

## Index of Symbols

---

$\deg_l(F)$ , 79, 163	$\log^+(a)$ , 29
$\Delta$ , 62, 123, 130	$M_K$ , 27
$\Delta'$ , 66, 123, 130	$[n]$ , 48, 49, 56, 70
$D_{K/\mathbb{Q}}$ , 10	$\text{Norm}_\pi(D)$ , 98
$h(F)$ , 84	$\text{Norm}_\pi(L)$ , 98
$\hat{h}_L(x)$ , 55	$\text{ord}_P(x)$ , 26
$h_K(x)$ , 29	$\partial^{a,b}$ , 89
$h_L(x)$ , 36	$\partial_I$ , 87
$h_\phi(x)$ , 35	$ja(x)$ , 62
$h^+(F)$ , 84	$\Phi_{N\theta}$ , 129
$h_K^+(x)$ , 29	$\Phi_{N\theta}^{ij}$ , 129
$h^+(x)$ , 30	$\Phi_{N\theta}^{ijk}$ , 129
$h(s)$ , 132	$\Theta$ , 63
$h(x)$ , 30	$\theta$ , 120, 129
$ I _{\mathbf{d}}$ , 85	$V(d_1, d_2, d)$ , 124, 130, 131
$I$ , 88	$ F _\infty$ , 79
$\text{ind}_{\mathbf{x}}(f; \mathbf{d})$ , 85	$ F _v$ , 77
$\text{ind}_{\mathbf{a}}(P; \mathbf{d})$ , 87	$ I $ , 34, 88
$J$ , 48, 61, 120	$ x _v$ , 26, 27
$j$ , 121	$ x $ , 121
$K(\mathbb{C})$ , 9, 74	$\ x\ _K$ , 74
$\langle x, y \rangle$ , 121	$X^I$ , 34, 77
$\langle x, y \rangle_L$ , 56	$x \otimes y$ , 31

## Index

---

- Abel–Jacobi map, 60, 62
  - injectivity of —s, 62, 121
- absolute value
  - archimedean —, 68
- algebraic integer, 9
- algebraic scheme, 164
- bidegree
  - of a polynomial, 101–5, 107, 109, 110, 131, 135, 137, 138, 140, 141, 143–45, 147, 163, 164
- centrally symmetric subset, 15
- convex body, 15
- curve, 164
  - algebraically equivalent —s, 51
  - Fermat — of degree  $n$ , 156
- Dedekind domain, 11, 69
- degree
  - of a polynomial, 79, 163
- difference, 23
- Diophantine geometry, 1, 4, 5, 25, 73, 118
- discrete valuation, 13
- discrete valuation ring, 12
- discriminant, 8
  - over  $\mathbb{Q}$ , 10
- $F$ -rational point, 112, 120, 164
- Faltings’s theorem, 73, 117–21, 129, 156, 157
- Fermat property, 157
- field of definition, 41
- formula
  - Jensen’s —, 81
  - product —, 27, 31, 68, 151
  - projection —, 51, 133, 144
  - fractional ideal (of a ring of integers), 12
- Gauss’s lemma, 78, 100, 103, 105, 109
- Gelfond’s inequality, 84
- genus (of a curve), 117, 157
- Grauert’s theorem, 46
- height
  - absolute (logarithmic) Weil —, 29, 32, 42, 84, 149, 156
  - canonical —, 55
  - associated to a line bundle, 36, 38, 149
  - of a global section of a Vojta divisor, 124, 125, 132
  - (logarithmic) — of a point in projective space, 30
  - of a polynomial, 84, 90, 99, 107, 144
  - (logarithmic) — of a vector, 29–31
  - naïve —, 32
  - Néron–Tate —, 55
  - Néron–Tate — pairing, 56, 66, 72, 121
  - positivity of —s, 38
- Hermite–Minkowski theorem, 19, 67, 69
- homomorphism (of group varieties), 44, 45, 55
- index
  - of a formal power series, 85
  - of a global section, 124, 125, 143, 146, 149, 150
  - of a polynomial, 87

