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978-1-108-93789-4 — Cambridge Connection Mathematics Level 8 Student's Book with AR APP, Poster, and CLP
Edited in consultation with Shaibal Mitra , Prakash Tiwari , Ishita Joshi
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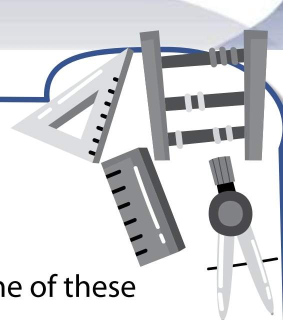
1 Rational Numbers

Warm-up



1. Which of the following is equivalent to $\frac{2}{5}$?
   a. $\frac{5}{2}$ b. $\frac{6}{15}$ c. $\frac{4}{5}$ d. None of these
2. We can find .... numbers between two rational numbers.
   a. one b. two c. three d. infinite
3. Rational numbers are not closed under ....
   a. addition b. multiplication c. subtraction d. division
4. Multiplicative inverse of –7 is ....
   a. $-\frac{1}{7}$ b. $\frac{1}{7}$ c. 1 d. 0
5. Additive inverse of $\left(\frac{2}{3} \times \frac{3}{7}\right)$ is ....
   a. $\frac{7}{2}$ b. $-\left(\frac{3}{2} \times \frac{3}{7}\right)$ c. $-\frac{2}{7}$ d. None of these

Introduction to Rational Numbers

The group of positive numbers which are countable are known as **natural numbers**, for example, 1, 2, 3, 4, … etc. In your previous classes, you must have come across simple equations like $x + 7 = 12$.

On solving this equation we get the solution $x = 5$, which is a natural number.

The group of natural numbers including 0 are known as **whole numbers**, for example, 0, 1, 2, 3, … etc. On solving the equation $x + 9 = 9$, we get the solution $x = 0$, which is a whole number.

The set of positive and negative numbers along with zero are known as **integers**. For example, –3, –2, –1, 0, 1, 2, 3, … etc.

On solving the equation like $7x + 9 = 0$, we get the solution $x = -\frac{9}{7}$, which is neither a natural number nor a whole number, nor an integer.

This leads us to the collection of **rational numbers**.

Learning Objectives

We shall learn about:

- *rational numbers*
- *equivalent rational numbers and their standard form*
- *properties of rational numbers*
- *representation of rational numbers on a number line*
- *different methods of finding a rational number between two rational numbers*

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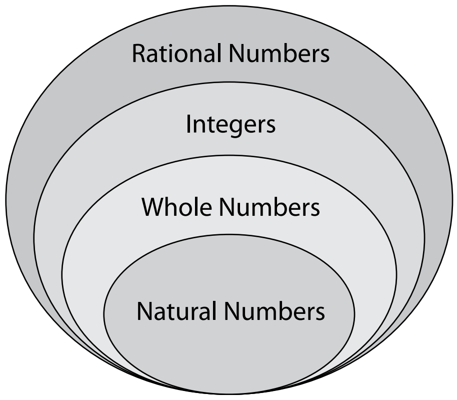
Thus, any number which can be expressed in the form of $\frac{p}{q}$, where p and q are integers, and $q \neq 0$ is called a rational number.

So, we can say that all fractions are rational numbers, but all rational numbers are not fractions.

In your previous classes you have learnt about fractions. Fraction is a part of whole, thus, a fraction can't be negative. For example, fractions $\frac{1}{2}, \frac{4}{9}, \frac{6}{5}$ are all rational numbers whereas, $\frac{-1}{3}, \frac{-2}{7}, \frac{-7}{9}$ are rational numbers but not fractions.



All terminating or recurring decimals (1.3, $2.3\overline{5}$, 9.76), natural numbers and integers are rational numbers.

Explain Your Thinking

Rahul says that if $\frac{a}{b}$ is fraction, then a and b can be integers as well as whole numbers. Is he right or wrong? Justify.

