Gödel, Tarski and the Lure of Natural Language

Is mathematics "entangled" with its various formalisations? Or are the central concepts of mathematics largely insensitive to formalisation, or "formalism free"? What is the semantic point of view and how is it implemented in foundational practice? Does a given semantic framework always have an implicit syntax? Inspired by what she calls the "natural language moves" of Gödel and Tarski, Juliette Kennedy considers what roles the concepts of "entanglement" and "formalism freeness" play in a range of logical settings, from computability and set theory to model theory and second order logic to logicality, developing an entirely original philosophy of mathematics along the way. The treatment is historically, logically and set-theoretically rich, and topics such as naturalism and foundations receive their due, but now with a new twist.

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Logical Entanglement, Formalism Freeness

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For Jouko

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Preface

This book is about *formalism freeness*, an idea stemming from Gödel's 1946 Princeton Bicentennial Lecture, or more precisely from the phrase "formalism independent" which appears in the opening paragraph of that lecture. The phrase "formalism independent" fell on fertile soil: The Helsinki Logic Group, my academic home for the last 20 years, is devoted to the semantic method in all its guises, whether it be the method of Ehrenfeucht–Fraïssé games, or Abstract Elementary Classes or other "logic-free" concepts, even as it is at the same time devoted to logic in its syntactic aspects. Secondly, the concept of "indifferentism", appearing in John Burgess's writings, solidified my interest in the constellation of ideas around formalism independence, formalism freeness, logic freeness, the pure semantic method and logics without syntax – ideas that pull away from foundationalism in ways that seemed puzzling to me at the time.

Gödel asks in his 1946 Lecture for notions of provability and definability that are formalism independent in the way he understands computability to be, given Turing's analysis of the intuitive notion: "human effective calculability following a fixed routine." Formalism independence in the case of computability involved confluence in the sense of transcendence with respect to a given class of formal systems. How to obtain confluence for definability? The following implementation suggests itself: given a notion of definability such as constructibility or on the other hand hereditary ordinal definability, the two notions Gödel considers in the lecture, why not vary the underlying logic with respect to these definability notions? Precisely: if, say, the constructible hierarchy is built over first order logic, then one can consider properties of the inner models (for set theory) one obtains from the constructible hierarchy, when built over other logics extending first order logic. In 2009 it was my great fortune to begin working with Menachem Magidor and Jouko Väänänen on exploring these inner models. This led to the paper [128], in which it was shown that

new inner models can be obtained in this way, which are generically absolute and which contain certain large cardinals, among other results. This work is ongoing. Most of Section 4.4 is excerpted from [128].

My 2013 [124] explored the philosophical and historical aspects of the implementation offered in [128] as well as of what would eventually be called formalism freeness generally, a concept which was intended to broaden Gödel's notion of formalism independence. This book both builds on and greatly expands [124], albeit with a somewhat more conservative view of the model theorist's break with syntax in the background, than is evinced in [124]. In particular we distinguish the first order case from the case of strong logics, i.e. logics extending first order logic, which play a central role in this book. In the so-called Abstract Elementary Class context the break with syntax is definitive, but even in that case one can argue for the presence of an implicit logic. Passages in Sections 2.0, 2.1 and 4.2 are excerpted from [124].

Computability is the primary example of a formalism independent concept for Gödel. The book's Chapter 3, anthologising [123], explores the evolution of the concept of computability in the 1930s with an emphasis on Gödel's evolving views. I thank Alisa Bokulich and Juliet Floyd, the editors of the volume *Philosophical Explorations of the Legacy of Alan Turing,* for their permission to anthologise [123] here.

A word about methodology. We are inspired here by the *encyclopaedic novel*. And just as Herman Melville makes free to halt the narrative of *Moby Dick* for the sake of giving extended disquisitions on its various topical aspects, so we will often take licence to lapse into the informational mode in this book – not by inserting long treatises on ship-building, or on the biology of whales, but by enumerating, for example, the various semantic characterisations of first order logic; or by giving a slew of equivalents of weakly compact cardinals.

