

1

Integers

Warm-up

- Arrange the following integers in ascending order.
 - $-10, -999, 16, -7614, 0, 4$
 - $1, -9, 9, 43, -908, -809$
- Arrange the following integers in descending order.
 - $20, -22, 11, -11, 222, -419$
 - $31, -77, 55, -63, 5, -8880, 903$
- Compare the following using $>$, $<$ or $=$ sign.
 - $(-30) + (-6)$ $(30) + (-6)$
 - $(-22) + (-190)$ $(-21) + (-109)$
- Compare the following pairs of numbers using a number line.
 - -9 and 10
 - -7 and -3

Introduction

You have learnt about integers in previous classes. In our daily lives, there is a need for negative integers and we need to understand how operations are performed on both negative and positive integers. Consider two shopkeepers who have bank accounts in the same bank. At the start of the first month of their operations, their balances in the account are: ₹ 12,600 and – ₹ 10,500. Who has better bank balance and what is the difference between their balances? How much is the total of their bank balances? To answer these questions, we need to learn positive and negative integers and their operations.

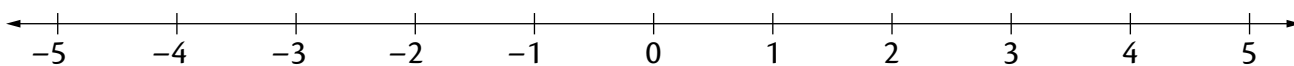
Integers

The set of integers includes whole numbers and negative numbers. It is denoted by **Z** and can be represented as $\mathbf{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.

Here, the numbers $1, 2, 3, 4, \dots$ are called positive integers whereas the numbers $-1, -2, -3, -4, \dots$ are called negative integers.

Representation of Integers on the Number Line

To represent integers on a number line, we draw a line and Mark '0' almost at the centre of the line. On the right hand side of 0, mark positive integers and on the left hand side of 0 mark negative integers as shown below:



Learning Objectives

We shall learn about:

- integers and their representation on a number line
- four operations and their properties
- simplification of numerical expressions

Remember

0 is neither negative nor positive.

As we have already learnt the rules of comparison of integers in our previous classes, so let us revisit them.

1. Every positive integer is always greater than every negative integer.
2. Zero is less than every positive integer and is greater than every negative integer.
3. If a and b are two positive integers such that $a > b$, then $-a < -b$.
4. By convention, the number occurring to the right is greater than that on the left of a number line.
5. -1 is the largest negative number.

Remember

There is no integer between any two consecutive integers, say, -13 and -14 .

Addition of Integers

To perform addition of integers, let us revise some of the rules of addition of integers.

Rule 1: When you add two integers having the same sign, add the numbers without the sign and apply the same sign as that of the two integers

For example, $(+6) + (+2) = +8$ $(-6) + (-2) = -8$

Rule 2: While adding a positive and a negative integer, we first consider the numbers without their signs, find the difference between the two and give the sign of the bigger number to the answer.

For example, $-4 + (+6) = +2$ $-12 + (+3) = -9$

Now, consider the earlier example of the two shopkeepers with bank balances ₹ 12,600 and $- ₹ 10,500$. If we need to find the total of their balances, we will need to find $12600 + (-10500)$.

Thus, $12600 + (-10500) = + (12600 - 10500) = ₹ 2100$

Properties of Addition of Integers

Property	Example
Closure property: The sum of two integers is always an integer, i.e., if $a \in \mathbf{Z}$ and $b \in \mathbf{Z}$, then $a + b \in \mathbf{Z}$.	$-4 + (+2) = -2$ $-4 + (+10) = 6$
Commutative property: For any two integers a and b , $a + b = b + a$	$-3 + (+7) = (+7) + (-3) = 4$
Associative property: For any three integers a , b , c , $(a + b) + c = a + (b + c)$	$[(-7) + (+5)] + (-2) = (-7) + [(+5) + (-2)] = -4$
Identity property: If $a \in \mathbf{Z}$, then $a + 0 = 0 + a = a$ Note: 0 is known as the additive identity element for the set of integers.	$(-5) + 0 = 0 + (-5) = (-5)$
Inverse property: For every $a \in \mathbf{Z}$, there exists an integer $-a \in \mathbf{Z}$ such that $a + (-a) = (-a) + a = 0$ Note: $(-a)$ is called the additive inverse of ' a ' for every $a \in \mathbf{Z}$.	$7 + (-7) = (-7) + 7 = 0$

Subtraction of Integers

To perform subtraction, let us revise the method for subtraction of integers.