Equivalent Rational Numbers

A rational number obtained by multiplying or dividing both numerator and denominator of a rational number by the same, non-zero integer, is said to be equivalent form of the given rational number. Equivalent rational numbers have the same value, even though they may look different.

For example: $\frac{-1}{2} = \frac{-2}{4} = \frac{-4}{8} = \frac{-8}{16}$.

Explain Your Thinking

Do you think $\frac{4}{8} = \frac{2}{1}$? If no why?

Properties of Rational Numbers

1. Closure Property

Operations	**Numbers**	**Remarks**
Addition	$\frac{-7}{3} + \frac{2}{3} = \frac{-5}{3}$, is a rational number. $\frac{-9}{2} + \left(\frac{-5}{2}\right) = -7$, is also a rational number. In general, for any two rational numbers: a and b; $a + b$ is a rational number.	Thus, rational numbers are closed under addition.
Subtraction	$\frac{9}{5} - \frac{2}{5} = \frac{7}{5}$, is a rational number. $\frac{-9}{5} - \frac{2}{5} = \frac{-11}{5}$, is also a rational number. In general, for any two rational numbers: a and b; $a - b$ is also a rational number.	Thus, rational numbers are closed under subtraction.

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Multiplication	$\frac{6}{7} \times 5 = \frac{30}{7}$, is a rational number. Similarly, $\frac{-6}{13} \times (-5) = \frac{30}{13}$, is also a rational number. In general, for any two rational numbers: a and b; $a \times b$ is also a rational number.	Thus, rational numbers are closed under multiplication.
Division	$\frac{7}{3} \div \frac{2}{3} = \frac{7}{2}$, is a rational number. $\frac{9}{2} \div 0$, is not a rational number.	Thus, rational numbers are not closed under division.

2. Commutative Property

Operations	Numbers	Remarks
Addition	$\frac{2}{5} + \frac{-7}{5} = \frac{-7}{5} + \frac{2}{5} = \frac{-5}{5}$ In general, for any two rational numbers: a and b; $a + b = b + a$.	Thus, rational numbers are commutative under addition.
Subtraction	$\frac{2}{5} - \frac{-7}{5} \neq \frac{-7}{5} - \frac{2}{5}$ In general, for any two rational numbers: a and b; $a - b \neq b - a$.	Thus, rational numbers are not commutative under subtraction.
Multiplication	$\frac{-2}{7} \times \frac{3}{5} = \frac{3}{5} \times \frac{(-2)}{7} = \frac{-6}{35}$. In general, for any two rational numbers: a and b; $a \times b = b \times a$.	Thus, rational numbers are commutative under multiplication.
Division	$\frac{2}{5} \div \frac{-7}{5} \neq \frac{-7}{5} \div \frac{2}{5}$ In general, for any two rational numbers: a and b; $a \div b \neq b \div a$.	Thus, rational numbers are not commutative under division.

3. Associativity

Operations	Numbers	Remarks
Addition	Is $\frac{2}{5} + \left(\frac{3}{7} + \frac{9}{4}\right) = \left(\frac{2}{5} + \frac{3}{7}\right) + \frac{9}{4}$? LHS $= \frac{2}{5} + \left(\frac{3}{7} + \frac{9}{4}\right) = \frac{2}{5} + \left(\frac{12 + 63}{28}\right) = \frac{2}{5} + \frac{75}{28} = \frac{56 + 375}{140} = \frac{431}{140}$ RHS $= \left(\frac{2}{5} + \frac{3}{7}\right) + \frac{9}{4} = \left(\frac{14 + 15}{35}\right) + \frac{9}{4} = \frac{29}{35} + \frac{9}{4} = \frac{116 + 315}{140} = \frac{431}{140}$ So, LHS = RHS In general, for any three rational numbers: a, b and c; $a + (b + c) = (a + b) + c$.	Thus, rational numbers are associative over addition.

