Essentials of Geophysical Data Processing

This textbook provides a concise introduction to geophysical data processing – including many of the techniques associated with the general field of time series analysis – for advanced students, researchers, and professionals. The treatment begins with calculus before transitioning to discrete time series via the sampling theorem, discussions of aliasing, the use of complex sinusoids, the development of the Discrete Fourier Transform from the Fourier series, and an overview of linear digital filter types and descriptions. Aimed at senior undergraduate and graduate students in geophysics, environmental science, and engineering with no previous background in linear algebra, probability, statistics or Fourier transforms, this textbook draws scenarios and data sets from across the world of geophysics and shows how data processing techniques can be applied to real-world problems using detailed examples, illustrations, and exercises. Exercises are mostly computational in nature and may be completed using MATLAB or a computing environment with similar capabilities.

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Essentials of Geophysical Data Processing

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Preface

At the beginning of 2007, I started preparing notes for an upper division course at the University of Texas at Austin (UT Austin) for geophysics majors, surveying time series and related data processing topics and illustrating the importance of linear systems and filter concepts in geophysical problems including instrument design, seismology, and gravity. The goal was to prepare students for career opportunities, often in the geophysical exploration industry, or for further study in graduate school.

This textbook has been developed from the course notes that have evolved over the years, and its key features are:

- Providing the foundation for a survey course on time series and data analysis methods suitable for upper division undergraduate science majors and early career graduate students.
- Surveying both time series analysis (the sampling theorem, the Discrete Fourier Transform, linear digital filters, and spectral analysis) and least squares – an essential tool in the analysis of time series and many other data types and in digital filter design.
- Bringing together the comprehensive range of topics required for a survey course suitable for undergraduates, from material in an extensive literature of textbooks, monographs, and journal articles.
- Offering a balanced mix of time series and data analysis methods and example applications, some from exploration geophysics but many from other geophysical problems.

Courses similar to the one I teach at UT Austin are common in undergraduate curricula emphasizing exploration geophysics, where reflection seismology is a main application. Yet the same methods are used in other disciplines, including meteorology, astronomy, oceanography, and many engineering fields. Given this, and the availability of time series and other data types on the World Wide Web and elsewhere, this text can serve as a resource supporting these other disciplines in the form of both organized courses and research at graduate and undergraduate levels.

In an ideal world, students who take this course would have a background in linear algebra, probability, statistics, and perhaps Fourier transforms of continuous functions. However, in my years of teaching, I have found that even students who have been exposed to some of these foundational topics in earlier course-work might not have fully grasped them. The first three appendices at the end of the book aim to address this issue.

Instructors teaching undergraduate science majors, who are typically prepared with at most two years of university calculus, should cover Chapters 1, 2, and 3, in that order. This should motivate the topics and ensure a proper understanding of the foundational notions of the sampling theorem, statistics, decibels, complex numbers, sinusoids, and Fourier series, which are needed to appreciate linear digital filters and the Discrete Fourier Transform.

Instructors teaching students who already have a good understanding of the topics in Chapters 1 through 3 might survey them quickly and then spend more time on Chapters 4 through 7, covering the Discrete Fourier Transform, linear digital filters, and least squares.
Chapters 8, 9, and 10 focus on linear filter design methods, applications, and spectral analysis. Some of these topics, such as linear filter design, are sufficient for an entire semester course, so might be only partially covered in a first semester with an emphasis on specific examples. In that case, these chapters might be a starting point for a second semester course that includes further discussion of linear filter design methods, the Fourier transform of continuous functions, as presented in Appendix B, and the underlying probability ideas in Appendix C.

Appendix D provides a list of monographs and texts for further reading on many of the topics covered in this textbook, often at a more advanced level. I have proactively decided not to include references within the text because virtually all topics in the book are well established in the recent literature. As evidence of this, most modern computational environments such as MATLAB, Mathematica, python, R, and others include functions to implement virtually all the algorithms and methods surveyed here.

I am grateful to have undertaken graduate studies at the Scripps Institution of Oceanography. Important lessons on many of the topics in this book were learned from my advisor Richard Haubrich and other faculty members, including Robert L. Parker, and dissertation committee members George Backus and Walter Munk. I also appreciate having been a colleague of Milo Backus at the University of Texas, who helped me understand many aspects of time series analysis related to exploration geophysics.

Clark R. Wilson