

Cambridge Elements

Elements in Philosophy and Logic

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HIGHER-ORDER LOGIC AND TYPE THEORY

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CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-108-98690-8 — Higher-Order Logic and Type Theory
John L. Bell
Frontmatter
[More Information](#)

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre,
New Delhi – 110025, India
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108986908
DOI: 10.1017/9781108981804

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First published 2022

A catalogue record for this publication is available from the British Library.

ISBN 978-1-108-98690-8 Paperback
ISSN 2516-418X (online)
ISSN 2516-4171 (print)

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Higher-Order Logic and Type Theory

Elements in Philosophy and Logic

DOI: 10.1017/9781108981804
First published online: March 2022

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Abstract: This Element is an exposition of second- and higher-order logic and type theory. It begins with a presentation of the syntax and semantics of classical second-order logic, pointing up the contrasts with first-order logic. This leads to a discussion of higher-order logic based on the concept of a type. Section 2 contains an account of the origins and nature of type theory and its relationship to set theory. Section 3 introduces local set theory (also known as 'higher-order intuitionistic logic'), an important form of type theory based on intuitionistic logic. In Section 4, a number of contemporary forms of type theory are described, all of which are based on the so-called 'doctrine of propositions as types'. The Element concludes with an Appendix in which the semantics for local set theory – based on category theory – is outlined.

Keywords: logic, type, proposition, set, constructive

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ISBNs: 9781108986908 (PB), 9781108981804 (OC)
ISSNs: 2516-418X (online), 2516-4171 (print)

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