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Joseph T. C. Liu
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1 Introduction

The motion of a fluid containing small solid particles has been of interest in many scientific and engineering disciplines since the turn of the previous century. In a paper on sand ripples in the desert, Theodore von Kármán (1947a) gave an invitation and pointed out the challenges to workers in fluid mechanics in elucidating problems in this field. Recent issues of aerospace engineering interest concern the aerodynamics of dusty planetary atmospheres. Early work recognized the presence of micron-sized dust particles in the atmospheres of Mars and Venus (Öpik 1962). More recent work on Martian atmospheric dust properties is presented in Appendix B; like the present continuum considerations of gas-particle flow, these are of current importance in planning human exploration of Mars (Levine and Winterhalter 2017).

Reviews of earlier, largely empirical works (Dalla Valle 1948; Hermans 1953; Othmer 1956; Torobin and Gauvin 1959) on such problems as atomization of liquids, fluidization, powder beds, and smoke and raindrop impingement and icing on airfoils (Serafini 1954; Gelder et al. 1956; Lewis and Brun 1956) are mainly concerned with trajectories of single droplets in an undisturbed aerodynamic flow field. The subjects subsumed by "heterogeneous flows" are sufficiently numerous that many books on the subjects have emerged – for example, Soo (1967), Rudinger (1980), Fan and Zhu (1998), Jackson (2000), Brenan (2005), Guazzelli and Morris (2012), and Michaelides (2014) to mention just a few.

The aeronautical and aerospace engineering interests in gas-particle flows stem from combustion instabilities in solid propellant rocket motors (Culick and Yang 1992) and their alleviation via embedded micron-sized aluminum oxide particles that damp out acoustical instabilities. Here, a host of gas-particle flow problems of aeronautical interest arise. Particle trajectories in a cascade flow were studied by Tabakoff and Hussein (1971) in a wind tunnel. The review by Hoglund (1962) of gas-particle flow in converging-diverging nozzles was largely taken up with tedious numerical procedures. Analytical descriptions of the nozzle problem were given by Rannie (1962) and Marble (1963). The normal shock wave in a gas containing small solid particles was first studied by Carrier (1958) and by Marble (1962), and it was also described in Rudinger (1964, 1980), Marble (1970), and Brenan (2005) and in reviews (e.g., Ingra and Ben-Dor 1988). The oblique shock wave on a wedge was studied by Miura and Glass (1986). Particle trajectories in a Prandtl–Meyer expansion gas flow were studied by Marble (1962) and in a gas-particle flow by Miura and Glass (1988). Work on weak waves over slender wedges was studied by Liu (1964) from a general small-disturbance theory point of view, and by Miura (1974). It seems that the relaxation wave equation was overlooked in Miura (1974).

The hydrodynamic stability of parallel laminar gas-particle flow was studied by Saffman (1962) and Michael (1964, 1965). Liu (1965) studied how the Stokes layer near the wall, which contributes to the Reynolds stress, is subject to frequency-dependent modification in gas-particle flow.

The discussion in the present work is necessarily modest and covers only a small segment of the vast subject of gas-particle flows. We deal with small solid particles in a gaseous medium, a "dusty gas," in which the small solid particles occupy negligible volume in a unit volume of the mixture. The solid particulate material density is ρ_S and the particle density per unit volume of the mixture is $\rho_P = \rho_S n_P$. Our range of interest is $\kappa = \rho_P/\rho = \vartheta(1)$, where the particulate phase momentum and thermal lag have an impact on the gas flow. When $\kappa \to 0$, the gas flow is not affected and the central interest is the study of particle trajectories in a known flow field (Glauert 1940; Probstein and Fassio 1970).

General wave problems in fluids are presented in Whitham (1974) and in Lighthill (1978).

