CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-84507-6 — Mathematical Logic through Python
Yannai A. Gonczarowski , Noam Nisan
Index
More Information

Index

∃, *see* existential quantifier
∀, *see* universal quantifier
⊕, *see* xor operator
→, *see* implies operator
$\overline{\&}$, *see* nand operator
$\overline{|}$, *see* nor operator
↔, *see* iff operator
&, *see* and operator
|, *see* or operator
⊢, *see* provability
⊨, *see* entailment
~, *see* not operator
3-CNF, 40

affineness of a Boolean function, 51
and operator
  semantics, 26, 34, 35, 49, 50, 52
  syntax, 14, 31, 34, 98, 104, 112
antecedent, 74
arity
  of a function, 125, 130
  of a relation, 126, 130, 136, 232, 233
  of an operator, 41, 43, 46, 49, 112
assignment (in predicate logic), 124–127, 133, 159, 170, 182
assumption
  of a proof
    in predicate logic, 94, 144, 160, 161, 163, 169, 170, 177, 253, 265
    in propositional logic, 60, 67, 74, 77, 89, 90
  of an inference rule, 53, 54, 60, 74, 77, 78, 80, 83, 92
axiom, *see also* axiomatic system
  in predicate logic, 160, 163, 169, 177, 178, 181, 182, 218, 255, 257, 258, 264
  in propositional logic, 76, 80, 84, 98, 173
axiom schema, *see* schema, *see also* axiom
axiomatic system
  for predicate logic, 178, 180, 218, 231, 250, 253, 255, 257, 258, 263, 265, 266
  for propositional logic, 74, 76, 84, 85, 87, 98, 172, 266, *see also* Hilbert axiomatic system, *see also* schema equivalents of a propositional axiomatic system

binary operator, 14, 41, 112, 225, *see also* arity of an operator
bound occurrence (of a variable name), 120, 127, 216, 222

Cantor's theorem, 252
capturing an assignment by a formula, 86, 89, 101
clause
  conjunctive, 31, 32
  disjunctive, 33, 34, 40
closed set of sentences, 232, 236, 237, 240, 241, 249–252, 265
closure of a set of sentences, *see* closed set of sentences
CNF, 34, 35, 39, 40
CNF theorem, 34, 35
coloring (of a graph), 36–40
compactness theorem
  for predicate logic, 253, 254
  for propositional logic, 95–98, 253, 254
compilation, 133
completeness
  of a proof system
    in predicate logic, 236, 252, 257
    in propositional logic, 84, 93, 94, 97, 257
  of a set of axioms, 257, 258, 263, 265
  of a set of operators, 46, 49–52, 84, 97, 112
completeness theorem
  for predicate logic, 231, 240, 249, 250, 252–255, 257
  for propositional logic, 90, 92–94, 97, 98, 253, 254, 257
computational search, 36, 39, 40
conclusion
  of a proof
    in predicate logic, 144, 160, 161
    in propositional logic, 60, 67, 77
  of an inference rule, 53, 54, 60, 74
conjunction operator, *see* and operator
conjunctive normal form, *see* CNF
consequent, 74
consistency, 80–83, 93–95, 97, 98, 214, 231, 236, 239, 241–245, 248–254, 265
consistency-preserving closure lemma, 241, 251, 252

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-84507-6 — Mathematical Logic through Python
Yannai A. Gonczarowski , Noam Nisan
Index
More Information

constant (in propositional logic), 14, *see also* nullary operator
constant name (in predicate logic)
  as a template (in a schema), 146, 181
  semantics, 122, 125, 232
  syntax, 110, 116, 233, 238, 242–245, 248, 249, 251
contradiction
  semantic, 30, 79, 80, 214
  syntactic, 79, 80, *see also* inconsistency, *see also* proof by way of contradiction
countable, *see* countably infinite
countably infinite, 22, 96, 121, 251–253

D axiom, 76–78, 83, 85, 172
deduction theorem
  for predicate logic, 212, 242
  for propositional logic, 78, 79, 82, 90, 101, 211
derivation tree, 15, 18, 21, 118, 119
diagonalization, 261
disjunction operator, *see* or operator
disjunctive normal form, *see* DNF
DNF, 32, 33, 35, 36, 39, 40, 46
DNF theorem, 32, 33, 46
double implication operator, *see* iff operator