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Subtraction	Is $\frac{5}{2}-\left(\frac{2}{3}-\frac{1}{4}\right)=\left(\frac{5}{2}-\frac{2}{3}\right)-\frac{1}{4}$? LHS $=\frac{5}{2}-\left(\frac{8-3}{12}\right)=\frac{5}{2}-\frac{5}{12}=\frac{30-5}{12}=\frac{25}{12}$ RHS $=\left(\frac{5}{2}-\frac{2}{3}\right)-\frac{1}{4}=\left(\frac{15-4}{6}\right)-\frac{1}{4}=\frac{11}{6}-\frac{1}{4}=\frac{22-3}{12}=\frac{19}{12}$ So, LHS $\neq$ RHS. In general, for any three rational numbers: a, b and c; $a-(b-c)\neq(a-b)-c$.	Thus, rational numbers are not associative under subtraction.
Multiplication	Is $\frac{2}{3}\times\left(\frac{1}{2}\times\frac{5}{6}\right)=\left(\frac{2}{3}\times\frac{1}{2}\right)\times\frac{5}{6}$? LHS $=\frac{2}{3}\times\frac{5}{12}=\frac{10}{36}$, RHS $=\frac{2}{6}\times\frac{5}{6}=\frac{10}{36}$ So, LHS $=$ RHS. In general, for any three rational numbers: a, b and c; $a\times(b\times c)=(a\times b)\times c$.	Thus, rational numbers are associative under multiplication.
Division	Is $\frac{2}{5}\div\left(\frac{1}{2}\div\frac{2}{3}\right)=\left(\frac{2}{5}\div\frac{1}{2}\right)\div\frac{2}{3}$? LHS $=\frac{2}{5}\div\left(\frac{1}{2}\times\frac{3}{2}\right)=\frac{2}{5}\div\frac{3}{4}=\frac{2}{5}\times\frac{4}{3}=\frac{8}{15}$ RHS $=\left(\frac{2}{5}\times\frac{2}{1}\right)\div\frac{2}{3}=\frac{4}{5}\div\frac{2}{3}=\frac{4}{5}\times\frac{3}{2}=\frac{6}{5}=1\frac{1}{5}$ In general, for any three rational numbers: a, b and c; $a\div(b\div c)\neq(a\div b)\div c$.	Thus, rational numbers are not associative under division.

4. Additive Identity

0 is called the additive identity for rational numbers because when a rational number is added by 0, the result is the rational number itself i.e., for any rational number a, $a+0=a$.

For example, $\frac{2}{3}+0=\frac{2}{3}=0+\frac{2}{3}$

$\therefore$ By adding 0, the rational number remains the same.

5. Multiplicative Identity

1 is known as the multiplicative identity for rational numbers because when a rational number is multiplied with 1, the result is the rational number itself i.e., for any rational number a, $a\times 1=a$.

For example: $\frac{4}{3}\times 1=\frac{4}{3}=1\times\frac{4}{3}$

6. Negative or Additive Inverse

Consider the rational number $\frac{3}{4}$.

$\frac{3}{4}+\left(\frac{-3}{4}\right)=0$. Also, $\left(\frac{-3}{4}\right)+\frac{3}{4}=0$.

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So, for a rational number $\frac{a}{b}$, we have $\frac{a}{b} + \left(-\frac{a}{b}\right) = \left(-\frac{a}{b}\right) + \frac{a}{b} = 0$

So, $\frac{-a}{b}$ is the additive inverse of $\frac{a}{b}$. And, $\frac{a}{b}$ is the additive inverse of $\left(-\frac{a}{b}\right)$.

7. Reciprocal/Multiplicative Inverse

Let's consider the rational number $\frac{7}{9}$.

$\frac{7}{9} \times \frac{9}{7} = 1$

Similarly, $\frac{-2}{3} \times \frac{3}{-2} = 1$

We say that a rational number $\frac{c}{d}$ is called the reciprocal or multiplicative inverse of rational number $\frac{a}{b}$, if $\frac{a}{b} \times \frac{c}{d} = 1$.

8. Distributive Property of Multiplication over Addition/Subtraction for Rational Numbers

According to this property:

$\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) + \left(\frac{a}{b} \times \frac{e}{f}\right)$

Let us verify the above property. For this, let us verify; $\frac{2}{3} \times \left(\frac{1}{2} + \frac{3}{4}\right) = \left(\frac{2}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{3}{4}\right)$.

