CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-84507-6 — Mathematical Logic through Python
Yannai A. Gonczarowski , Noam Nisan
Table of Contents
More Information

Contents

Preface *page* xi

0 Introduction and Overview 1
0.1 Our Final Destination: Gödel's Completeness Theorem 2
0.2 Our Pedagogical Approach 4
0.3 How We Travel: Programs That Handle Logic 5
0.4 Our Roadmap 8

Part I Propositional Logic

1 Propositional Logic Syntax 13
1.1 Propositional Formulas 13
1.2 Parsing 18
1.3 Infinite Sets of Formulas 21
1.A Optional Reading: Polish Notations 22

2 Propositional Logic Semantics 24
2.1 Detour: Semantics of Programming Languages 24
2.2 Models and Truth Values 25
2.3 Truth Tables 28
2.4 Tautologies, Contradictions, and Satisfiability 30
2.5 Synthesis of Formulas 31
2.A Optional Reading: Conjunctive Normal Form 33
2.B Optional Reading: Satisfiability and Search Problems 35

3 Logical Operators 41
3.1 More Operators 41
3.2 Substitutions 43
3.3 Complete Sets of Operators 46
3.4 Proving Incompleteness 49

4 Proof by Deduction 53
4.1 Inference Rules 53
4.2 Specializations of an Inference Rule 56
4.3 Deductive Proofs 59

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-84507-6 — Mathematical Logic through Python
Yannai A. Gonczarowski , Noam Nisan
Table of Contents
More Information

4.4 Practice Proving 64
4.5 The Soundness Theorem 66

5 Working with Proofs 69
5.1 Using Lemmas 69
5.2 Modus Ponens 73
5.3 The Deduction Theorem 76
5.4 Proofs by Way of Contradiction 79

6 The Tautology Theorem and the Completeness of Propositional Logic 84
6.1 Our Axiomatic System 84
6.2 The Tautology Theorem 86
6.3 The Completeness Theorem for Finite Sets 92
6.4 The Compactness Theorem and the Completeness Theorem for Infinite Sets 94
6.A Optional Reading: Adding Additional Operators 97
6.B Optional Reading: Other Axiomatic Systems 101

Part II Predicate Logic

7 Predicate Logic Syntax and Semantics 109
7.1 Syntax 110
7.2 Semantics 121

8 Getting Rid of Functions and Equality 129
8.1 Getting Rid of Functions 129
8.2 Getting Rid of Equality 138

9 Deductive Proofs of Predicate Logic Formulas 143
9.1 Example of a Proof 144
9.2 Schemas 145
9.3 Proofs 160
9.4 Getting Rid of Tautology Lines 171

10 Working with Predicate Logic Proofs 178
10.1 Our Axiomatic System 178
10.2 Syllogisms 184
10.3 Some Mathematics 195

11 The Deduction Theorem and Prenex Normal Form 211
11.1 The Deduction Theorem 211
11.2 Prenex Normal Form 215

12 The Completeness Theorem 231
12.1 Deriving a Model or a Contradiction for a Closed Set 236
12.2 Closing a Set 240

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-84507-6 — Mathematical Logic through Python
Yannai A. Gonczarowski , Noam Nisan
Table of Contents
More Information

12.3 The Completeness Theorem 252
12.4 The Compactness Theorem and the "Provability" Version of the Completeness Theorem 253

13 Sneak Peek at Mathematical Logic II: Gödel's Incompleteness Theorem 256
13.1 Complete and Incomplete Theories 256
13.2 Gödel Numbering 258
13.3 Undecidability of the Halting Problem 260
13.4 The Incompleteness Theorem 262

Cheatsheet: Axioms and Axiomatic Inference Rules Used in This Book 266

Index 268

© in this web service Cambridge University Press & Assessment www.cambridge.org