EI axiom schema, 179, 181, 185, 216
encoding an inference rule by an axiom, 74, 86, 92
entailment
  in predicate logic, 169, 170, 254, 257
  in propositional logic, 54, 55, 66, 67, 73, 92, 93, 97
equality relation
  semantics, 126, 138, 139, 141, 179
  syntax, 112, 116, 138, 142, 231, 232, 238
equality-free analog
  of a formula, 138
  of a model, 139–141
  of a set of formulas, 142
ES axiom schema, 179, 181, 185, 216
evaluation
  of a formula, *see* truth value
  of a term (in predicate logic), *see* value
exclusive or operator, *see* xor operator
existential derivation, 192
existential introduction, *see* EI axiom schema
existential quantifier
  semantics, 126, 127, 171, 178, 236
  syntax, 112, 179, 215–217, 233, 237–239, 248, 249, 251, 252
existential simplification, *see* ES axiom schema
existential witness, 233, 245, 248, 249, 251
existentially closed set of sentences, 232, 236, 245, 248, 249, 252
expression (in predicate logic), 121

F constant (nullary operator)
  semantics, 26, 49–52
  syntax, 13, 79, 98
field, 205, 257
first-order logic, *see* predicate logic
first-order predicate logic, *see* predicate logic
formula
  in predicate logic, 112, 115, 116, 119, 121, 126–128, 132–134, 137, 142, 159, 167, 170, 214, 216, 222, 231, 238, 249, 251–254, 258, 259, 260, *see also* sentence
  in propositional logic, 13, 15, 16, 18, 20–22, 25, 26, 29, 30, 32–36, 39, 40, 49, 53, 54, 56, 60, 80, 83, 86, 87, 92–97, 167
`Formula` class
  of the `predicates` package (predicate logic), 112, 115
  of the `propositions` package (propositional logic), 15, 16
free instantiation, 198
free occurrence (of a variable name), 120, 126, 127, 170, 171, 181, 216, 222
free variable name, 120, 126, 133, 159, 170, 171, 180, 212, 214, 216, 231, 238, 254
`@frozen` decorator, 16, 115
function invocation
  semantics, 125, 130, 133
  syntax, 110, 129, 132, 133, 137, 231, 232, 238
function name, 110, 122, 125, 130, 131, 133
function-free analog
  of a formula, 132–134
  of a model, 130, 131, 133
  of a set of formulas, 137

Gödel numbering, 258, 259
group, 195, 256, 257

$\mathcal{H}$, *see* Hilbert axiomatic system
halting (by a program), 259–264
halting problem, 261–264
Henkin constant, *see* witnessing constant name
Hilbert axiomatic system, 86, 89, 92–94, 97, 99, 101, 103, 104, 177
hypothetical syllogism, 78, 101

I0 axiom, 65, 76–78, 80, 83, 85, 172
I1 axiom, 76–78, 83, 85, 87, 172
I2 axiom, 80, 81, 83, 85, 87, 172
iff operator
  semantics, 42, 50, 51
  syntax, 42
implication operator, *see* implies operator
implies operator
  semantics, 26, 27, 49, 50, 73, 170
  syntax, 14, 74, 77, 78, 87, 112
incompleteness, *see* completeness of a set of axioms
incompleteness theorem, 258, 261–263
inconsistency, *see* consistency
induction axiom schema, 206
inference (in predicate logic), 169, 170, 182, 189

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-84507-6 — Mathematical Logic through Python
Yannai A. Gonczarowski , Noam Nisan
Index
More Information

inference rule
  in predicate logic, 160, 164, 166, 264
  in propositional logic, 53–56, 60, 66, 67, 71, 72, 74, 76, 78, 92, 189
`InferenceRule` class, 53
instance (of a schema), 145–148, 152, 159, 161, 172, 173, 181, 245
instantiation (of a schema), *see* instance
interpretation, 122, 130, 136, 138, 232

lemma, 69, 71, 72
lemma theorem, 71, 72, 77, 78, 83, 86, 99, 101, 103, 104
Löwenheim–Skolem theorem, 252