2 Conservation Equations of Gas-Particle Flows

The motion of a gas containing small solid particles (a dusty gas) was discussed in rather general terms by Kiely (1959); he stated the equations of motion and applied the techniques of irreversible processes for small departures from thermodynamic equilibrium to deduce the forms of the particle-fluid interaction force. However, he did not point out the dissipative mechanism resulting from the work done owing to particle-fluid momentum interaction, and this omission is perpetuated in the subsequent calculation of entropy sources. This was remedied by Chu and Parlange (1962), also from an irreversible thermodynamic point of view (Prigogine 1961), who obtained laws for momentum and thermal interaction between the two phases. As expected, these laws are linear in the velocity and temperature differences, which are the forms that Stokes's law takes. In this context, the linear interaction laws are on the same footing as the Newtonian and Fourier linear relations between fluxes of momentum and of heat and the gradients of velocity and temperature, respectively, for small departures from thermodynamic equilibrium.

The conservation equations were also discussed by Van Deemter and Van der Laan (1961) and Hinze (1962), but only for mass, momentum, and kinetic energy. The omission of thermodynamic considerations of thermal energy renders them incomplete. Then Marble discussed the general continuum conservation equations on the basis of particle distribution function (Marble 1962), and the effect of phase change (Marble 1969). In the present work, we discuss in detail inert particles with only momentum and thermal interactions between the two phases. Consider metallic solid particles for which the ratio of mass density of the solid material to that of a gas at standard conditions is estimated to be $\rho_S/\rho = \vartheta(10^3)$. Our interest is in the range in which the mass density of the solid phase $\kappa = \rho_P/\rho = n_P\rho_S/\rho = \vartheta(1)$, for which there is mutual influence between the two phases. In this case, the total volume occupied by the solid phase in a unit volume of the mixture is $\vartheta(10^{-3})$. Thus the per-unit volume of the mixture is synonymous with the per-unit volume of the gas due to the negligible volume occupied by the solid particles. Furthermore, for $\kappa = \vartheta(1)$, if the average particle radius is $r_P = 1, 10$ *micron*, then $n_P = \vartheta(10^5)$ and the interparticle distance would be about 10^{-2} *mm*. For $r_P = 10$ *micron*, these would be, respectively, $n_P = \vartheta(10^3)$ and 10^{-1} *mm*. Thus one can certainly define a macroscopic "point" the size of a fraction of a millimeter over which an average quantity of particle cloud can be suitably defined. Concurrently, the interparticle distance is sufficiently large compared to the size of the particles so as to render particle-particle interactions secondary compared to particle-fluid interactions. For particles of disparate size, "collisions" become unavoidable so that this situation is a separate consideration (Marble 1970). Here, we consider only particles of a single averaged size. The particle-fluid interactions are continuous since the mean free path of the gaseous medium is of the order of 5×10^{-5} *mm* and the ratio of the mass of a single particle to that of a gas molecule is of the order of 10^{11} and 10^{14} for $r_P = 1$ and 10 micron, respectively.

We thus consider a perfect gas coexisting with small solid particles that we assume to be spherical and sufficiently rare and non-mutually interacting. Within a macroscopic point, in general, the individual particles may have different velocities, directions, and temperatures. Owing to the large number of particles within the macroscopic point, averaged local properties such as density, velocity, and temperature are defined, and these are point functions as in ordinary gas dynamics. The conservation equations are:

Conservation of mass – In the absence of mass exchange between the two phases, the continuity equation for the gas phase is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0 \qquad (2.1),$$

and for the particle phase, it is:

$$\frac{\partial \rho_P}{\partial t}+\frac{\partial\left(\rho_P u_{Pj}\right)}{\partial x_j}=0 \qquad (2.2).$$

Conservation of momentum – The momentum equation for the gas phase takes the usual Navier–Stokes form, but is augmented by the force per unit volume exerted on the gas by the particle phase F_{Pi}:

$$\frac{\partial(\rho u_i)}{\partial t}+\frac{\partial\left(\rho u_j u_i\right)}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{\partial \tau_{ji}}{\partial x_j}+F_{Pi} \qquad (2.3).$$

The usual viscous stress tensor τ_{ji}, which is linearly related to the gas phase rate of strain tensor, refers to smooth derivatives of the gas velocities. In this case, the detailed disturbance on the gas velocity due to passage of the particles is neglected.

