THE NUCLEAR COOPER PAIR

This monograph presents a unified theory of nuclear structure and nuclear reactions in the language of quantum electrodynamics-Feynman diagrams. It describes how two-nucleon transfer reaction processes can be used as a quantitative tool to interpret experimental findings with the help of computer codes and nuclear field theory. Making use of Cooper pair transfer processes, the theory is applied to the study of pair correlations in both stable and unstable exotic nuclei. Special attention is given to unstable, exotic halo systems, which lie at the forefront of the nuclear physics research being carried out at major laboratories around the world. This volume is distinctive in dealing in both nuclear structure and reactions and benefits from comparing the nuclear field theory with experimental observables, making it a valuable resource for incoming and experienced researchers who are working in nuclear pairing and using transfer reactions to probe them.

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THE NUCLEAR COOPER PAIR
STRUCTURE AND REACTIONS

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Preface

The elementary modes of nuclear excitation are vibrations and rotations, single-particle motion, and pairing vibrations and rotations. The reactions that specifically probe them are inelastic scattering plus Coulomb excitation and single- and two-particle transfer processes, respectively.

The interweaving of the elementary modes of excitation leads to the renormalization of energy, radial wavefunction, and particle content of the single-particles. It also leads to the renormalization of energy, width, and collectivity of vibrations and rotations. This implies renormalization of transition densities and formfactors, as well as of deformation (order) parameters, both in 3D and in gauge space. A consequence is the emergence of a variety of properties, such as generalized rigidity and long-range correlations in connection with collective pairing states, implying, for example, that pair transfer is dominated by the successive tunneling of entangled nucleons and, consequently, the need to go beyond lowest-order distorted wave Born approximation, that is, to second-order DWBA.

Within this context one can posit that nuclear structure (bound) and reactions (continuum) are but two aspects of the same physics. Even more concerning is the study of light exotic halo nuclei, in which case the distinction between bound and continuum states is, to a large extent, blurred. This is also the reason why these two aspects of nuclear physics are treated in the present monograph on equal footing, within the framework of a unified nuclear field theory of structure and reactions $(\mathrm{NFT})_{(\mathrm{s+r})}$.

This theory provides the (graphical) rules to diagonalize in a compact and economic way the nuclear Hamiltonian for both bound and continuum states. It does so in terms of Feynman diagrams, which describe the coupling of elementary modes of excitation, correcting for the overcompleteness of the basis (structure) and for the nonorthogonality of the scattering states (reaction), as well as for Pauli principle violation. The outcome connects directly with observables: absolute reaction cross sections and decay probabilities.

$(\mathrm{NFT})_{(\mathrm{s+r})}$ focuses on the scattering amplitudes that determine the absolute cross sections for the variety of physical processes, involving also those in which (quasi) bosons and fermions are created or annihilated, connecting such processes to form-factors and transition densities, processes where one set of particles with given energies, momenta, angular momenta, etc. go in and another group (or the same) comes out.

Pairing vibrations and rotations, closely connected with nuclear superfluidity, are paradigms of quantal nuclear phenomena. They play an important role within the field of nuclear structure. It is natural that two-nucleon transfer plays a similar role concerning the probing of the structure of the nucleus.

At the basis of fermionic pairing phenomena[1] one finds Cooper pairs[2] – weakly bound, very extended, strongly overlapping (quasi-) bosonic entities made out of pairs of nucleons dressed by collective vibrations and interacting through the exchange of these vibrations as well as through the bare NN-interaction. Cooper pairs change, under certain conditions,[3] the statistics of the nuclear stuff around the Fermi surface and, condensing, the properties of nuclei close to their ground state. They also display a rather remarkable mechanism of tunneling between target and projectile in direct two-nucleon transfer reactions.

