

CAMBRIDGE TRACTS IN MATHEMATICS

General Editors

J. BERTOIN, B. BOLLOBÁS, W. FULTON, B. KRA,  
I. MOERDIJK, C. PRAEGER, P. SARNAK, B. SIMON, B. TOTARO

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**226 The Mordell Conjecture**

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**The Mordell Conjecture**  
**A Complete Proof from Diophantine Geometry**

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## Preface

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This book originated from course notes for “Topics in Algebra” taught by Atsushi Moriawaki to senior undergraduate students and beginning graduate students at Kyoto University in 1996. Shu Kawaguchi, then a graduate student, attended the course.

The purpose of the course was to give a self-contained and detailed proof of the Mordell conjecture (Faltings’s theorem) by following Vojta’s and Bombieri’s papers [5, 29], while touching on several important theorems and techniques from Diophantine geometry.

We have fully revised and expanded the course notes into this book, and some of the explicit and detailed computations presented here may be appearing in the literature for the first time. This book will also provide an introduction to Diophantine geometry.

We assume that the reader is familiar with basic concepts of algebraic geometry and has good knowledge of undergraduate algebra and analysis. Some basics of algebraic number theory are included in Chapter 2.

For the reader who is familiar with the basics of Diophantine geometry, and is interested only in the proof of Faltings’s theorem, we suggest starting from Chapter 5 while referring to Chapter 4. Otherwise, we suggest starting with Chapters 2 and 3 while referring, if necessary, to books on algebraic geometry and algebraic number theory (e.g., [11, 23]), and then reading Chapter 5 while referring to Chapter 4.

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