The particle phase momentum equation is:

$$\frac{\partial(\rho_P u_{Pi})}{\partial t}+\frac{\partial\left(\rho_P u_{Pj} u_{Pi}\right)}{\partial x_j}=-F_{Pi} \qquad (2.4),$$

which makes no contribution to its own pressure and viscous stresses according to our discussions (Marble 1970). The force per unit volume exerted on the particle phase by the gas is $-F_{Pi}$.

Conservation of energy – The energy equation for the gas phase in the form of the first law of thermodynamics for the internal energy per unit mass is:

$$\frac{\partial(\rho e)}{\partial t}+\frac{\partial\left(\rho u_j e\right)}{\partial x_j}+p\frac{\partial u_j}{\partial x_j}=-\frac{\partial q_j}{\partial x_j}+\Phi+Q_P+\Phi_P \qquad (2.5),$$

where Q_P is the rate of heat transferred per unit mass from the particle phase to the gas phase, Φ_P is the dissipative mechanism owing to the work done on the gas by the particle phase and is $(u_{Pi}-u_i)F_{Pi}$, q_i is the Fourier heat flux vector (again, it refers to smooth derivatives of the gas temperature in the Fourier assumption), and Φ is the rate of viscous dissipation. The particle phase energy equation, in terms of the particle phase internal energy per unit mass, $e_p = c_s T_P$, is:

$$\frac{\partial(\rho_p e_P)}{\partial t}+\frac{\partial\left(\rho_P u_{Pj} e_P\right)}{\partial x_j}=-Q_P \qquad (2.6).$$

The rate at which heat is transferred per unit volume from the gas to the particle phase is $(-Q_P)$.

Equation of state – The gas phase for moderate temperatures and pressures is assumed to be a perfect gas, satisfying

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$$p = \rho R T \qquad (2.7).$$

The particle phase is not constrained by an equation of state owing to its lack of volume and randomizing effects.

Particle phase-gas phase interaction force and heat transfer – Irreversible thermodynamic considerations (Prigogine 1961; Chu and Parlange 1962) provide the interactions in the form of linear differences between the velocities and temperatures for momentum and heat transfer, respectively, between the phases. The "coefficients" of such differences are a separate matter, much like the viscosity and thermal conductivity in the linear relation between stress and rates of strain and between the heat flux and the temperature gradient. A way of evaluating the "coefficients" is Stokes's law, which has already yielded the linear differences.

Since the particle phase occupies negligible volume, the force exerted on the particle phase due to the pressure gradient in the gas, as well as that due to virtual mass, is neglected. The interaction force per unit volume for the particle phase-gas phase interaction for a linear relation in the relative velocity is:

$$F_{Pi} = \frac{\rho_P}{\tau_V}(u_{Pi} - u_i) \qquad (2.8).$$

If we assume that Stokes's law holds, then the relation gives the velocity relaxation time as

$$\tau_V = m_P/(6\pi\mu r_P) \qquad (2.9).$$

The particles are assumed noninteracting in the unit volume of the mixture, as discussed earlier in this Element. Similarly, the heat transferred to the particle phase is represented as a linear dependence on the relative temperature difference between the two phases:

$$Q_P = \frac{\rho_P}{\tau_T} c_S (T_P - T) \qquad (2.10),$$

where the thermal relaxation time for particles obeying Stokes's law is:

$$\tau_T = \frac{m_P c_S}{4\pi r_P k} \qquad (2.11).$$

For metallic particles, the two relaxation times are of the same order.

It has been tacitly assumed, since the interparticle distance in our range of interest is much larger than the particle size, that the interaction between the particle phase in a unit volume of mixture is equal to the number of particles in that volume times the corresponding effect of a single particle.

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Rubinow and Keller (1961) showed that the transverse force on a sphere in shear flow is solely due to its spin. However, when a sphere originally lacks spin, it continues to lack it thereafter. This is assumed to be the case, and the transverse force is not included in our considerations.

2.1 Incompressible Gas Flow

The reduction of the gas-phase conservation equations to an incompressible form generally follows from Lagerstrom (1996), where flow quantities undergo a double expansion for small-temperature loading and small Mach numbers. The former expansion renders negligible the variation in gas and transport properties; the latter expansion makes the rate of viscous dissipation and the work done by the pressure gradients unimportant.