When we subtract two integers, we add the opposite of the subtrahend (additive inverse) to the minuend, i.e., $a - b = a + (-b)$.

Consider the earlier example and find the difference between the balances of the two bank accounts. Thus, here we need to find $12600 - (-10500)$.

Thus, $12600 - (-10500) = 12600 + 10500$ [additive inverse of -10500 is 10500]
 $= 23100$.

So the difference in the bank balances is ₹ 23,100.

Properties of Subtraction of Integers

Property	Example
Closure property: The difference between two integers is always an integer, i.e., if $a \in \mathbf{Z}$, $b \in \mathbf{Z}$, then $a - b \in \mathbf{Z}$	$-5 - (+2) = -7$ $-6 - (-4) = -2$
Commutative property: For any two integers a and b , $a - b \neq b - a$	$-6 - (-7) = 1$; $(-7) - (-6) = -1$ $\therefore -6 - (-7) \neq (-7) - (-6)$
Associative property: For any three integers a , b , c , $(a - b) - c \neq a - (b - c)$	$[9 - (-5)] - 4 = 14 - 4 = 10$; $9 - [-5 - 4] = 9 - (-9) = 18$ $\therefore [9 - (-5)] - 4 \neq 9 - [-5 - 4]$

Explain Your Thinking

Maria performs a subtraction operation on integers as:

$$-5 - (+3) = (5 - 3) = 2$$

She feels her answer is incorrect, as subtracting a positive number should lead to the answer being smaller than the first number.

1. Do you think her calculation is correct?
2. Do you agree with her method of checking? Is her answer correct or wrong?

Solved Examples

Example 1: Solve $[(-39) + (-11) - 45 - 55 + 50]$ by applying properties of integers.

Solution: $[(-39) + (-11)] + (-45 - 55) + 50$ (using commutative and associative property)
 $= (-50) + (-100) + 50$
 $= [(-50) + 50] + (-100)$ (using associative property)
 $= 0 + (-100) = -100$

Example 2: Verify: $(-15) + [(-3) + (-12)] = [(-15) + (-3)] + (-12)$

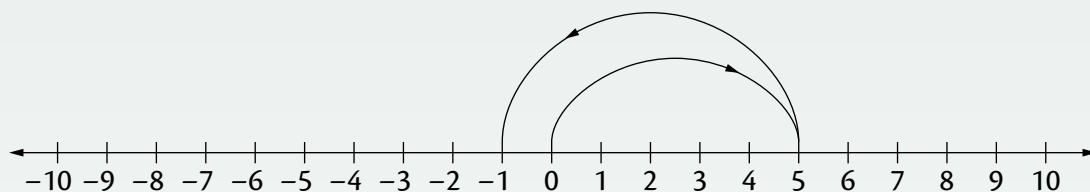
Solution: LHS $= (-15) + [(-3) + (-12)]$
 $= (-15) + (-15) = (-30)$
 RHS $= [(-15) + (-3)] + (-12)$
 $= (-18) + (-12) = (-30)$
 LHS = RHS, hence verified.

Example 3: Draw a number line and represent each of the following on it:

a. $5 + (-6)$ b. $(-8) - (-3)$

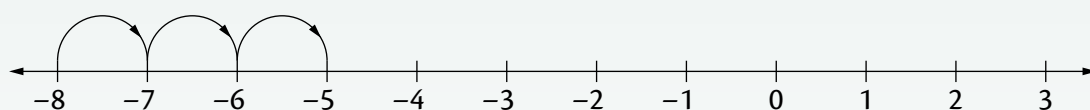
Solution: a. $5 + (-6)$

We start from 5. We need to add -6 . This is the same as subtracting 6. So we move 6 steps to the left, to reach -1 .



b. $-8 - (-3)$

We start from -8 . We need to subtract -3 from it. This is the same as adding 3. So we move 3 steps to the right, to reach -5 .



Example 4: The temperature inside a refrigerator is -4°C . When the electricity supply is turned off, the temperature rises by 3°C every hour. What is the temperature in the refrigerator 3 hours after the electricity is turned off?

Solution: Initial temperature $= -4^\circ\text{C}$

After 3 hours, the temperature in the refrigerator $= -4 + 3 + 3 + 3 = -4 + 9 = 5^\circ\text{C}$

Example 5: An elevator is on the 14th floor. If it goes up 7 floors and then comes down 19 floors, then on which floor is the elevator now?

