Look through the contents page and choose a topic for your workshop. As well as the mathematical content each workshop will emphasise a teaching strategy. These strategies are introduced in the following pages. They are arranged in two themes. The first is practical approaches to learning and the second is communication in the classroom.

Strategy 1: Visual and practical learning styles

Visual and practical learning styles is used as a strategy in chapters: 1, 3, 7, 8, 12, 14, 15, 16, 17, 19.

Presenting a visual stimulus and really encouraging learners to look is a powerful way to focus attention in the classroom and get the learners thinking. The workshops are full of activities which use practical activities to get the learners involved and interested. Simple, inexpensive equipment such as string and counters and paper are used wherever possible. Often a short practical activity is used to develop understanding before moving to more formal tasks. **See page 4.**

Strategy 2: People maths

People maths is used as a strategy in chapters 1, 4, 12, 14, 20.

Sometimes the resource used is the learners themselves. Learners can make a bar chart, demonstrate reflection in a mirror or demonstrate subtraction of a negative number. **See page 5.**

Note

You could return to these pages after you have tried several workshops to discuss which strategies you have found most useful in your classroom.

How do you remember a cell phone number? Do you 'see' it in your mind's eye?

Do you recall it by saying it?

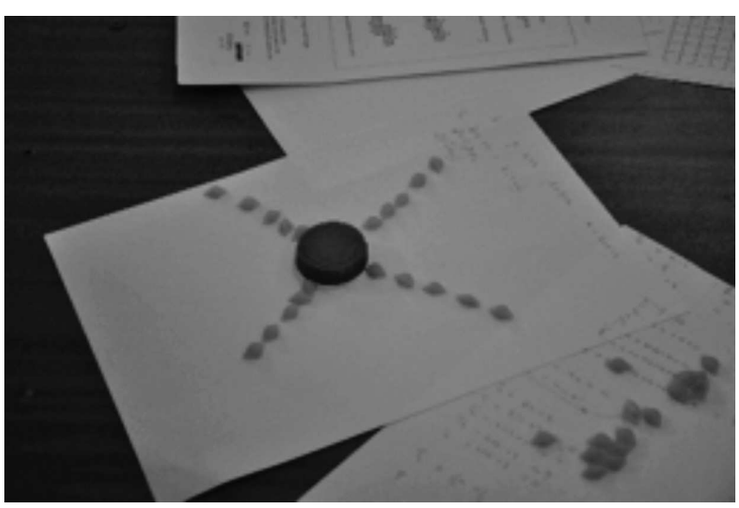
Do you need the cell phone in front of you to tap it out?

Many people have a preference for a learning style – visual, auditory, or kinaesthetic. What this means is that, given a choice, they prefer to learn new information through seeing or hearing or doing. Of course that doesn't mean that they can't use their other senses – unless they have some specific disability they will be using all three (and others) all the time. They just learn better in one way, that's all.

So what are the implications of this for educators?

Firstly, be aware of whether you have a learning style preference and if so, what it is. (Trying the cell phone test can give you an idea, or go to the website below). The reason it is good to know is that educators tend to teach in the way that they themselves prefer to be taught. So if you prefer learning through listening you're likely to lecture your learners, if you prefer learning by using pictures and diagrams you're likely to demonstrate through illustration, and if you prefer learning by doing you're likely to set up activities where your learners have 'hands-on' experiences. Actually, of course, you should strive for a balance of instructional methods so that all your learners meet a variety of experiences. So whatever you are teaching, try to make sure that you:

- Provide some visual clues and use diagrams and pictures. Ask learners to draw their own representations of the maths you are doing together. Ask them to close their eyes and answer questions about a mathematical object they can visualise, such as a 3 dimensional shape, or the number line, or a graph...
- Give a good verbal explanation, expect learners to talk about what they are doing, and ask them to explain their ideas to you and to each other... (you can't do maths if you don't talk maths).
- Make sure that your learners have to DO something – with resources, or perhaps by moving their own bodies. There are lots of everyday objects that can be used to model mathematics... buttons, bottle tops, stones, string, straws....