For gas-particle flows, additionally the relative temperature differences between the phases and the Mach number based on the relative velocities are small. The first restriction is to keep the gas density and transport properties relatively constant; the second is to render negligible the heat source in the gas due to work done on the gas by the force exerted by the particle phase. Since the particle phase is not constrained by an equation of state, such as those for an ideal gas for the gas phase, the particle density per unit volume of the mixture, ρ_P, is governed solely by mass conservation considerations and the particle-phase continuity equation does not have an incompressible form.

For incompressible gas-particle flows, then,

$$\begin{aligned}&\frac{\partial u_j}{\partial x_j}=0\\&\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho_0}\frac{\partial p}{\partial x_i}+\nu_0\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\frac{\rho_P}{\rho_0}\frac{1}{\tau_{V0}}(u_{Pi}-u_i)\end{aligned}\qquad(2.12),$$

where the Navier–Stokes relation for the viscous stress tensor has been used and the subscript 0 denotes free-stream conditions, $\tau_{V0}=m_P/6\pi\mu_0 r_P$.

The corresponding particle phase continuity and momentum equations become:

$$\begin{aligned}&\frac{\partial\rho_P}{\partial t}+\frac{\partial(\rho_P u_{Pj})}{\partial x_j}=0\\&\frac{\partial(\rho_P u_{Pi})}{\partial t}+\frac{\partial(\rho_P u_{Pj}u_{Pi})}{\partial x_j}=-\frac{\rho_P}{\tau_{V0}}(u_{Pi}-u_i)\end{aligned}\qquad(2.13).$$

The "incompressible" form of the gas-particle flow equations forms the basis of boundary-layer studies (Saffman 1962; Marble 1962, 1970; Michael 1964; Liu 1966, 1967). Viscous boundary layer problems are essentially "transverse

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wave" problems in which the diffusion process relaxes from frozen diffusion via $\nu = \mu/\rho$, to equilibrium diffusion via $\nu_e = \mu/(1+\kappa)\rho$.

The thermal problems are uncoupled from the above, in which the momentum problem is considered solved and used as input in the thermodynamic problem. The energy equations become, for the heat transfer problem,

$$\begin{aligned}
&\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \mathrm{K}_0 \frac{\partial^2 T}{\partial x_j^2} + \frac{\rho_P c_S}{\rho_0 c_P} \frac{1}{\tau_{T0}} (T_P - T) \\
&\frac{\partial T_P}{\partial t} + u_j \frac{\partial T_P}{\partial x_j} = -\rho_P c_S \frac{1}{\tau_{T0}} (T_P - T)
\end{aligned} \qquad (2.14),$$

where $\tau_{T0} = m_P c_S / 4\pi k_0 r_P$ and the thermal diffusivity is $K_0 = k_0/\rho_0 c_P$. For the insulated wall problem or the thermometer problem, dissipative effects would be included.

2.2 Inviscid Compressible Flows

In gas-particle flow fields of aeronautical interest, those resulting from the obstacles in the stream, the classification of certain regions in the flow field is similar to that in gas dynamics for gases with negligible friction, since only transport effects of the gas are considered. In other words, shearing stresses and heat conduction are confined to thin boundary layers adjacent to solid boundaries when the flow Reynolds number, based on some macroscopic characteristic length, is large. Outside such regions, the gas-particle momentum interaction that arises due to gas viscosity is more important compared to the viscous forces acting on a solid boundary, and similarly for other transport effects such as heat conductivity and mass diffusion. Loosely speaking, then, outside certain thin boundary layer regions, we may consider the "inviscid" flow in gas-particle flows.