ME axiom schema, 181, 185, 196, 200, 202
meaning of equality axiom schema, *see* ME axiom schema
model
  in predicate logic, 122, 123, 125, 126, 128, 130, 131, 133, 137, 140–142, 159, 169–171, 182, 231, 232, 236, 237, 239, 249, 252–254, 260, 265
  in propositional logic, 25–30, 32, 34, 36, 37, 54, 55, 67, 73, 86, 87, 89, 90, 94–98, 101, 170, 254
`Model` class, 123
modus ponens, *see* MP inference rule
monotonicity of a Boolean function, 50
MP inference rule, 73, 74, 77, 78, 80, 83, 85, 86, 160, 161, 164, 169–171, 173, 177, 190, 245
multiplexer operator, *see* mux operator
mux operator, 43, 49, 50, 52

N axiom, 82, 83, 85, 172
n-ary function, *see* arity of a function
n-ary operator, *see* arity of an operator
n-ary relation, *see* arity of a relation
nand operator
  semantics, 42, 49
  syntax, 42
negation operator, *see* not operator
NI axiom, 85, 87, 172
NN axiom, 85, 87, 172
non-implication operator, 52
nor operator
  semantics, 42, 49
  syntax, 42
not operator
  semantics, 26, 30, 35, 49, 51
  syntax, 13, 31, 33, 40, 79, 80, 86, 87, 103, 112, 214, 224, 233, 236, 239, 241, 242, 251, 254
notation
  infix, 22, 115
  polish, 22, 115
  postfix, *see* notation, reverse polish
  prefix, *see* notation, polish
  reverse polish, 22, 115

NP-completeness, 39, 40, 79
nullary operator, 41, *see also* arity of an operator

operator, 14, 41, 43, 46, 49, 52, 97, 112, 238
or operator
  semantics, 26, 32, 35, 50, 52
  syntax, 14, 32, 33, 40, 98, 104, 112

parametrized formula, 147, 149, 178–181, 217
parsing, 15, 18, 20, 117, 262
Peano arithmetic, 206, 258–260, 264, 265
predicate logic, 109
prefix-free, 20, 116
prenex normal form, 215, 216, 236, 237, 241, 249, 251
prenex normal form theorem, 216, 217, 230, 231, 252
primitive sentence, 232, 233, 236, 238, 239, 241, 242, 249
primitively closed set of sentences, 232, 236, 241, 242, 249, 252
program, 258–260, 262–264
proof, *see also* provability
  in predicate logic, 144, 160, 161, 169, 171, 172, 177, 182, 245, 251, 253, 255, 258, 262–265
  in propositional logic, 60, 61, 66, 67, 69, 70, 74, 77, 87, 89, 90, 92, 94, 171, 172
proof by way of contradiction, 79, 80, 82, 83, 213, 214
`Proof` class
  of the `predicates` package (predicate logic), 161, 182
  of the `propositions` package (propositional logic), 61, 69
propositional formula, *see* formula in propositional logic
propositional logic, 13
propositional skeleton
  of a predicate-logic formula, 167, 170–173, 180, 240, 245
  of an inference (in predicate logic), 189
provability, *see also* proof
  in predicate logic, 169, 170, 177, 182, 214, 216, 246, 250, 254, 257, 264
  in propositional logic, 60, 66, 71, 72, 74, 77, 78, 80, 82, 83, 85, 89, 90, 92, 93, 97, 101, 172
`Prover` class, 182

quantification
  existential, *see* existential quantifier
  universal, *see* universal quantifier
Quine arrow operator, *see* nor operator

R axiom, 85, 90, 172
reduction, 39, 40
redundance of equality theorem, 142, 231, 232, 238
redundance of functions theorem, 137, 231, 232, 238
reflexivity
  axiom schema, *see* RX axiom schema
  of equality, 138