Everyday objects used for sequences

Try thinking about a recent lesson you have taught and see what learning styles you and your learners used. You might be surprised!

Richard Felder 2007 www4.ncsu.edu/unity/lockers/users/f/felder/public/Learning_Styles.html

Have you ever: sung in a choir, played sport in a team, played in a band, acted in a play?

Such activities are often very memorable; we get pleasure from being involved, from being active. Young people are full of energy and we need to harness that energy to assist their learning. Throughout this book you will find activities that get the learners out of their seats and taking part in a mathematical activity. These are called People Maths activities.

Some examples of people maths activities are:

- Making string patterns based on factors (see page 17 in chapter 1)
- Using showboards for an activity on subtracting negative numbers (see page 41 in chapter 4)
- Working with brackets (see page 96 in chapter 11)
- Turning through an angle and angles in a polygon (see pages 104 and 105 and in chapter 12)
- Acting out reflection in a mirror (see page 121 in chapter 14)
- Forming a grouped bar graph or a pie chart (see pages 168 and 169 in chapter 20)

Once you have tried a few of these activities and found the ones which are most effective in involving your learners you will find that you can use people maths when you are teaching lots of different topics. The activities are designed to help the learners understand the maths. When you have moved to form a reflection in a mirror you do not forget that the image is the same distance from the mirror as the object. Wherever you use counters or draw diagrams you could instead use people and perhaps a few props like string to bring the maths to life.

Learners hold 5 equally spaced pieces of string to show angles at centre of regular polygon. See chapter 13

Strategies 3 to 6 are grouped around the theme of communication in the classroom. These strategies will help you to develop a whole range of teaching skills so that you will be better able to judge what your learners understand and to adjust your teaching to help them develop their understanding.

Strategy 3: Questioning

Questioning is used as a strategy in chapters 4,13 and Key questions are included in each chapter.

What sort of questions are useful and why? What type of question will encourage the learner to explain? Some workshops take questioning as the main strategy but in all workshops you will find suggestions for two or three Key Questions to use as you try to help the learners develop their mathematical thinking. **See page 7.**

Strategy 4: Discussion

Discussion is used as a strategy in chapters: 2, 3, 5, 10, 13, 18.

How do you get the learners to talk to each other about the mathematics? First you need an activity that is interesting enough for the learners to want to talk about it and then questions to focus the discussion. Working in pairs is often the easiest to arrange and can be very effective. **See page 8.**

Strategy 5: Getting feedback

Getting feedback is used as a strategy in chapters 4 and 8 and whenever the use of showboards is suggested.

Add to the ways in which you find out what the learners understand during the lesson so that you can adapt your teaching effectively. **See page 10.**

Strategy 6: Starting from a problem not a technique

Problem not a technique is used as a strategy in chapters: 6, 9 and whenever an activity clearly leads on to a technique in the curriculum.

Communication isn't going to work if the learners don't see the point of what they are learning. Presenting a problem that interests the learners can be used to show the need for a particular technique. **See page 12.**

One way of categorising factual questions is whether they are closed or open.

A closed question has a definite answer e.g.

"What is the solution of the equation $2x + 3 = 17$?"

An open question has many possible answers e.g.

"Can you tell me an equation which has as solution $x = 7$?

Open questions can tell us much more about what learners know. They are also very economical because they allow learners to give you different possible answers, all of which can be correct! Here are some open questions that help learners to think about facts and the structure of the mathematics they are learning about.

Give me an example of a square number

What makes $x(2 + x)$ an example of a quadratic equation?

What is the same and different about 2, 4, 6, 8,10.... and 5, 7, 9, 11, 13, ... ?

What needs to be changed so that (2,4) (3, 5) (5, 10) (6, 12) are all points on the same straight line?