Solution: Initial floor of elevator $= 14$

Position after going 7 floors up $= 14 + 7 = 21$

Position after coming 19 floors down $= 21 - 19 = 2$

\therefore Current position of elevator $= 2\text{nd floor}$

Practice 1.1

1. Find the value of the following.

a. $(-657) + 345$

b. $(-101) + 202$

c. $(-99) + 97 + (-95)$

d. $93 + (-91) + 89$

e. $81 - (-45) + (-165) + 5$

f. $(-34) + 56 - 89 + (-23)$

g. $24 + (-11) + 59$

h. $102 - (-755) + 67$

i. $(-251) + 561 + (-90)$

2. State whether the following statements are true or false. Give reasons.

a. $(-21) + 13 = 13 + (-21)$

b. $34 - (9 - 15) = (34 - 9) - 15$

c. $[(-5) + 7] + (-18) = 7 + [(-5) + (-18)]$

d. $876 + (-876) = (-876) - (-876)$

3. Find the value of the following, using properties.

a. $34 - (-16) + (+59) - (+29)$

b. $(-50) - (+15) + (-82) - 8$

c. $(-144) - (-48) - (-2) - (-54) + 17$

d. $(-150) + (-135) + 15 + (-80)$

4. Solve using number line:

a. $-10 - 8$

b. $8 - (-6)$

c. $-9 - 8$

d. $-7 + (-1)$

e. $13 + (-6)$

f. $(-15) + (-5)$

g. $11 - 10$

h. $2 - 2$

5. From the sum of (-156) and 67 , subtract the sum of (-109) and 109 .

6. The sum of two integers is -15 . If one of them is 87 , then find the other.

7. A submarine is situated 950 feet below the sea level. If it ascends 335 feet, what is its new position?

8. In Leh, the temperature was -5°C in the afternoon. If the temperature dropped by 9°C , what is the temperature now?

9. At sunrise, the outside temperature was 3°C below zero. By lunch time, the temperature rose by 21°C and then fell by 11°C by night. What was the temperature at the end of the day?

10. A submarine hovers at 310 m below sea level. If it descends 180 m and then ascends 275 m, what is its new position?



Maths Genius

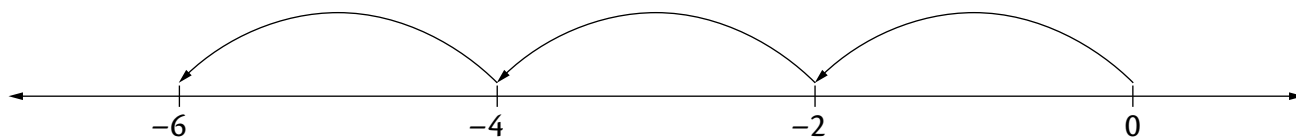
Fill the missing values such that sum of the integers of every row and column is equal to -10 .

2		-14		0
8		-8	-1	1
-11	-9			
		4		-12
-4	3	10	-13	

Multiplication of Integers

We know that multiplication is repeated addition of the same number, i.e., $2 + 2 + 2 = 3 \times 2 = 6$

Similarly, we can find the value of 3×-2 i.e. $(-2) + (-2) + (-2)$.



Thus, $3 \times (-2) = (-6)$

Now, let us find $(-2) \times 3$.

For this, observe the following pattern.

We have $2 \times 3 = 6$

$$1 \times 3 = 3 = 6 - 3$$

$$0 \times 3 = 0 = 3 - 3$$

$$-1 \times 3 = -3 = 0 - 3$$

$$-2 \times 3 = -6 = -3 - 3$$

Thus, we have $3 \times (-2) = (-2) \times 3 = -6$

Thus, while multiplying a positive integer with a negative integer, we get a negative integer.

Now, let us find $(-3) \times (-4)$.

For this, observe the following pattern.

We know that, $(-3) \times 4 = -12$

$$(-3) \times 3 = -9 = (-12) - (-3)$$

$$(-3) \times 2 = -6 = (-9) - (-3)$$

$$(-3) \times 1 = -3 = (-6) - (-3)$$

$$(-3) \times 0 = 0 = (-3) - (-3)$$

$$(-3) \times (-1) = 0 - (-3) = 3$$

$$(-3) \times (-2) = 3 - (-3) = 6$$

Thus, we have $(-3) \times (-2) = 6$

Thus, by multiplying two negative integers, we get a positive integer.

