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LECTURES ON RANDOM LOZENGE TILINGS

Over the past 25 years, there has been an explosion of interest in the area of random tilings. The first book devoted to the topic, this timely text describes the mathematical theory of tilings. It starts from the most basic questions (which planar domains are tileable?) before discussing advanced topics about the local structure of very large random tessellations. The author explains each feature of random tilings of large domains, discussing several different points of view and leading on to open problems in the field. The book is based on upper-division courses taught to a variety of students, but it also serves as a self-contained introduction to the subject.

Test your understanding with the exercises provided, and discover connections to a wide variety of research areas in mathematics, theoretical physics, and computer science, such as conformal invariance, determinantal point processes, Gibbs measures, high-dimensional random sampling, symmetric functions, and variational problems.

Vadim Gorin is a faculty member at the University of Wisconsin–Madison and a member of the Institute for Information Transmission Problems at the Russian Academy of Sciences. He is a leading researcher in the area of integrable probability, and has been awarded several prizes, including the Sloan Research Fellowship and the Prize of the Moscow Mathematical Society.

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Preface

These are lecture notes for a one-semester class devoted to the study of random tilings. It was 18.177 taught at the Massachusetts Institute of Technology during the spring of 2019. The brilliant students who participated in the class,[1] Andrew Ahn, Ganesh Ajjanagadde, Livingston Albritten, Morris (Jie Jun) Ang, Aaron Berger, Evan Chen, Cesar Cuenca, Yuzhou Gu, Kaarel Haenni, Sergei Korotkikh, Roger Van Peski, Mehtaab Sawhney, and Mihir Singhal, provided tremendous help in typing the notes.

Additional material was added to most of the lectures after the class was over. Hence, when using this review as a textbook for a class, one should not expect to cover all the material in one semester; something should be left out.

I would like to thank my wife, Anna Bykhovskaya, for her advice, love, and support. I also thank Amol Aggarwal, Alexei Borodin, Richard Kenyon, Christian Krattenthaler, Arno Kuijlaars, Igor Pak, Jiaming Xu, Marianna Russkikh, and Semen Shlosman for their useful comments and suggestions. I am grateful to Christophe Charlier, Maurice Duits, Sevak Mkrtchyan, and Leonid Petrov for the help with the simulations of random tilings.

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[1] In alphabetical order by last name.