These are usually questions about thinking. It is often useful to ask learners how they worked something out, by asking questions such as How did you...? What if you....? Why did you...? The answers learners give are useful in knowing how well they understand what they are doing.

When teachers set out to give activities to learners whereby the learners will investigate mathematical ideas and discover relationships and meanings for themselves, the teachers need to ask the right sorts of questions to guide the learners. In each workshop we suggest key questions you can use to help the learners to develop understanding and so that the teacher can judge how best to guide that individual.

Usually the teacher asks the questions and the learners give the answers. Learners should be given opportunities to ask questions as well as to answer them. Perhaps you could begin a new topic by asking the learners to write down topic questions they would like to know the answer to. At the end of a topic you could ask the learners to pose their own questions for each other. These sorts of activities can help you as a teacher to know more about what your learners know and understand.

Mason, J. (2010) Effective Questioning and Responding in the Mathematics Classroom. mcs.open.ac.uk/jhm3/Selected%20Publications/Effective%20Questioning%20&t%20Responding.pdf

A group of teachers at a workshop can use the one, two, four, more approach. Working individually at the start gives everyone time and space to read the introduction to the task and to think quietly so as to sort out their own ideas. After a few minutes it can help to work in pairs to do the activity together and to discuss it. Later if two pairs make up a foursome they may find that they have taken different approaches and have valuable ideas to exchange with the other pair. Finally each foursome will have something worthwhile to contribute to a whole group discussion.

At the start of the lesson, the teacher introduces a task and asks learners to spend a few minutes 'doing' the task on their own. The task should be such that everyone can make a start. This stage is important so that nobody can get the idea that they just wait for someone else to do the work, which is one of the problems that can arise with group work in class.

Then, at a signal from the teacher, the learners work in pairs and 'talk' about the task. They should discuss the methods they have been using. Is anyone stuck? Can they help each other? Are they thinking the same way? Did they think of the same method or different ways of tackling the task? What can they learn from each other? They should do the task together by at least one method, record their work and check their answers. This paired work should take up at least half the lesson. Even in the largest class, with limited space, it is usually possible to organize paired work.

Paired discussion gives everyone a chance to speak

When the teacher decides that the moment is right she instructs each pair of learners to work with another pair to make up a foursome. For smooth transitions the teacher may decide that the learners should sit in the same places in all the mathematics lessons, work with the same partners and make up the same groups of four. For example, if the class are not sitting around tables half the pairs might turn around to work with the pair behind

them, to talk about the work they have done. Again they can compare methods and check to see if they have got the same answers or equivalent answers. Again they might try to do the task by an alternative method and record their work.

A group of four discuss a matching activity

This is the whole class discussion at the end of the lesson when the teacher builds on what the learners have discovered for themselves. The teacher calls on representatives of different groups to report to the whole class on how they tackled the task and what they have found out, then summarizes the ideas and perhaps adds some explanation, for example on the connections between the different approaches used or the different points of view taken.

Ray, R. Paired and group work for secondary school learners in mathematics. School Improvement Service. East Riding of Yorkshire Council.
http://www.eriding.net/maths/tl_resources_sec.shtml

During a lesson a teacher needs to get as much feedback as possible on what the learners understand and how many of them are confused. A traditional method of getting feedback is to ask learners to put their hand up if they want to respond to a question. The teacher can then choose learners to answer. There are other ways of getting feedback on what learners think that enable the teacher to get more information about more learners.

Show me thumbs up if you think this statement is true, and thumbs down if you think it is false. The teacher uses a question that can be answered by yes or no. The learners think for a moment and, at the teacher's signal, everyone show thumbs up or thumbs down. A variation is to allow learners to hold their thumbs horizontal to show they are unsure. The teacher can follow up by asking several learners to explain why they chose their response.

Disadvantage	The teacher has to think of useful yes/no questions.
Advantage	The teacher can scan the room and get some feedback about all the learners.

In many countries children in primary schools use individual chalkboards to practice writing and to show teachers their answers. This traditional method is being revived in some European, Asian and African countries with some interesting variations.