Cooper pair partners, being weakly bound ($\ll \epsilon_F$, Fermi energy), are correlated over distances (correlation length ξ) much larger than nuclear dimensions ($\gg R$, nuclear radius). On the other hand, Cooper pairs – building blocks of the so-called abnormal (pair) density – are forced to be confined within regions in which normal, single-particle density is present, that is, within nuclear dimensions. Said differently, the mean field acts on Cooper pairs as a strong external field, distorting their spatial structure.

The correlation length paradigm comes into evidence, for example, when two nuclei are set into weak coupling in a direct nuclear reaction with distance of closest approach $D_0 \lesssim \xi$. In such a case, the partner nucleons of a Cooper pair have a finite probability to be confined each within the mean field of a different nucleus, equally well pairing correlated than when both nucleons are in the same nucleus. It is then natural that a Cooper pair can also tunnel between target and projectile, equally well correlated, through simultaneous rather than through successive transfer processes. Because of the weak binding of the Cooper pair, let alone the fact that tunneling probability falls off exponentially with increasing mass, successive is the dominant transfer process.

1 Bardeen et al. (1957a,b).
2 Cooper (1956).
3 Value of the intrinsic excitation energy, rotational frequency, and distance of closest approach (D_0) in Cooper pair tunneling between superfluid nuclei in nuclear reactions, smaller than the correlation length (ξ).

Although one does not expect supercurrents in nuclei, one can study long-range pairing correlations in terms of individual quantal states and of the tunneling of single Cooper pairs. Such (weak coupling) Cooper pair transfer reminds the tunneling mechanism of electronic Cooper pairs across a barrier (e.g., a dioxide layer of dimensions much smaller than the correlation length) separating two low-temperature, metallic superconductors and known as a Josephson junction.[4]

In the nuclear time-dependent junction transiently established in direct two-nucleon transfer process, only one or sometimes none of the two weakly interacting nuclei is superfluid. On the other hand in nuclei, a paradigmatic example of finite quantum many-body systems (FQMBS), zero-point fluctuations (ZPF) in general, and those associated with pair addition and pair subtraction modes known as pairing vibrations in particular, are, as a rule, much stronger than in condensed matter. Thus, pairing correlations based on even a single Cooper pair can lead to distinct effects in two-nucleon transfer processes.

Nucleonic Cooper pair tunneling has played and is playing a central role in the probing of these subtle quantal phenomena, both in the case of light exotic nuclei and for medium and heavy nuclei lying along the stability valley. They have been instrumental in shedding light on the subject of pairing in nuclei at large, and on nuclear superfluidity in particular. Consequently, and as already said, the subject of two-nucleon transfer reactions occupies a central place in the present monograph, both concerning the conceptual and the computational aspects of the description of nuclear pairing, as well as regarding the quantitative confrontation of the theoretical results with the experimental findings, in terms of absolute differential cross sections.

Concerning exotic nuclei, experimental studies carried out at TRIUMF, Vancouver (Canada),[5] have provided the basis for what can be considered a nuclear embodiment[6] of the Cooper pair model: a pair of fermions (nucleons N) moving in time reversal states on top of the Fermi surface and interacting through the short-range, bare NN- and the long range, induced-pairing interaction.[7] This last one results from the exchange of a long-wavelength dipole vibration (quasi-boson), leading to an extended, weakly bound system.

Regarding medium heavy nuclei lying along the stability valley, studies of heavy ion reactions between superfluid nuclei carried out at energies around and below the Coulomb barrier at the National Laboratory of Legnaro (Italy)[8] have

4 Josephson (1962); Anderson (1964b).
5 Tanihata et al. (2008).
6 Barranco et al. (2001); Potel et al. (2010).
7 Fröhlich (1952); Bardeen and Pines (1955).
8 Montanari et al. (2014).

provided a measure of the neutron Cooper pair size (mean square radius or correlation length[9]).

In the present monograph, interdisciplinarity[10] is employed as a tool to attack concrete nuclear problems and, making use of the unique laboratory provided by the atomic nucleus,[11] to shed light on condensed matter results in terms of analogies involving individual, quantal states and tunneling of single Cooper pairs.