For the gas phase, which can be compressible, the conservation equations take the form:

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0 \\
&\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_j u_i)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + F_{P_i} \\
&\frac{\partial (\rho e)}{\partial t} + \frac{\partial (\rho u_j e)}{\partial x_j} + p \frac{\partial u_j}{\partial x_j} = Q_P + (u_{Pi} - u_i) F_{Pi} \\
&p = \rho R T
\end{aligned} \qquad (2.15).$$

In terms of the gas static enthalpy per unit mass, $h = e + p/\rho$, the more convenient energy equation for flow problems is:

$$\frac{\partial(\rho h)}{\partial t}+\frac{\partial\left(\rho u_j h\right)}{\partial x_j}=\frac{\partial p}{\partial t}+u_j\frac{\partial p}{\partial x_j}+Q_P+(u_{Pi}-u_i)F_{Pi} \qquad (2.16).$$

The corresponding conservation equation for the particle phase remains the same as (2.2), (2.4), and (2.6). The enthalpy of the particle phase is the same as its internal energy, $h_P = c_S T_P = e_P$.

The entropy (per unit mass) equations for the two phases are, following standard definitions,

$$\begin{aligned}\rho\frac{\partial s}{\partial t}+\rho u_j\frac{\partial s}{\partial x_j}&=\frac{1}{T}\left[Q_P+(u_{Pi}-u_i)F_{Pi}\right]\\ \rho_P\frac{\partial s_P}{\partial t}+\rho_P u_{Pj}\frac{\partial s_P}{\partial x_j}&=\frac{1}{T_P}\left[-Q_P\right]\end{aligned} \qquad (2.17).$$

The pioneering work of Ackeret (1925), Glauert (1928), Kármán and Moore (1932), and Kármán (1941) and the lectures of Busemann (1935), Prandtl (1935), and Kármán (1935) at the Fifth Volta Congress for High Speed Aeronautics held in 1935 in Rome essentially established the foundations of small disturbance theory in supersonic flows. One only need to consult the two subsequent general lectures of Kármán, the Tenth Wright Brothers Lectures of 1947 (Kármán 1947b), and the Fifth Guggenheim Memorial Lecture of 1958 (Kármán 1959) to gain perspective of the developments of supersonic flow. Sears (1954), among others, connected acoustics (Rayleigh [1894] 1945) with aerodynamics through the assumption that the thin body emits acoustic waves as it travels, transforming the acoustic wave equation into the aerodynamics of unsteady supersonic flow (Miles 1959). What is more, in the special case of steady flow, the Prandtl–Glauert–Ackeret equation emerges. These ideas are followed here from acoustic wave propagation into steady supersonic flow over thin bodies in gas-particle flow.

The inviscid, compressible conservation equations are given in Section 2. They are the "Euler equations" in gas-particle flows that are to be linearized, leading to the formulation of small-disturbance perturbation theory. In gas dynamics, the development of acoustical theory, in terms of small perturbations about a stationary medium (Rayleigh [1894] 1945), is unified with aerodynamics of slender or thin bodies, in small perturbations from a uniform stream, the Prandtl–Glauert perturbation equation, as reviewed by Sears (1954) concerning the historical evolution of small perturbation theory in gas dynamics. The physical implication of linearized thin airfoil theory in a compressible gas is that the flow is governed by the equations of acoustics in the frame of reference fixed on the airfoil and where the perturbation of the gas velocities is assumed to be small compared to the acoustic propagation speed. This general idea is adopted in what follows, proceeding from acoustic propagation in a gas-particle medium originally at rest and in thermal equilibrium (Chu and Parlange 1962). The Prandtl–Glauert–Ackeret form of the equation in gas-particle flow (obtained by Marble 1962, 1970) through the perturbation of a uniform stream is applied to the flow over a wavy wall (Marble 1962). As in gas dynamics (Sears 1954), the gas-particle flow equivalent of the Prandtl–Glauert equation is obtained in the special case when the situation is steady in the moving frame of reference. This alternate derivation essentially connects the acoustics and aerodynamic concepts of small perturbation theory in gas-particle flows.

In the process of obtaining the small perturbation theory, (a) the role of entropy production, if any and, (b) the possibility of expressing the perturbation velocities of both gas and particle phases in terms of the gradients of their respective velocity potentials, both arise. The latter was apparently overlooked in Chu and Parlange (1962).

