

Introduction

The aim of this discussion is to chart salient but abstract responses to apparent contradiction in christian theology. The chart is drawn as simply, concisely, and minimally as possible compatible with user-friendliness. Along these lines, the focus is on abstract responses to contradiction versus historically occupied responses, although, where appropriate, pointers to possible historical occupants of canvassed positions are provided.

One motivation behind spelling out the abstract framework – and illustrating over and again with different examples – is to highlight the need for theologians and theology-focused philosophers to spell out the entailment relations that are inexorably involved in their would-be responses to theological contradiction. Theology, at least qua truth-seeking theory, aims at a would-be true theory of theological reality. ‘Systematic’ theologies cannot be adequately evaluated or even understood without spelling out at least a few of the basic entailment relations that govern them.

Our hope is that this Element provides an abstract framework through which theology-directed work may identify – and precisely spell out – at least the basic entailment relations on which their would-be theories (i.e., theologies) rest.

One final administrative note: as mentioned, we aim to be very concise, providing tools and basic illustrations – and nothing more. Minimal but adequate references towards actual theories (however loosely spelled out) are cited for purposes of pointing interested readers towards sufficient bibliographies.

TOOLS, TERMINOLOGY, AND BASIC IDEAS

1 Entailment, Contradiction, and Theories

The terminology of ‘contradiction’ is closely related to that of ‘entailment’ and ‘consequence’. These terms are defined, for present purposes, as follows.

1.1 Entailment

An *entailment relation* is a lack-of-counterexample relation between sentences of a language. Here, counterexamples are ‘possibilities’ recognized by the given relation. For example, while there are logical possibilities in which the actual physical laws are broken by physical objects, such possibilities are not recognized as physical possibilities – or genuine counterexamples – by the entailment relation(s) governing true physics. While there are logical possibilities in which polytheism is true, such merely logical possibilities are not

treated as genuine theological possibilities by standard christian theological theory; they are not genuine counterexamples to the theological necessity of monotheism. The point, for present purposes, is that refutation of a would-be entailment claim demands a counterexample (viz., a relevant possibility in which the would-be *entailing* sentences are true but the would-be *entailed* sentence is untrue); however, the counterexample needs to be one that is recognized within – or within the range of or scope of – the target entailment claim (i.e., in the range of the target entailment relation). Again, pointing to logical possibilities in which physics is different from true (actual) physical theory is irrelevant if such merely logical possibilities aren't recognized as physical possibilities by the entailment relation of true physics.

Let R be an entailment relation in the foregoing sense, say, the entailment relation over all physical possibilities, or the entailment relation over all theological possibilities, or so on. When a sentence A of the relevant language (e.g., the language of physics, or of theology, or etc.) R -entails a sentence B of the language (i.e., A entails B according to the relation R), the sentence B is said to be a *consequence* of sentence A according to R . Example: in standard christian theology, the sentence 'Christ is holy' is a *consequence* of the sentence 'Christ is divine', since, according to standard christian theology, the divinity of a person entails the holiness of the person. There may be logical possibilities in which divinity and holiness come apart, but those logical possibilities are not relevant possibilities – and, hence, not relevant counterexamples – to the given entailment claim from divinity to holiness, at least according to the entailment relation involved in standard christian theology.

Further illustration of such terms is provided in subsequent sections wherein the terms are put to use. The definitions of target terms run as follows.

Definition 1 (Entailment relation: sentence–sentence) *Let A and B be sentences of some language. A entails B according to relation R (equivalently, A R -entails B) iff there's no relevant possibility in which A is true but B untrue.*

Definition 2 (Entailment relation: set–sentence) *Let X be a set of sentences of some language, say, $\{A_1, \dots, A_n\}$, and let B be a sentence of the given language. X entails B according to entailment relation R iff there is no relevant possibility in which all elements of X are true but B untrue.*

1.2 Some Special Entailment Relations

Logical entailment is a special relation in the sense that it's universal and topic-neutral. This is the entailment relation over all 'logical possibilities', the

broadest set of possibilities, governing the very sparse set of so-called logical vocabulary – the ‘topic-neutral’ or ‘universal’ vocabulary in all true theories. (See Appendix A.)