LHS $= \frac{2}{3} \times \left(\frac{1}{2} + \frac{3}{4}\right) = \frac{2}{3} \times \left(\frac{2+3}{4}\right) = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$

RHS $= \left(\frac{2}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{3}{4}\right) = \frac{1}{3} + \frac{1}{2} = \frac{2+3}{6} = \frac{5}{6}$

LHS = RHS

So, we can say that rational numbers follow distributive property of multiplication over addition.

Similarly, $\frac{a}{b} \times \left(\frac{c}{d} - \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) - \left(\frac{a}{b} \times \frac{e}{f}\right)$

Solved Examples

Example 1: Find:

a. $\frac{3}{5} + \frac{-4}{13} + \frac{4}{5} + \frac{5}{13}$

b. $\frac{-4}{15} \times \frac{3}{7} \times 1\frac{1}{4} \times 2\frac{1}{3}$

Solution: a. $\frac{3}{5} + \frac{-4}{13} + \frac{4}{5} + \frac{5}{13} = \left(\frac{3}{5} + \frac{4}{5}\right) + \left(\frac{-4}{13} + \frac{5}{13}\right)$ (using commutative property)

$= \frac{7}{5} + \frac{1}{13} = \frac{91+5}{65} = \frac{96}{65}$.

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b. $\frac{-4}{15} \times \frac{3}{7} \times 1\frac{1}{4} \times 2\frac{1}{3} = \frac{-4}{15} \times \frac{3}{7} \times \frac{5}{4} \times \frac{7}{3}$

$= \left(\frac{-4}{15} \times \frac{5}{4}\right) \times \left(\frac{3}{7} \times \frac{7}{3}\right)$ (using commutative property)

$= \frac{-1}{3} \times 1 = \frac{-1}{3}$

Remember

Additive inverse means to find the negative of the number.

Example 2: Find the additive inverse of the following:

a. $\frac{-2}{3}$

b. $\frac{1}{121}$

Solution: a. $\frac{2}{3}$ is the additive inverse of $\frac{-2}{3}$ because $\frac{2}{3} + \left(\frac{-2}{3}\right) = 0$

b. $\frac{-1}{121}$ is the additive inverse of $\frac{1}{121}$ because $\frac{-1}{121} + \frac{1}{121} = 0$.

Example 3: Find:

a. $\frac{3}{4} \times \frac{-3}{5} - \frac{5}{6} - \frac{5}{7} \times \frac{-3}{5}$

b. $\frac{6}{8} \times \frac{4}{5} + \frac{1}{4} + \frac{4}{5} \times \frac{-2}{3}$

Solution: a. $\frac{3}{4} \times \frac{-3}{5} - \frac{5}{6} - \frac{5}{7} \times \frac{-3}{5} = \frac{3}{4} \times \frac{-3}{5} - \frac{5}{7} \times \frac{-3}{5} - \frac{5}{6}$ (by commutative property)

$= \frac{-3}{5} \times \left(\frac{3}{4} - \frac{5}{7}\right) - \frac{5}{6}$ (by distributive property)

$= \frac{-3}{5} \times \left(\frac{21-20}{28}\right) - \frac{5}{6} = \frac{-3}{5} \times \frac{1}{28} - \frac{5}{6}$

$= \frac{-3}{140} - \frac{5}{6} = \frac{-9-350}{420} = \frac{-359}{420}$

b. $\frac{6}{8} \times \frac{4}{5} + \frac{1}{4} + \frac{4}{5} \times \frac{-2}{3}$

$= \frac{6}{8} \times \frac{4}{5} + \frac{-2}{3} \times \frac{4}{5} + \frac{1}{4}$ (by commutative property)

$= \frac{4}{5} \times \left[\frac{6}{8} + \left(\frac{-2}{3}\right)\right] + \frac{1}{4}$ (by distributive property)

$= \frac{4}{5} \times \left(\frac{18-16}{24}\right) + \frac{1}{4}$

$= \frac{4}{5} \times \frac{2}{24} + \frac{1}{4}$

$= \frac{1}{15} + \frac{1}{4}$

$= \frac{4+15}{60} = \frac{19}{60}$

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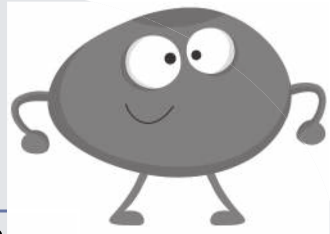
Mental Maths



Complete the following table.