This book, long in the writing, owes a debt to many. Conversations essential to me during the writing of my 2013 [124] were had with John Baldwin, who recognised the interest of formalism freeness early on and then ran with it in his own book. Throughout the writing of this book Andres Villaveces and Norma Claudia Yunez provided essential input and perspective. I first spoke about formalism freeness in an ASL special session devoted to Gödel on the occasion of Gödel's centenary, to which session I was invited by Scott Weinstein. In addition to conversations over the years Scott read the manuscript carefully in fall of 2019 and made deep and interesting comments, also as to suggestions for work going forward. Andy Arana, whose clarity and creativity as a philosopher I have often benefitted from over the years, opened up the discussion with a number of observations, some of which made their way into this

book. In the last year or so of writing, John Baldwin, Patricia Blanchette, John Burgess, Walter Dean, the late Mic Detlefsen, Sebastian Gandon, Tapani Hyttinen, Sandra Laugier, Penelope Maddy, Ofra Magidor, Maryanthe Malliaris, Juliet Mitchell, Gil Sagi, Zeynep Soysal, Silvia de Toffoli, Boban Velickovic, Dag Westerståhl and Mark Wilson encouraged, listened and in some cases contributed to the final shaping of the ideas. For pressing me on various weak points I am indebted to Anand Pillay for a long conversation we had one rainy night in New York City's financial district. In connection with the writing of the computability paper anthologised here I am very grateful to Wilfried Sieg, who made important comments and corrections to an early draft, and to Juliet Floyd for helpful comments, for her support of this project, and for essential conversations and friendship over the years. I would also like to express my gratitude to my editor Hilary Gaskin, for her Job-like patience, also to the friends and family who have supported me throughout the writing of this book, of which I mention Robert Disch, Kathrin Hilten, Roope Rissanen and Amy Sandback.

I am privileged to be surrounded by an exceptional community of logicians and set theorists. For memorable conversations and correspondence about the foundations of mathematics, stretching over decades and sometimes under extreme weather conditions, I would especially like to thank Joan Bagaria, Mirna Džamonja, Wilfrid Hodges, Saharon Shelah, John Steel, Boban Velickovic, Philip Welch and Hugh Woodin.

Much of this book was finalised in the fall of 2019 when I was a visitor at the Institute for Advanced Study in Princeton. I wish to take this opportunity to express my gratitude to Peter Goddard of the IAS for his support of my visits to the IAS over the years, and also to the School of Historical Studies at the IAS for providing such a supportive and warm scholarly environment during the years 2011–12 and subsequently. I am also grateful to the librarians managing the Gödel archive at the IAS, especially Marcia Tucker, for their help in archival matters and also for granting permission to quote from Gödel's unpublished Max Phil notebooks.

This book was finished while I was a guest of the Hebrew University in the late fall of 2019. The logical and set-theoretical atmosphere of the seminars at the Hebrew University is like that of no other, and it has been a great privilege for me to take part in them through numerous visits in recent years.

As one of many who have been the object of his generosity, both intellectual and personal, I would like to express my deepest gratitude to Menachem Magidor. It would offend Menachem's modesty to express my admiration for him in the terms I would like to do here. Suffice it to say that for me he personifies everything that is good – both in academic life and outside of it, as a human being.

Finally I would like to thank the dedicatee of this book, my husband, Jouko Väänänen. His vast knowledge of logic and set theory, his logical morality – the "border-crossing point of view" as I called it in a paper for his 60th birthday volume – and finally his sense for the depths in foundational practice, has profoundly shaped my own thinking. Our conversations about strong logics in particular were especially important for the development of the ideas of this book, and indeed, more than any of the few whose work and whose conversations with me are cited here, it is his voice which is heard in these pages – and how could it be otherwise? Such is love.