In general, whenever we multiply two integers, the numerical value (value without considering its sign) of the product is equal to the product of numerical values of the multiplicand and multiplier.

Now let's revisit the various rules to be applied while multiplying two integers.

Rule 1: The product of two positive integers is a positive integer.

For example, $(+7) \times (+3) = (+21)$

Rule 2: The product of a positive integer and a negative integer is a negative integer.

For example, $(+6) \times (-2) = -12$

Rule 3: The product of two negative integers is a positive integer.

For example, $(-7) \times (-3) = (+21)$

Properties of Multiplication of Integers

Property	Example
Closure property: The product of any two integers is always an integer, i.e., if $a \in \mathbf{Z}$ and $b \in \mathbf{Z}$, then $a \times b \in \mathbf{Z}$.	$(-3) \times (-2) = (+6)$ $(-3) \times (+2) = (-6)$
Commutative property: For any two integers a and b , $a \times b = b \times a$	$(-7) \times 5 = 5 \times (-7) = -35$
Associative property: For any three integers a , b and c , $a \times (b \times c) = (a \times b) \times c$	$(-7) \times [(-3) \times 9] = [(-7) \times (-3)] \times 9 = 189$
Distributive property: For any three integers a , b and c , $a \times (b + c) = a \times b + a \times c$	$(-5) \times [(-7) + (-3)] = (-5) \times (-7) + (-5) \times (-3) = 50$
Identity property: If $a \in \mathbf{Z}$, then $a \times 1 = 1 \times a = a$ Note: 1 is known as the multiplicative identity for the set of integers.	$(-7) \times 1 = 1 \times (-7) = -7$
Zero property: For every $a \in \mathbf{Z}$, $a \times 0 = 0 \times a = 0$	$(-5) \times 0 = 0 \times (-5) = 0$

Explain Your Thinking

Tilak doesn't understand how to multiply negative numbers with negative numbers. So he says, he will try using properties of multiplication.

$$-4 \times (-3 + 3) = (-4 \times -3) + (-4 \times 3)$$

$$0 = (-4 \times -3) + -12$$

So he says, that $-4 \times -3 = 12$. Do you agree with his method?

Solved Examples

Example 1: Find the value of the following.

a. $(-11) \times 4 \times (-3)$ b. $(-16) \times 17 + (-16) \times (-18)$ c. $0 \times (-1) \times 2(-3) \times 4 \times (-5)$

Solution:

a. $(-11) \times 4 \times (-3) = [(-11) \times 4] \times (-3) = (-44) \times (-3) = 132$

b. $(-16) \times 17 + (-16) \times (-18) = (-16) \times [17 + (-18)]$ {Using distributive property}
 $= (-16) \times (17 - 18) = (-16) \times (-1) = 16$

c. $0 \times (-1) \times 2(-3) \times 4 \times (-5) = 0$ $\{\therefore 0 \times a = 0, \text{ for any integer } a \in \mathbf{Z}\}$

Example 2: Take three integers $a = -3$, $b = 4$ and $c = -2$.

Check if $a \times (b + c) = a \times b + a \times c$.

Solution:

Here, we have $a = (-3)$, $b = 4$ and $c = -2$.

$$\text{LHS} = a \times (b + c) = (-3) \times [4 + (-2)] = (-3) \times 2 = (-6)$$

$$\text{RHS} = a \times b + a \times c = (-3) \times 4 + (-3) \times (-2) = (-12) + (+6) = (-6)$$

Thus, LHS = RHS, hence proved.

Remember

$$\underbrace{(-) \times (-) \times (-) \times \dots}_{\text{odd}} = (-)$$

$$\underbrace{(-) \times (-) \times (-) \times \dots}_{\text{even}} = (+)$$

Example 3: In a class test containing 20 questions, 2 marks are given for every correct answer and (-1) mark is given for every incorrect answer. If Maneet attempts all questions and 15 of her answers are correct, what is her total score?

Solution:

Marks given for every correct answer = 2

$$\text{So, marks given for 15 correct answers} = 2 \times 15 = 30$$

Marks given for an incorrect answer = -1

$$\text{So, marks given for 5 (= 20 - 15) incorrect answers} = (-1) \times 5 = -5$$

$$\text{Therefore, Maneet's total score} = 30 + (-5) = 25$$

Example 4:

A merchant makes a profit of ₹ 2000 per piece on the sale of a laptop and a loss of ₹ 550 per piece on the sale of a mobile. What is his profit or loss if he sells 20 laptops and 10 mobiles?