Because of the central role the interweaving of the variety of elementary modes of nuclear excitation plays in nuclear superfluidity, the study of Cooper pair tunneling in nuclei requires a consistent description of nuclear structure in terms of dressed quasiparticles and, making use of the resulting renormalized wavefunctions (formfactors), of single-nucleon transfer processes.[12] This is similar to the situation encountered in superconductors, in connection with strongly renormalized systems,[13] studied through one-electron tunneling experiments.[14]

In the present monograph the general physical arguments and technical computational details concerning the calculation of absolute one-and two-nucleon transfer differential cross sections within the framework of DWBA, making use of state-of-the-art NFT structure input, are discussed in detail.

As a result of this approach, it is expected that both theoretical and experimental nuclear practitioners can use the present monograph at profit. To help this use, the basic nuclear structure formalism, in particular that associated with single-particle and collective motion in both normal and superfluid nuclei, is economically introduced through general physical arguments. This is also in keeping with the availability in the current literature of detailed discussions of the corresponding material. Within this context, the monographs *Nuclear Superfluidity* by Brink and Broglia and *Oscillations in Finite Quantum Systems* by Bertsch and Broglia, published also by Cambridge University Press, can be considered companion volumes to the present one. This volume shares with those a similar aim: to provide a broad physical view of central issues in the study of finite quantal many-body nuclear systems accessible to motivated students and practitioners. However, neither the present one nor the other two are introductory texts. In particular, in the present one, an attempt at unifying structure and reactions is made. On the other hand, unifying discrete (mainly structure) and continuum (mainly reactions) configuration spaces implies that one will be dealing with those structure results that can be tested

[9] Potel et al. (2021).
[10] Quoting de Gennes (1974): "what a theorist can and should systematically introduce is comparison with other fields."
[11] Broglia (2020) (overview).
[12] In other words, one recognizes the difficulties of extracting spectroscopic factors from experiment, in terms of single-particle transfer cross sections calculated making use of mean field wavefunctions.
[13] Eliashberg (1960).
[14] Giaever (1973).

by means of experiment, a fact that makes the subject of the present monograph a chapter of quantum mechanics, and thus open to a wide range of practitioners.[15]

Concerning the notation, we have divided each chapter into sections. Each section may, in turn, be broken down into subsections. Equations and figures are identified by the number of the chapter and that of the section. Thus (5.1.33) labels the thirty-third equation of Section 1 of Chapter 5. Similarly, "Fig. 5.1.2" labels the second figure of Section 1 of Chapter 5. Concerning the appendices, they are labeled by the chapter number and by a Latin letter in alphabetical order, e.g., App. 2.A, App. 2.B. Concerning equations and figures, a sequential number is added. Thus (2.B.2) labels the second equation of Appendix 2.B of Chapter 2, while "Fig. 2.B.1" labels the first figure of Appendix 2.B of Chapter 2. References are called in terms of the author's surname and publication year and are found in alphabetic order in the bibliography at the end of the monograph.

A methodological approach used in the present monograph concerns a certain degree of repetition. Similar but not the same issues are dealt with more than once using different but equatable terminologies. This approach reflects the fact that useful concepts like reaction channels or correlation length, let alone elementary modes of excitation, are easy to understand but difficult to define.[16] This is because their validity is not exhausted by a single perspective. But even more important, it is because their power in helping at connecting[17] seemingly unrelated results and phenomena is difficult to fully appreciate the first time around, spontaneous symmetry breaking and associated emergent properties providing an example of this fact.