- inequality
  - Gelfond's —, 84
  - Vojta's —, 121
- inner product, 14
- integral basis, 10, 18
- Jensen's formula, 81
- Kronecker's theorem, 42
- lattice, 14, 18, 28
  - volume of a —, 15
- Leibniz rule, 114, 116, 150
- lemma
  - Gauss's —, 78, 100, 103, 105, 109
  - rigidity —, 44
  - Roth's —, 88, 89, 91, 92, 143, 145, 146
  - Siegel's — (for  $\mathbb{Z}$ ), 74
  - Siegel's — (for algebraic integers), 76, 141
- length
  - of a polynomial with respect to  $v$ , 77, 90
- line bundle
  - even — (on an abelian variety), 49, 55, 56, 63, 66
  - odd — (on an abelian variety), 49, 55
  - Poincaré —, 61, 65
- local Eisenstein theorem, 112, 146, 153
- Mahler measure, 79
- Minkowski's convex body theorem, 16, 58
- Minkowski's discriminant theorem, 19
- Mordell–Weil theorem, 58, 67, 71, 121
- norm
  - (of an element of an extension field), 6
  - of a Cartier divisor, 98
  - of an invertible sheaf, 95, 96, 98
- Northcott's finiteness theorem, 39, 41, 42, 56, 58, 68, 72, 121, 123
- number field, 9
- Picard group, 164
- place
  - archimedean —, 79, 84, 101, 103–5, 109, 110, 141, 143, 147, 148, 152, 153
  - nonarchimedean —, 31, 78, 84, 100, 103, 105, 109, 143, 147, 148, 151, 153
- polynomial
  - bihomogeneous —, 107, 131, 135–38, 141, 144, 164
- product formula, 27, 68, 151
- projection formula, 51, 133, 144
- $R$ -valued point, 9, 164
- ramification index, 22
- ramified, 22–24, 69
- residue degree, 22, 24
- Riemann–Roch theorem, 133, 134
- rigidity lemma, 44
- ring of integers, 9, 11–13
- Roth's lemma, 88, 89, 91, 92, 143, 145, 146
- seesaw theorem, 45
- Serre's duality theorem, 133
- Siegel's lemma (for  $\mathbb{Z}$ ), 74
- Siegel's lemma (for algebraic integers), 76, 141
- theorem
  - Faltings's —, 73, 117–21, 129, 156, 157
  - Grauert's —, 46
  - Hermite–Minkowski —, 19, 67, 69
  - Kronecker's —, 42
  - local Eisenstein —, 112, 146, 153
  - Minkowski's convex body —, 16, 58
  - Minkowski's discriminant —, 19
  - Mordell–Weil —, 25, 58, 67, 71, 121
  - Northcott's finiteness —, 39, 41, 42, 56, 58, 68, 72, 121, 123
  - Riemann–Roch —, 133, 134
  - seesaw —, 45
  - Serre's duality —, 133
  - of the cube, 47
  - of the square, 50
  - weak Mordell–Weil —, 70, 72
- theorem of the cube, 47
- theorem of the square, 50
- theta divisor, 60, 63, 66, 121
- torsion point, 56
- trace (of an element of an extension field), 6
- form, 8
- unramified, 22–24, 69
- variety
  - abelian —, 43–45, 47, 49–52, 54–56, 61, 69–71

## Index

169

- algebraic —, 164
- geometrically irreducible algebraic —, 43
- group —, 43, 44
- Jacobian —, 48, 59–61, 120
- Picard —, 61
- Vojta divisor, 124, 126, 127, 130, 131, 133
- sequence of polynomials expressing a global section of a —, 132
- Vojta's inequality, 121
- weak Mordell–Weil theorem, 70, 72
- Wronskian of a polynomial, 88