Solution:

Profit is represented by positive integer and loss is represented by negative integer.

$$\text{Profit on one Laptop} = ₹ 2,000$$

$$\text{Profit on 20 laptops} = 2,000 \times 20 = ₹ 40,000$$

$$\text{Loss on one mobile} = - ₹ 550$$

$$\text{Loss on 10 mobiles} = 10 \times (-550) = (-5500)$$

$$\text{Total amount received} = 40,000 + (-5500) = ₹ 34,500$$

So, he gained ₹ 34,500.

Division of Integers

Let us learn about how to divide integers.

We know that division is the reverse of multiplication, i.e., $6 \times 2 = 12$ gives $12 \div 2 = 6$ and $12 \div 6 = 2$.

Similarly, $(-6) \times 2 = -12$ gives $(-12) \div 2 = (-6)$ and $(-12) \div (-6) = 2$.

Thus, we get the following rules while dividing integers.

Rule 1: The quotient of two positive integers is a positive integer.

For example, $8 \div (+2) = +4$

Rule 2: The quotient of two negative integers is a positive integer.

For example, $(-8) \div (-2) = +4$

Rule 3: The quotient of a positive integer and a negative integer is negative.

For example, $12 \div (-4) = -3$

Properties of Division of Integers

Property	Example
Closure property: The quotient of two integers is not always an integer, i.e., if $a \in \mathbf{Z}$, $b \in \mathbf{Z}$, then $a \div b \notin \mathbf{Z}$	$(-5) \div (-20) = \frac{1}{4}$, which is not an integer
Commutative property: For any two integers a and b , $a \div b \neq b \div a$	$(-16) \div (-4) = 4$; $(-4) \div (-16) = \frac{1}{4}$ $\therefore (-16) \div (-4) \neq (-4) \div (-16)$
Associative property: For any three integers a , b and c , $(a \div b) \div c \neq a \div (b \div c)$	$[12 \div (-3)] \div 4 = (-4) \div 4 = -1$; $12 \div [-3 \div 4] =$ $12 \div \left(\frac{-3}{4}\right) = 12 \times \frac{4}{(-3)} = -16$ $\therefore [12 \div (-3)] \div 4 \neq 12 \div [-3 \div 4]$
Division of an integer by itself: For any integer a , $a \div a = 1$	$(-4) \div (-4) = 1$
Zero property: For any integer a , $0 \div a = 0$	$0 \div (-9) = 0$
Division of an integer by its additive inverse and - 1: If a ($a \neq 0$) is any integer, then $a \div (-a) = -1$, $(-a) \div a = -1$, $a \div (-1) = -a$	$\frac{4}{-4} = -1$; $\frac{-3}{(-1)} = 3$

Solved Examples

Example 1: Find the value of the following.

a. $361 \div (-19)$

b. $(-45) \div (-5)$

Solution: a. $(361) \div (-19) = \frac{361}{(-19)} = -19$

b. $(-45) \div (-5) = \frac{(-45)}{(-5)} = 9$

Example 2: The product of two integers is -960 . If one of them is -32 , find the other.

Solution: One number = -32

Product = -960

Other number = $(-960) \div (-32) = 30$

Example 3: The quotient when the integer -1170 is divided by a number is 65 . Find the divisor.