Numbers are closed under	**Addition**	**Subtraction**	**Multiplication**	**Division**
Rational numbers	yes	....	....	....
Integers	....	....	....	No
Whole numbers	yes	....	....	....
Natural numbers	....	No	....	....

Practice 1.1

1. Name the property used.

   a. $\frac{-7}{11}\times 1=\frac{-7}{11}=1\times\frac{-7}{11}$

   b. $\frac{-2}{3}\times\left(\frac{1}{2}\times\frac{-5}{6}\right)=\left(\frac{-2}{3}\times\frac{1}{2}\right)\times\frac{-5}{6}$

   c. $\frac{-1}{2}+\left(\frac{7}{2}+\frac{3}{4}\right)=\left(-\frac{1}{2}+\frac{7}{2}\right)+\frac{3}{4}$

2. Using properties, find:

   a. $-\frac{5}{6}\times\frac{7}{8}+\frac{2}{9}+\left(\frac{-2}{3}\right)\times\frac{7}{8}$

   b. $\left\{\frac{9}{16}\times\frac{4}{12}\right\}+\left\{\frac{9}{16}\times\frac{-3}{9}\right\}$

3. Verify each of the following.

   a. $\frac{-13}{24}\times\left\{\frac{-12}{5}\times\frac{35}{36}\right\}=\left\{\frac{-13}{24}\times\frac{-12}{5}\right\}\times\frac{35}{36}$

   b. $\frac{-15}{4}\times\left\{\frac{3}{7}+\left(\frac{12}{-5}\right)\right\}=\left\{\frac{-15}{4}\times\frac{3}{7}\right\}+\left\{\frac{-15}{4}\times\frac{-12}{5}\right\}$

   c. $\frac{-16}{7}\times\left\{\frac{-8}{9}+\frac{-7}{6}\right\}=\left\{\frac{-16}{7}\times\frac{-8}{9}\right\}+\left\{\frac{-16}{7}\times\frac{-7}{6}\right\}$

   d. $\left\{\frac{-8}{3}+\frac{-13}{12}\right\}\times\frac{5}{6}=\left\{\frac{-8}{3}\times\frac{5}{6}\right\}+\left\{\frac{-13}{12}\times\frac{5}{6}\right\}$

4. For $a=\frac{5}{8}$, $b=\frac{-3}{4}$ and $c=\frac{-1}{2}$, check whether the following properties hold true.

   a. Associative property of addition

   b. Distributive property of multiplication over subtraction

   c. Associative property of subtraction

   d. Distributive property of multiplication over addition

5. Find the additive inverse of the following.

   a. $\frac{2}{7}$ b. $\frac{3}{16}$ c. $\frac{-17}{2}$ d. $\frac{-29}{8}$ e. $\frac{-3}{5}$ f. $1\frac{6}{7}$

6. Find the multiplicative inverse of the following.

   a. -22 b. $\frac{22}{3}$ c. $\frac{4}{7}$ d. $\frac{1}{9}$ e. $\frac{-1}{18}$ f. $1\frac{5}{6}$

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7. Verify that $-(-x) = x$ for the given value of x.

   a. $\frac{17}{11}$ b. $\frac{-22}{15}$ c. $\frac{2}{5}$ d. $-\left(\frac{-8}{9}\right)$ e. $\frac{-7}{12}$ f. $1\frac{4}{7}$

8. Fill in the blanks.

   a. $\frac{2}{3}\times\left(\frac{7}{8}-\frac{1}{3}\right)=\frac{2}{3}\times\frac{7}{8}-\frac{2}{3}\times$ ....

   b. $\frac{9}{8}\times$ .... $=\frac{-9}{8}$.

   c. Reciprocal of $\frac{-3}{7}$ is ....

   d. $\left(\frac{6}{7}+3\right)+\frac{4}{9}=$ .... $+\left(3+\frac{4}{9}\right)$.

   e. The product of two rational numbers is always a ....

   f. The numbers .... and .... are their own reciprocals.

   g. The product of a positive rational number and a negative rational number is always ....

