used, and few would dare contend that he had not ample grounds for the opinion. Warner and Coutts both launched damaging attacks, yet to-day it is a surprising fact that

the net premium method of valuation for the purpose of determining the amount of cash surplus available for distribution is more firmly entrenched than ever before. In view of its admitted artificiality under modern conditions, this fact is so remarkable as to merit serious consideration.

15. The fact that the net premium method automatically produces a reserve of zero at entry is a great convenience and avoids the necessity of testing for and eliminating negative values (rightly considered inadmissible) which may arise with any form of gross premium valuation, but other reasons for the persistence of the method must be sought. A pointer may be obtained from the fact that the defenders of the net premium method invariably used offensive methods against the principal alternative—the bonus reserve method—instead of putting forward a direct defence of the net premium method. May it not be the absence of a practicable alternative to the net premium method which leaves it in undisputed possession of the field? It must be clear from reading the papers dealing with the bonus reserve method that severe practical difficulties lie in the way of employing it. Moreover, the net premium method has acquired in the eyes of the public a reputation as a standard of undoubted strength and the adoption of an alternative, perhaps just as stringent, would be difficult in these days of unbridled competition. Further, the use by most companies of the same valuation system facilitates comparisons which would be nearly impossible if bonus reserve valuations, suited to the circumstances of each company, were the rule. Whether these comparisons are of much use is an open question, especially when, as will be seen in a later chapter, it is realized that the strength of a certain net premium valuation may be less than might be thought owing to the methods employed in determining the ages of the lives and the durations of the contracts.

Finally Coutts, in a paper to be referred to later, states that the method leaves nothing in effect to individual judgment. While this suggestion may not be complimentary to actuaries, there is no denying that the net premium standard is a powerful safeguard against any temptation to distribute bonus on too liberal a scale.