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GÖDEL'S INCOMPLETENESS THEOREMS

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Gödel's Incompleteness Theorems

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Abstract: This Element takes a deep dive into Gödel's 1931 paper giving the first presentation of the Incompleteness Theorems, opening up completely passages in it that might possibly puzzle the student, such as the mysterious footnote 48a. It considers the main ingredients of Gödel's proof – arithmetization, strong representability, and the Fixed Point Theorem – in a layered fashion, returning to their various aspects – semantic, syntactic, computational, philosophical, and mathematical – as the topic arises. It samples some of the most important proofs of the Incompleteness Theorems, for example, due to Kuratowski, Smullyan, and Robinson, as well as newer proofs, also of other independent statements, due to Friedman, Weiermann, and Paris-Harrington. It examines the question whether the incompleteness of, for example, Peano Arithmetic immediately gives the undecidability of the *Entscheidungsproblem*, as Kripke has recently argued. It considers set-theoretical incompleteness, and finally considers some of the philosophical consequences considered in the literature.

Keywords: Gödel, First Incompleteness Theorem, Second Incompleteness Theorem, Fixed Point Theorem, Diagonalization

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