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Lenny Taelman

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Lectures on the Woods Hole Trace Formula

LENNY TAE LMAN

Universiteit van Amsterdam



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