Definition 3 (Logical entailment) *Let X be a set of sentences of some language, say, $\{A_1, \dots, A_n\}$, and let B be a sentence of the given language. Then, X logically entails B iff there’s no logical possibility in which everything in X is true but B untrue. (Terminology: we sometimes abbreviate ‘logical entailment’ to **logic**.)*

Again, the space of *logical possibilities* is the widest space of possibilities; it’s the space from which other entailment relations select relevant possibilities. (Again, there are logical possibilities in which all sorts of physically impossible things happen; however, the true theory of physics rules out such logical possibilities and treats them as irrelevant or, as in Section 1.1, *physically impossible*.)

Another special entailment relation (or, better, family of relations) is *predicate entailment*.

Definition 4 (Predicate-entailment relation) *Let P and Q be predicates in some language. Then, P predicate-entails (‘ p -entails’ for short) Q iff there’s no relevant possibility in which P is true of something but Q isn’t true of the given something.*

Note that, in general, p -entailment and R -entailment (whether the R is logical entailment or otherwise) are intimately related but nonetheless distinct. Each is tied to a particular space of possibilities (*viz.*, the space that the relation looks at for potential ‘counterexamples’). On one hand, any p -entailment relation is (or inevitably delivers) an R -entailment relation. In particular, let the space of possibilities over which R -entailment is defined just be the space over which p -entailment is defined. In that case, predicate P p -entails Q just if Pc R -entails Qc , where c is any ‘singular term’ (a name or a refers-to-an-individual-object term). Moreover, provided that R -entailment is defined over a language with predicates (and any serious language has them), the converse direction also applies: Pc R -entails Qc iff P p -entails Q – at least where c is an arbitrary term.

1.3 Theories: Open and Closed

Truth-seeking theorists generally aim to advance not only the truth but the full truth; they aim to truly describe their target phenomenon and to do so as fully as possible. The resulting theory contains not only some scattered truths; the

theory contains all *consequences* (or ‘implications’) of such truths, and all consequences of all such consequences of such truths, and so on. In this way, truth-seeking theorists, at least those after the full truth, have a twofold task constructing their target theories: first, they must put truths about the target phenomenon in their initial (say, ‘seed’) theory; second, they must ‘close’ the theory under an entailment (or consequence) relation for the theory.¹

Definition 5 (Subset) *Let X be any set of objects (e.g., any set of sentences, or apples, or tractors, or what have you). Then, Y is a subset of X iff everything in Y is also in X . (Moreover, Y is a proper subset of X iff Y is a subset of X but there’s also some element of X which is not an element of Y .)*

Definition 6 (Set closed under entailment) *Let X be any set of sentences from some language. Let R be an entailment (or consequence) relation for (or on) the given language. Then, X is closed under R (or R -closed) iff there’s no subset of X that entails something that’s not also in X . (In other words: X is closed under R iff there’s no subset Y of X , and no sentence A of the given language, such that Y entails A according to R but A is not in the set X .)*

R -closed sets of sentences are so called because they are ‘full’ (or ‘complete’ or ‘closed to the brim’) with respect to the set’s R -consequences (i.e., the consequences that, according to entailment relation R , follow from some subset of the set): if there’s some claim entailed by something in the set, then the set, if closed under the given entailment relation, contains that claim. If the given set is true – that is, all sentences of the set are true – and if the set is closed under all appropriate entailment relations (more on this in Section 3), the set delivers not only the truth about its target domain of phenomena but the *full truth*, at least the full truth according to the given entailment relation.

In general, every language (and, hence, every language of every theory) has numerous entailment relations on it. (This will become clearer with examples, although we focus, for simplicity, on only two salient such relations, namely *logical* and *predicate* entailment relations – more on which in Section 3.) One can think of a language L having some associated (non-empty) set \mathcal{R} of relevant entailment relations on it. From these ingredients come R -open theories and R -closed theories:

¹ Note well: the terms ‘open set’ and ‘closed set’ have very specific meanings in particular branches of mathematics (e.g., topology). The following use of such terms is *not* to be conflated with any such commonly used terminology in maths; the terms in this discussion mean just what their definitions say – nothing more.