9. Is $\frac{1}{0}$ the multiplicative inverse of 0? Why or why not?

10. Multiply $\frac{3}{13}$ by the reciprocal of $\frac{9}{26}$.

11. Find $(x + y) \div (x - y)$, if $x = \frac{1}{4}$ and $y = \frac{3}{2}$.

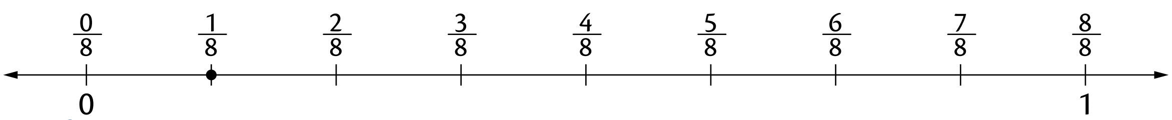
Maths Genius

All integers are rational numbers. Are all rational numbers integers? Justify your answer with an example.

Representation of Rational Numbers on the Number Line

Let us represent $\frac{2}{3}$ on the number line. For this, we know that $\frac{2}{3}$ lies between 0 and 1. So, we draw the number line and divide this into 3 equal parts. 1 can be written as $\frac{3}{3}$, 0 as $\frac{0}{3}$ and accordingly we make $\frac{1}{3}$ and $\frac{2}{3}$ and then we can label $\frac{2}{3}$ on the number line.

Similarly, if we need to label $\frac{1}{8}$ on the number line. We divide the number line between 0 and 1 into 8 equal parts. Now, 0 is $\frac{0}{8}$ and 1 is $\frac{8}{8}$ and then we can make $\frac{1}{8}$, $\frac{2}{8}$ and so on. And finally, we label $\frac{1}{8}$ on the number line.



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Inserting a Rational Number Between Two Rational Numbers

How many whole numbers are there between 4 and 9? *There are four whole numbers, they are 5, 6, 7 and 8.*

How many integers are there between –3 and 1? *There are four integers, they are –2, –1 and 0.*

Thus, we find that there are finite number of whole numbers between two whole numbers and finite number of integers between two integers.

Now, how many rational numbers are there between $\frac{1}{5}$ and $\frac{4}{5}$? One may say, they are $\frac{2}{5}, \frac{3}{5}$, i.e., two rational numbers.

But we know that $\frac{1}{5}$ can be written as $\frac{10}{50}$ and $\frac{4}{5}$ as $\frac{40}{50}$. Now, we see that the rational numbers $\frac{11}{50}, \frac{12}{50}, \frac{13}{50}, ..., \frac{39}{50}$ lie between $\frac{10}{50}$ and $\frac{40}{50}$.

Again, $\frac{1}{5}$ can be written as $\frac{100}{500}$ and $\frac{4}{5}$ as $\frac{400}{500}$. Now, we see that the rational numbers $\frac{101}{500}, \frac{102}{500}, \frac{103}{500}, ..., \frac{399}{500}$ lie between $\frac{100}{500}$ and $\frac{400}{500}$.

Thus, we can insert as many rational numbers between any two rational numbers, i.e., there are infinite rational numbers between any two rational numbers.

Now, in order to find a rational number between any two given rational numbers, we use the following methods:

Method 1: To find a rational number between two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, then $\frac{\left(\frac{a}{b}+\frac{c}{d}\right)}{2}$ is a rational number lying between $\frac{a}{b}$ and $\frac{c}{d}$.