3 Small Perturbation Equations in a Stationary Frame

Consider perturbations about a gas-particle medium at rest and in thermodynamic equilibrium. Denote the stationary coordinate system by $(\widetilde{x}_j, \widetilde{t})$; the undisturbed gas and particle phase quantities are at rest and are uniform, with subscript 0. The small perturbation expansion, about the stationary constant state,

$$\begin{array}{c} p = p_0 + p',\ \rho = \rho_0 + \rho',\ T = T_0 + T',\ u_i = 0 + u_i' \\ \rho_P = \rho_{P0} + \rho_P',\ T_P = T_0 + T_P',\ u_{Pi} = 0 + u_{Pi}' \end{array} \tag{3.1},$$

then yields, for the gas phase conservation equations for continuity, momentum, and state from (2.5), for energy from (2.6) (after dropping the primes for convenience), and in stationary coordinates,

$$\begin{aligned} &\frac{\partial \rho}{\partial \widetilde{t}} + \rho_0 \frac{\partial u_j}{\partial \widetilde{x}_j} = 0 \\ &\rho_0 \frac{\partial u_i}{\partial \widetilde{t}} = -\frac{\partial P}{\partial \widetilde{x}_i} + \frac{\rho_{P0}}{\tau_{V0}} (u_{Pi} - u_i) \\ &\rho_0 c_P \frac{\partial T}{\partial \widetilde{t}} = \frac{\partial P}{\partial \widetilde{t}} + \frac{\rho_{P0}}{\tau_{T0}} c_S (T_P - T) \\ &\frac{p}{p_0} = \frac{\rho}{\rho_0} + \frac{T}{T_0} \end{aligned} \tag{3.2}.$$

The heat capacity of the gas is taken as a constant averaged quantity. Substituting the perturbation expansion into the particle conservation equations

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(2.2), (2.5), and (2.6) leads to the following first-order perturbation equations in stationary coordinates:

$$\begin{aligned}
&\frac{\partial \rho_P}{\partial \widetilde{t}} + \rho_{P0}\frac{\partial u_{Pj}}{\partial \widetilde{x}_j} = 0 \\
&\rho_{P0}\frac{\partial u_{Pi}}{\partial \widetilde{t}} = \frac{\rho_{P0}}{\tau_{V0}}(u_{Pi} - u_i) \\
&\rho_{P0}c_S\frac{\partial T_P}{\partial \widetilde{t}} = -\frac{\rho_{P0}}{\tau_{T0}}c_S(T_P - T)
\end{aligned} \qquad (3.3).$$

3.1 The Role of Entropy

It is particularly important to discuss the mechanism of net entropy production, if any, in terms of small perturbation theory. There are in general two sources for the net production of entropy when the particle and gas phases are not in momentum and thermal equilibrium. One is the work done on the gas by velocity slip between the two phases and is the second term on the right side of the gaseous entropy equation from (2.17):

$$(u_{Pi} - u_i)F_{Pi}/T > 0,$$

which is a positive definite quantity. This dissipative work enters in the second order only: it is absent from first-order small perturbation theory.

The second entropy production mechanism is due to the inter-phase heat transfer Q_P. It occurs on the right sides of both the gaseous and particle phase entropy equations in (2.17). Normally heat transfer to the gas is received by the gas at its local temperature T and the local increase or decrease of gaseous entropy per unit volume is Q_P/T. Similarly, the local increase or decrease of particle phase entropy per unit volume is $-Q_P/T_P$, and the transfer of heat to and from the particle phase occurs at its temperature, T_P. To fix ideas, suppose that locally, $T > T_P$, and heat is locally transferred from the gas to the particle phase. In other words, the transfer of heat from the gaseous phase takes place at T and is received by the particle phase at a lower temperature, T_P. In this process, net entropy is produced. But for first-order small perturbation theory, the heat transfer takes place at the free-stream temperature, T_0, and thus the mechanism for net entropy production is absent from the first-order theory.

3.2 Velocity Potentials

We proceed again from the situation when the gas and particle phases are initially at rest and in thermodynamic equilibrium – that is, from the acoustical situation. It is now convenient to use vector rather than index notation. The