

Cambridge Elements

Elements in Philosophy and Logic

edited by

Bradley Armour-Garb

SUNY Albany

Frederick Kroon

The University of Auckland

SET THEORY

John P. Burgess

Princeton University



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-108-98691-5 — Set Theory
John P. Burgess
Frontmatter
[More Information](#)

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre,
New Delhi – 110025, India
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108986915
DOI: 10.1017/9781108981828

© John P. Burgess 2022

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2022

A catalogue record for this publication is available from the British Library.

ISBN 978-1-108-98691-5 Paperback
ISSN 2516-418X (online)
ISSN 2516-4171 (print)

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Set Theory

Elements in Philosophy and Logic

DOI: 10.1017/9781108981828
First published online: January 2022

John P. Burgess
Princeton University

Author for correspondence: John P. Burgess, jburgess@princeton.edu

Abstract: Set theory is a branch of mathematics with a special subject matter, the infinite, but also a general framework for all modern mathematics, whose notions figure in every branch, pure and applied. This Element will offer a concise introduction, treating the origins of the subject, the basic notion of set, the axioms of set theory and immediate consequences, the set-theoretic reconstruction of mathematics, and the theory of the infinite, touching also on selected topics from higher set theory, controversial axioms and undecided questions, and philosophical issues raised by technical developments.

Keywords: sets, infinity, continuum, cardinals, ordinals

© John P. Burgess 2022

ISBNs: 9781108986915 (PB), 9781108981828 (OC)
ISSNs: 2516-418X (online), 2516-4171 (print)

Contents

1	Historical Roots	1
2	The Notion of Set	4
3	The Zermelo–Fraenkel Axioms	9
4	Immediate Consequences	14
5	Number Systems Within Set Theory	22
6	Infinities	29
7	The Axiom of Choice	38
8	Topics in Higher Set Theory	41
9	Metamathematics of Set Theory	49
10	Large Cardinals and Determinacy	61
11	Concluding Philosophical Remarks	66
	References	69