Method 2: Write the given rational numbers as equivalent rational numbers by making the denominator same which is equal to the LCM of both the denominators. Insert required number of rational numbers by choosing numbers lying between the rational numbers.

For example, let us find out eight rational numbers between $\frac{2}{5}$ and $\frac{2}{3}$. First of all make the denominator same by taking LCM.

So, $\frac{2}{5} = \frac{6}{15}$ and $\frac{2}{3} = \frac{10}{15}$.

Now, multiply the numerator and denominator of both the rational numbers by 10.

$\frac{6}{15} = \frac{6 \times 10}{15 \times 10} = \frac{60}{150}$ $\qquad$ $\frac{10}{15} = \frac{10 \times 10}{15 \times 10} = \frac{100}{150}$

Thus, eight rational numbers between $\frac{2}{5}$ and $\frac{2}{3}$ are $\frac{61}{150}, \frac{62}{150}, \frac{63}{150}, \frac{64}{150}, \frac{65}{150}, \frac{66}{150}, \frac{67}{150}, \frac{68}{150}$.

Solved Examples

Example 1: Insert 10 rational numbers between $\frac{-3}{11}$ and $\frac{8}{11}$.

Solutions: We know that $-3 < -2 < -1 < 0 < 1 < 2 < 3 < 4 < 5 < 6 < 7 < 8$.

$\therefore$ 10 rational numbers between $\frac{-3}{11}$ and $\frac{8}{11}$ are $\frac{-2}{11}, \frac{-1}{11}, \frac{0}{11}, \frac{1}{11}, ..., \frac{7}{11}$.

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Example 2: Insert 3 rational numbers between 4 and 4.5.

Solution: A rational number between 4 and 4.5 is $= \frac{4+4.5}{2} = 4.25$

A rational number between 4 and 4.25 is $= \frac{4+4.25}{2} = 4.125$

A rational number between 4 and 4.125 is $= \frac{4+4.125}{2} = 4.0625$

Therefore, three rational numbers between 4 and 4.5 are 4.25, 4.125 and 4.0625.

Example 3: Find three rational numbers between –2 and –5.

Solution: We know that, $-2 = \frac{-2}{1}$ and $-5 = \frac{-5}{1}$.

Now, $\frac{-5}{1} \times \frac{2}{2} = \frac{-10}{2}, \frac{-2}{1} \times \frac{2}{2} = \frac{-4}{2}$

Now, we need to find out 3 rational numbers between $\frac{-10}{2}$ and $\frac{-4}{2}$.

If we look at the numerators, we can find out 3 rational numbers between –10 and –4.

So, the three rational numbers are $\frac{-9}{2}$, $\frac{-8}{2}$ and $\frac{-7}{2}$.

Practice 1.2

1. Represent the following rational numbers on the number line.

   a. $\frac{-7}{8}$ b. $\frac{9}{8}$ c. $\frac{-1}{5}$ d. $\frac{15}{9}$ e. $-\frac{8}{5}$ f. $\frac{-4}{7}$

2. Insert a rational number between $\frac{5}{7}$ and $\frac{2}{3}$.
3. Insert a rational number between $\frac{6}{5}$ and $\frac{7}{2}$.
4. Insert three rational numbers between 6 and 6.6.
5. Insert five rational numbers between 1 and 2.
6. Insert six rational numbers between 3 and 4.
7. Insert five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.
8. Write seven rational numbers which are smaller than 3.
9. Write ten rational numbers which are greater than –7.

Word Problems

Solved Examples

Example 1: From a rope 11 m long, two pieces of lengths $2\frac{3}{5}$ m and $3\frac{3}{10}$ m are cut off. What is the length of the remaining rope?

Solution: Here, the rope is 11 m long.

$2\frac{3}{5} = \frac{13}{5}$ m and $3\frac{3}{10} = \frac{33}{10}$ m are being cut off.

So, the length of the remaining rope $= 11 - \left(\frac{13}{5} + \frac{33}{10}\right) = 11 - \left(\frac{26+33}{10}\right)$

$= 11 - \frac{59}{10} = 11 - 5.9 = 5.1$ m