[15] Within this context let us mention the intimately correlated subjects of Random Phase Approximation (RPA) and Particle Vibration Coupling (PVC) not found in a fourth-year curriculum. They are explained and referred to in a number of places throughout the present monograph, starting from a pedestrian level and for both surface (particle-hole) and pairing (particle-particle and hole-hole) vibrations (Sect. 1.2, Fig. 1.2.3 (inset) and Sect. 1.3), and then extended to include further details and facets (see Sects. 1.5, 1.6, and 2.3, Fig. (2.3.1), Sect. 2.3.1 (Fig. 2.3.9), App. 2.B, and Sect. 3.1 (Fig. 3.1.6)). Furthermore, in the case of RPA of pairing vibrations around the closed shell system ^{208}Pb, one provides in Sect. 3.5 detailed documentation of the numerical calculation of the associated wavefunctions (X- and Y-amplitudes) at the level of an exercise in a fourth-year course. A similar situation is encountered in connection with the subject of the Distorted Wave Born Approximation (DWBA), again a subject not found in fourth-year curricula. It is treated at the pedestrian level in Sects. 5.6 and 6.4 in connection with one-particle and two-particle transfer reactions, respectively. And, once more in full detail, without eschewing complexities, but again in the style of an example exercise in Sects. 5.1 and 6.2 (within this context we quote, "Sometimes one has to say difficult things, but one ought to say them as simply as one knows how" (G. H. Hardy, *A mathematician's apology*, Cambridge University Press, Cambridge (1969)).

[16] "Gentagelsen den er virkeligheden, og Tilværelsens Alvor" (Repetition is the reality and the seriousness of life: S. Kierkegaard Gjentagelsen (1843)).

[17] "The concepts and propositions get 'meaning' viz. 'content,' only through their connection with sense-experiences. . . . The degree of certainty with which this connection, viz., intuitive combination, can be undertaken, and nothing else, differentiates empty fantasy from scientific 'truth.'. . . A correct proposition borrows its 'truth' from the truth-content of the system to which it belongs" (A. Einstein, Autobiographical notes, in *Albert Einstein*, Ed. P. A. Schilpp, Vol I, Harper, New York (1951) p. 13.).

Throughout, a number of footnotes are found. This is in keeping with the fact that footnotes can play a special role within the framework of an elaborated presentation. In particular, they are useful to emphasize relevant issues in an economic way. Being outside the main text, they give the possibility of stating eventual important results, without the need of elaborating on the proof, but referring to the corresponding sources. Within this context, and keeping the natural distances, one can mention that in the paper in which Born[18] introduces the probabilistic interpretation of Schrödinger's wavefunction, the fact that this probability is connected with its [modulus] squared and not with the wavefunction itself is only referred to in a footnote.

A large fraction of the material contained in this monograph has been the subject of the lectures of the fourth-year course Nuclear Structure Theory, which RAB delivered throughout the years at the Department of Physics of the University of Milan, as well as at the Niels Bohr Institute (Copenhagen) and at Stony Brook (State University of New York). Part of it was also presented by the authors in the course Nuclear Reactions held at the PhD School of Physics of the University of Milan.

GP wants to thank the tutoring of Ben Bayman concerning specific aspects of two-particle transfer reactions. Discussions with Ian Thompson and Filomena Nunes on a variety of reaction subjects are gratefully acknowledged. RAB acknowledges the essential role the collaboration with Francisco Barranco and Enrico Vigezzi has played concerning the variety of nuclear structure aspects of the present monograph. His debt to the late Aage Winther regarding the reaction aspects of it is difficult to express in words. The overall contributions of Daniel Bès, Ben Bayman and the late Pier Francesco Bortignon are only too explicitly evident throughout the text and constitute a daily source of inspiration. GP and RAB have received important suggestions and comments regarding concrete points and the overall presentation of the material discussed below from Ben Bayman, Pier Francesco Bortignon, Willem Dickhoff, Vladimir Zelevinsky, and the late David Brink, who are here gratefully acknowledged. We are especially beholden to Elena Litvinova and Horst Lenske for much constructive criticism and many suggestions.

[18] Born (1926). Within this context, it is of note that the extension of Born probabilistic interpretation to the case of many-particle systems is also found in a footnote (Pauli (1927)), footnote on p. 83 of this reference; see Pais (1986)).