Definition 7 (R-Open Theory) *Let L be some language; let \mathcal{R} contain entailment relations on L (i.e., all entailment relations over the same fragment of L); let R be one such entailment relation in \mathcal{R} . An R-open theory in (or for or on) the language L is any set of L sentences that is not closed under R .*

Definition 8 (R-Closed Theory) *Let L be some language; let \mathcal{R} contain entailment relations on L (i.e., all entailment relations over the same fragment of L); let R be one such entailment relation in \mathcal{R} . An R-closed theory in (or for or on) the language L is any set of L sentences closed under R .*

Note that, with respect to Definition 8, a theory might be R -closed but not R' -closed, where R and R' are different entailment relations governing the same (fragment of) a language. This fact is not only relevant but important when it comes to the target topic of contradiction in christian theology (or, more explicitly, christian-theological theories).

1.4 Contradiction

Definition 9 (Formal Contradiction) *A sentence A is a formal contradiction iff it's of the form*

It is true that . . . and it is false that . . .

where both occurrences of '. . .' are replaced by one and the same sentence, the 'it is true that', 'it is false that' and the 'and' are logical vocabulary.²

Using logical notation (from Appendix A), a *formal contradiction* has the explicit form

$$\dagger A \wedge \neg A$$

where \dagger is logic's (logically redundant) truth connective (sometimes 'logical nullation'); \neg is logic's falsity connective (sometimes 'logical negation'); and \wedge is logical conjunction. Given the logical redundancy of logic's truth connective, any formal contradiction has the implicit form

$$A \wedge \neg A.$$

Definition 10 (Contradiction) *A sentence A is a contradiction in a theory iff A entails, according to the theory's relevant entailment relation, a formal contradiction.*

² See Appendix A for logical vocabulary.

Definition 11 (Contradictory) *A sentence A is contradictory in a theory iff A entails, according to the theory's relevant entailment relation, a contradiction. A set X of sentences is contradictory (in a theory) iff X entails a contradiction (in said theory).*

2 The Threat of Contradiction

The principal threat of contradiction for a would-be true theory is 'triviality', which may be understood via so-called trivial theories.

2.1 Trivial Theories

Triviality is the uncontroversial paradigm of an absurd theory:

Definition 12 (Trivial Theory) *Let L be some language. Then, T^\perp (pronounced 'T-bottom') is the trivial theory in L iff T^\perp contains all L sentences (i.e., all sentences in the language are true according to T^\perp).*

There's a general fact concerning *some* entailment relations and any would-be true (closed) theory. The target fact is straightforward given the idea of a *logically explosive* or *logically exclusive* entailment relation:

Definition 13 (Logically Explosive/Exclusive Entailment) *Let R be an entailment relation over language L , and let A and B be L sentences. Finally, let \neg and \wedge be logical negation and logical conjunction (i.e., logical 'and'), so that, for example, $\neg A$ is the logical negation of A , and $A \wedge B$ is the logical conjunction of A and B . Then, R is logically explosive (equivalently, logically exclusive) iff arbitrary $A \wedge \neg A$ R -entails arbitrary B .³*

The terminology reflects the fact that, according to any such entailment relation R , one cannot have both A and its logical negation $\neg A$ in a theory without having *all* sentences of the language in the R -closure of that theory; the theory 'explodes' into the trivial one for the language in the presence of contradiction. This, in the end, is the target general fact:

Fact 1 (Exploding Theories into Triviality) *Let R be a logically explosive entailment relation. Let T be a theory closed under R . Let the language of T*

³ More generally, the definition is that $\{A, \neg A\}$ R -entails arbitrary B iff R is logically explosive/exclusive; however, for present purposes, wherein all canvassed accounts of logical conjunction have various features, the given general account is equivalent to the otherwise more limited one. We stick with the latter because it simplifies discussion of contradiction.

contain all logical vocabulary. Then, T contains a formal contradiction iff the R -closure of T is T^\perp , the trivial theory in T 's language.