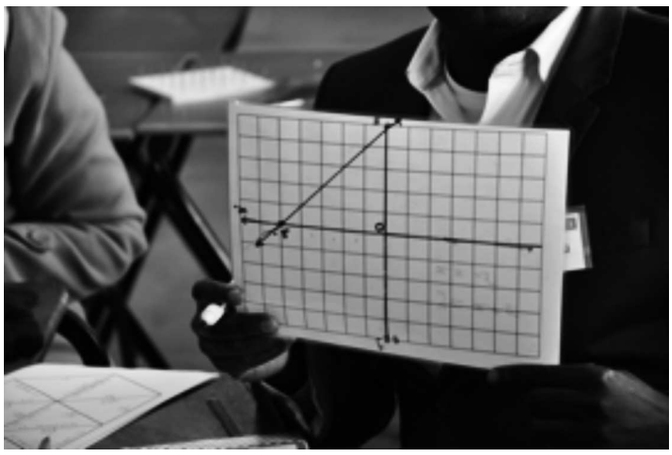
- It is being used with learners of all ages from 5 to 18.
- The showboards may be chalkboards or mini whiteboards, or laminated sheets of paper with marker pens that can be rubbed out.

Graphs on showboards

Showboards are particularly useful for open questions. An open question is a question that can have many answers, for example: show me a number that is divisible by 3.

All learners can answer the question; more confident learners can give numbers such as 120, or 72000 to show their understanding. The teacher can look around the class and, to encourage discussion, can ask several learners to show their boards to the whole class or choose two boards e.g. one with an odd and one with an even number.

Another example might be: show me a sketch of a graph with a gradient of +1.



Graph with a gradient of +1

Disadvantage	The need to convince colleagues that these boards are just as valuable with older learners as with young learners and to buy or make a class set of boards.
Advantage	Each learner must be active and the teacher can quickly judge the understanding of all the learners. The boards can be used to get responses from groups in large classes.

Standards Unit Improving learning in mathematics: challenges and strategies Malcolm Swan University of Nottingham https://www.ncetm.org.uk

In the curriculum, there will be a list of topics such as: Multiplication of fractions, Solving linear equations, Angles in a triangle. This could suggest a teaching style based on a rather mechanical format.

- Stage 1 Demonstrate the technique for solving linear equations
- Stage 2 Practice the technique with a set of exercises gradually increasing in difficulty.

Syllabuses describe the solutions not the problems so the learners never get to understand WHY the techniques were developed. With teaching in this 'transmission' style, learners may be able to carry out routine procedures successfully but they will struggle with complex procedures and problem solving.

Teachers everywhere, over the centuries have developed and collected interesting problems that lead up to important curriculum topics. This makes possible a 'transformative' teaching style:

- Stage 0 Work on a problem that leads up to the technique
- Stage 1 Develop some techniques with the pupils
- Stage 2 Practice with interesting and varied problems.

This book contains many suggestions for Stage 0 work and interesting problems for Stage 2. We hope you will find some useful activities in this book that will help you to develop your teaching styles so that you can help learners to understand and enjoy their mathematics and to tackle mathematical problems with determination and confidence.

Discuss the value of this approach. Here are some statements to stimulate discussion.

Which ones do you agree with?

- I'm short of time already. I don't have time to add an additional stage.
- A few weeks after a topic my learners have forgotten everything. I have to start all over again. Approaches they would remember would be really helpful.
- If learners develop ideas for themselves, and understand why techniques work, they will not need so much revision and repetition.
- When learners come across a slightly different problem in an exam, problem-solving techniques can give them confidence in tackling it.
- I want my learners to find mathematics more satisfying.
- When teaching equations I have to teach each slightly different type of equation separately.
- I try to teach general principles that the learner can apply to many different problems.

Murray, H., Olivier, A., Human, P. Learning Through Problem Solving, University of Stellenbosch, South Africa. www.academic.sun.ac.za/mathed