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CLASSICAL FIRST-ORDER LOGIC

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Classical First-Order Logic

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Abstract: One is often said to be reasoning well when one is reasoning logically. Many attempts to say what logical reasoning is have been proposed, but one commonly proposed system is classical first-order logic. This Element will examine the basics of classical first-order logic and discuss some surrounding philosophical issues. The first half of the Element develops a language for the system, as well as a proof theory and model theory. The authors provide theorems about the system they developed, such as unique readability and the Lindenbaum lemma. They also discuss the meta-theory for the system, and provide several results there, including sketching a proof of the soundness and completeness theorems. The second half of the Element compares classical first-order logic to other systems: classical higher-order logic, intuitionistic logic, and several paraconsistent logics which reject the law of *ex falso quodlibet*.

Keywords: classical logic, first-order logic, model theory, proof theory, meta-theory higher-order logic, intuitionistic logic, paraconsistent logic

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