2.2 Theology, Contradiction, and Triviality

Given Fact 1 (in Section 2.1), it's clear that any would-be true theology is either

- *the trivial theology* (for the given language) *or*
- contains no contradictions *or*
- is not closed under a logically explosive entailment relation.

The same applies to any would-be true theory (of anything), but christian theism is the target topic.⁴

3 Outline of Target Abstract Responses in General

For present purposes, we simplify discussion by assuming that contradictions follow, if at all, from axioms in a given theory, where *axioms* are simply core truths of the theory – not in any way necessarily ‘self-evident’ or the like. In the case of theological theories, axioms often take the form of central doctrines. (Examples are given below.)

⁴ The claim is in fact true only given our simplifying assumption that theology's predicate-entailment relation does not treat standard axioms (e.g., the humanity and divinity of Christ; the trinitarian identity of Father, Son, Spirit; etc.) as ‘explosive’ in the following sense.

Definition 14 (*R*-explosive sentence) *Let A and B be sentences in language L . Let R be a relevant entailment relation for theory T . A is R -explosive iff A R -entails B for all B in L (where ‘ A R -entails B ’ just means that A entails B according to entailment relation R).*

Definition 15 (*R*-explosive set of sentences) *Let X be a set of sentences in language L , and A any sentence in L . Let R be a relevant entailment relation for theory T . X is R -explosive iff X R -entails A for all A in L (where ‘ X R -entails A ’ just means that X entails A according to entailment relation R).*

Given all of this, the true general claim is that, given Fact 1, any would-be true theology is either

- *the trivial theology* (for the given language) *or*
- contains no contradictions *or*
- is not closed under a logically explosive entailment relation *or*
- either contains no R -explosive sentences or is not R -closed.

In the face of derived contradiction, one response is to reject one or more of the initial axioms. For our purposes – focussing on christian theological theory – we do not discuss that approach. Our aim is to discuss responses that preserve the basic axioms (i.e., preserve central doctrines, as illustrated in subsequent sections).⁵

Suppose that a contradiction is derived from given axioms of a given theory (e.g., on our focus, a theology). For present purposes, there are basically three avenues of response, each with different avenues of implementation.⁶

- *Partial Theology*. Theology is not closed under otherwise governing entailment relations (e.g., logical or the relevant predicate-entailment relations); it is *R*-open with respect to at least one salient, relevant entailment relation *R*. (Examples in subsequent sections.) Accordingly, the would-be entailment of apparent contradiction need be no threat to the given theory so long as the theory omits relevant entailments or consequences that deliver the contradiction.
- *Robust Theology (1)*. *Closed under standard logic and non-standard predicate entailment*. Theology is closed under both (theological) predicate entailment and logical entailment, where logical entailment is (logically) explosive but theological predicate entailment, contrary to the standard relation, does not deliver the given contradiction. Hence, the apparent contradiction is merely apparent.
- *Robust Theology (2)*. *Closed under non-standard logic and standard predicate entailment*. Theology is closed under both (theological) predicate entailment and logical entailment, where theological predicate entailment delivers the given contradiction but logical entailment is *not* (logically) explosive. Hence, the principal threat from the theory's contradiction(s) is merely apparent.

Each of the avenues of response enjoys different avenues of implementation. For present purposes, at most one route towards implementing given responses is canvassed, letting that route be a representative (though, to repeat, the given representatives neither exhaust the options nor necessarily realize historically occupied responses).

⁵ Rejecting the axioms is tantamount to rejecting the given theory/theology. This is what so-called theological heresies do: reject the standard theory by rejecting one or more axioms. Our aim in this Element is only to cover options for retention of the axioms as far as possible.

⁶ There are other combinatorial options but we focus only on the following three salient ones.

TWO PRINCIPAL EXAMPLES

4 One Central Example: Incarnation

In what follows, the term ‘axiom’ is not intended to convey any epistemic status (e.g., ‘self-evident’ or ‘obvious’ or what have you); the term is used simply to flag that, at least in the standard christian theology (e.g., at least up through the 451 CE Council of Chalcedon), these claims are taken to be fundamental – even partly definitive – of the target phenomenon. Epistemic grounds for such claims is a different issue, one on which this discussion remains neutral (beyond noting that the grounds usually involve revelation as recorded in christian scriptural records and in the catholic christian church).