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-108-84507-6 — Mathematical Logic through Python
Yannai A. Gonczarowski , Noam Nisan
Index
More Information

relation invocation
  syntax, 112
relation corresponding to function, 130, 131, 133, 135, 136
relation invocation
  semantics, 126, 130, 232
  syntax, 133, 136, 233, 238
relation name, 112, 122, 126, 130, 131, 133, 138, 251
  as a template (in a schema), 147, 181
Russell's paradox, 209
RX axiom schema, 179, 181, 185, 196, 200

satisfiability, 30, 36, 37, 39, 40
schema, 145–147, 159, 169, 177–182, 231, 250, 252, 253, 255, 258, 264
`Schema` class, 145–147
schema equivalents of a propositional axiomatic system, 171, 172, 177, 178, 240, 250, 266
scope of quantification, 112, 127, 216
sentence, 180, 212, 231, 236–239, 241, 242, 244, 248, 249, 251, 254, 257, 260, 262, 265
Sheffer stroke operator, *see* nand operator
Skolem's paradox, 252
soundness
  of a predicate-logic formula, 159, 170
  of a propositional formula, *see* tautology in propositional logic
  of a schema, 159, 170, 172, 181
  of an inference (in predicate logic), 169, 170, 182
  of an inference rule, 54, 55, 66, 67, 73, 74, 76, 80, 82, 85, 92–94, 98, 190
soundness theorem
  for predicate logic, 170, 172, 182, 231, 249, 252–254, 257, 264
  for propositional logic, 66, 67, 84, 92, 93, 98, 254
  of proofs by way of contradiction
    for predicate logic, 214, 254
    for propositional logic, 82, 83, 93, 97
specialization (of an inference rule), 56, 57, 60, 66, 67, 71, 173
specialization provability lemma, 70, 71
specialization soundness lemma, 66, 67, 159
syllogism, 144, 184, 187, 191, *see also* hypothetical syllogism
symmetry of equality, 138, 196

T constant (nullary operator)
  semantics, 26, 50, 51
  syntax, 13, 98
T-preservation, 50
tautological implication, 188, 190, 191, 214, 240
tautology
  in predicate logic, 160, 161, 167, 169–172, 177, 180, 214, 231, 240, 245, 250
  in propositional logic, 30, 36, 39, 55, 73, 90, 92, 160, 167, 171, 172
tautology theorem
  predicate-logic version, 176, 177, 240
  propositional-logic version, 90, 92, 143, 172
template (in a schema), 145–147
term (in predicate logic), 110, 111, 116, 118, 121, 125
`Term` class, 110, 111
ternary operator, 43, *see also* arity of an operator
testing, 7, 8
transitivity of equality, 138, 202
truth table, 29, 32–35, 43, 46, 49, 84
truth value
  of a predicate-logic formula, 126, 128, 133, 170, 180, 182, 240
  of a propositional formula, 25–30, 32–35, 54, 67, 89, 97, 101, 170

UG inference rule, 160, 161, 166, 169–171, 180, 212, 214, 232, 245, 250
UI axiom schema, 178, 180, 181, 185, 216, 232, 249, 254
unary operator, 14, 41, 112, *see also* arity of an operator
undecidability theorem, 261–263
unique readability theorem
  of predicate-logic formulas, 119
  of propositional formulas, 21
  of terms (in predicate logic), 118
uniquely named variables, 222
universal generalization, *see* UG inference rule
universal instantiation
  axiom schema, *see* UI axiom schema
  of a predicate-logic formula, 186, 233, 243, 244, 249, 251
universal quantifier
  semantics, 126, 127, 170, 171, 178, 180, 182, 236
  syntax, 112, 144, 178, 179, 215–217, 232, 233, 237–239, 244, 249, 251, 252, 254
universal simplification, *see* US axiom schema
universally closed set of sentences, 232, 236, 243, 244, 249, 252
universe (of a model), 122, 130, 141, 232
US axiom schema, 179, 181, 185, 212–214, 216

value (of a term in predicate logic), 125, 139, 182
variable name
  in predicate logic
    as a template (in a schema), 146, 181
    semantics, 125–127, 133, 182
    syntax, 110, 112, 116, 133, 217, 222, 238
  in propositional logic
    semantics, 25, 26, 28, 29, 32–37, 86, 97, 170
    syntax, 13, 20, 22, 31, 33, 40, 56, 57, 86, 96, 167, 189

witnessing constant name, 245, 248

xor operator
  semantics, 42, 51, 52
  syntax, 42

ZF, 210
ZFC, 210, 252, 253, 255, 258

© in this web service Cambridge University Press & Assessment www.cambridge.org