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PARACONSISTENCY IN MATHEMATICS

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Paraconsistency in Mathematics

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Abstract: Paraconsistent logic makes it possible to study inconsistent theories in a coherent way. From its modern start in the mid-twentieth century, paraconsistency was intended for use in mathematics, providing a rigorous framework for describing abstract objects and structures where some contradictions are allowed, without collapse into incoherence. Over the past decades, this initiative has evolved into an area of nonclassical mathematics known as inconsistent or paraconsistent mathematics. This Element provides a selective introductory survey of this research program, distinguishing between "moderate" and "radical" approaches. The emphasis is on philosophical issues and future challenges.

Keywords: nonclassical mathematics, inconsistent mathematics, paraconsistent logic, paraconsistent mathematics, nonclassical logic

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