1. Christ is divine.
Source: theological axiom.
2. Christ is human.
Source: theological axiom.
3. Christ is omniscient.
Source: from (1) by theology’s *predicate-entailment* relation.
4. Christ is non-omniscient.
Source: from (2) by theology’s *predicate-entailment* relation.⁷
5. It’s false that Christ is omniscient.
Source: (4) by theology’s *predicate-entailment* relation.
6. Christ is omniscient and it’s false that Christ is omniscient.
Source: from (3)–(5) by *logical* entailment.

5 Partial Theology: Responses to the Incarnation

For simplicity, focus exclusively on the apparent contradiction in Section 4. One family of responses to the apparent contradiction is to pursue a true theology but not the full truth. There are other responses (viz., so-called QUA or ‘reduplicative’ responses) that try to retain the axioms by claiming either that they’re equivocal or that they are implicitly other than what is explicit – not just ‘Christ is ignorant’ but rather ‘Christ-qua-human is ignorant’ or etc. (Cross, 2011; Senor, 2002). We do not discuss such approaches; rather, we discuss

⁷ For any who think that *being human* does not entail *being ignorant* (i.e., ignorant of at least some things), the claim is also standardly supported via scriptural revelation in christian theology (e.g., Mark 13:32). For ease of exposition, claim (5) in the given derivation is taken to follow from claim (2) via standard meanings of ‘human’ and ‘non-omniscient’. (Some might say that the meaning of ‘human’ doesn’t entail – necessitate – non-omniscience; however, the standard christian tradition is in tension with the genuine possibility that humans are completely on par with the omniscient God, and this is one of many reasons that the target apparent contradiction involved in the incarnation doctrine is so common.)

just those options that keep the axioms as they are.⁸ Of course, inasmuch as, according to the standard christian theology, divine reality remains mysterious in some respects to any but divine beings, ‘the full truth’ in this context just means *as full as possible*. That the full truth is beyond the capacities of non-divine theorists is entirely compatible with a theology that records the full truth as far as possible within the bounds of possibility open to non-divine beings.

Partial theology, per Definition 7, is simply theology that aims only at *R*-open theories, theologies – theories of the divine – that are not closed under one or more of the otherwise salient ‘governing’ entailment relations *R*. For present purposes, there are two salient entailment relations involved in the target contradiction (see Section 4): logical entailment and the standard predicate-entailment relation, where the latter validates the entailments marked ‘predicate entailment’ in the guiding example. A relevant open theology is closed under at most one of the two said entailment relations.

Open theology, as herein discussed, comes in three basic approaches, depending on the entailment relations in play. The following three approaches do not exhaust the combinatorial possibilities; they rather represent the options that are most natural were one to pursue partial theology in response to contradiction. (If one’s theology were closed under a non-standard account of logical entailment, especially one, as in Section 6.3, that accommodates contradictions, the motivation for partial theology may be diminished.)

5.1 Closed Under No Entailment At All

This is just a set of claims without their respective consequences (or, at least, without all of the consequences). In particular, a natural implementation of the *closed-under-none* approach takes the theological theory to contain *just* the axioms (1) and (2) of Section 4 and none of the other claims that *otherwise* follow from (1) and (2) by the salient entailment relations otherwise governing the full truth of Christ.

5.1.1 Historical Examples

No historical examples of such an explicitly *closed-under-none* approach are known. Coakley (2002) argues that the relevant ecumenical councils – especially up through Chalcedon 451 – lay down apparent contradiction (or ‘paradox’) without explicitly cashing out the consequences; however, it is

⁸ Of course, some of the given responses that we do not discuss are framed as uncovering the implicit forms of the ‘axioms as they are’, but we treat such responses more as alternative semantics (especially of singular terms but perhaps also predicates or even the exemplification relation) rather than attending to salient entailment relations.