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MATHEMATICS AND METAPHILOSOPHY

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Mathematics and Metaphilosophy

Elements in the Philosophy of Mathematics

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Abstract: This Element discusses the problem of mathematical knowledge and its broader philosophical ramifications. It argues that the challenge to explain the (defeasible) justification of our mathematical beliefs ("the justificatory challenge") arises insofar as disagreement over axioms bottoms out in disagreement over intuitions. And it argues that the challenge to explain their reliability ("the reliability challenge") arises to the extent that we could have easily had different beliefs. The Element shows that mathematical facts are not, in general, empirically accessible, contra Quine, and that they cannot be dispensed with, contra Field. However, it argues that they might be so plentiful that our knowledge of them is unmysterious. The Element concludes with a complementary "pluralism" about modality, logic, and normative theory, highlighting its revisionary implications. Metaphysically, pluralism engenders a kind of perspectivalism and indeterminacy. Methodologically, it vindicates Carnap's pragmatism, transposed to the key of realism.

Keywords: philosophy of mathematics, knowledge, realism, objectivity, pluralism

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