Solution: Quotient = 65 and dividend = -1170

$$\therefore \text{Divisor} = (-1170) \div 65 = -18$$

Practice 1.2

- Find the product.
 - $(-31) \times (-3)$
 - $(-1) \times 2 \times (-3) \times 4 \times (-5) \times 6$
 - $(-1) \times (-1) \times (-1) \dots 199$ times
 - $(-12) \times 5 \times (-8)$
 - $11 \times (-13) \times 15 \times 17$
 - $(-4) \times 5 \times (10)$
 - $24 \times (-5) \times (-12) \times (-1)$
 - $(-1) \times (-2) \times (-6) \times (-4)$
- Find the value.
 - $(+544) \div (+8)$
 - $(-102) \div (-17)$
 - $(-174) \div (-6)$
 - $1120 \div (-224)$
 - $(-346) \div 2$
 - $(-160) \div (-40)$
 - $(-110) \div (-5)$
 - $(-2655) \div 15$
 - $(-1545) \div (-5)$
- Simplify and state the property used in each case.
 - $(-11) \times (-5) - (-17) \times (-5)$
 - $(-343) \times 0$
 - $(-561) \div (-561)$
 - $(-91) \times [6 - (-9)]$
 - $0 \div 819$
 - $(-41) \times 2 - 10 \times 2$
- State true or false.
 - When any negative integer is divided by (-1) , the quotient is additive inverse of the integer.
 - The product of an integer with zero is always the given integer.
 - The product of 11 negative numbers and 10 positive numbers is positive.
 - The quotient of any integer and its additive inverse is 1.
 - Multiplicative identity of integers is -1 .
- Take three integers $a = (-4)$, $b = (-2)$ and $c = (-1)$ and check the following properties.
 - Associative property of multiplication
 - Distributive property of division
- The product of two even integers is -216 . If one of the integers is -54 , then find the other.
- The quotient on dividing two integers is 65 . If the dividend is -845 , then find the divisor.
- Mrs Kapoor has a negative balance of ₹ 1500 in her bank account at the start of June. After she deposited ₹ 450 for 3 months, what is her new balance?
- During an 18-hour period, the temperature which dropped by a fixed value every hour came down to 54°C . By how many degrees did the temperature drop each hour?
- The origin of a spring is 12 feet below the ground level. If a machine can dig 3 feet at a time, then how many times would the machine have to be used to reach the origin of the spring?
- A video game player receives ₹ 60 for every correct point and pays ₹ 54 for every time he fails. After a new game of 25 shots, he misses 13. Did he receive any money?
- On a rainy day, the amount of rain in a rain gauge increased by 4 inches over a 24-hour period, how much will the amount of rain in the gauge increase by over a 9-day period?

13. Smitha answered 34 questions correctly in a quiz. According to the rule, every correct answer is awarded 3 points and (-1) is deducted for every wrong answer. Find the number of questions she attempted, if she scored 95 marks.
14. Geetha had 12 toffees with her in the morning. She gave 3 to her brother and by afternoon her uncle gave her 5 more and then she gave 4 more to her brother. By evening, she received 6 toffees from her grandfather and again gave 7 more toffees to her brother. Find the number of toffees left with her by the end of the day, if she eats 3 of them.

Simplification of Numerical Expressions

Vinculum or Bar

It is a bar drawn on two or more terms of an expression. In any expression involving vinculum, we first simplify the terms involving vinculum and then perform the other operations.

Brackets: Given below are the three types of brackets. The order in which they should be used is as follows.

1. $()$ are known as parentheses, round brackets or small brackets.
2. $\{ \}$ are known as braces, curly brackets or flower brackets.
3. $[]$ are known as square brackets, box brackets or big brackets.

When an expression is enclosed within a bracket, we simplify it separately. This is called the removal of the brackets.

In some situations, more than one operation is used in simplification. Consider the following example:

$$\{[11 - (14 - 18 \div 2)] \times 4 - 3\} \text{ and } (-5)[(-6) - \{-5 + (-2 + 1 - 3 + 2)\}]$$

How do we simplify this?

Which operation would you perform first?

While solving such problems, we follow a rule known as BODMAS. This rule tells the order that we must follow while simplifying. The order is as given below:

B – Brackets O – of D – Divide M – Multiply A – Add S – Subtract

All the above operations must be taken up only in this order.

Solved Examples

Example 1: Simplify: $11 + [-7 - \{-3 + 8 \times (6 - 2 - \overline{3 + 5 - 4}) \div 17\}]$

Solution: Let's simplify.

Step 1: First simplify the terms under the bar.

$$\text{That is, } 3 + 5 - 4 = 8 - 4 = 4$$

$$\therefore 11 + [-7 - \{-3 + 8 \times (6 - 2 - \overline{3 + 5 - 4}) \div 17\}] = 11 + [-7 - \{-3 + 8 \times (6 - 2 - 4) \div 17\}]$$

Step 2: To remove the round brackets, simplify the terms within these brackets.

$$\text{That is, } 6 - 2 - 4 = 6 - 6 = 0$$

$$\therefore 11 + [-7 - \{-3 + 8 \times (6 - 2 - 4) \div 17\}] = 11 + [-7 - \{-3 + 8 \times 0 \div 17\}]$$