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Introduction

Toute théorie physique est fondée sur l'analogie qu'on établi entre des choses malconnues et des choses simples.

Every physical theory is based on the analogy which one establishes between things not well known and things that are simple.

Simone Weil

1.1 Views of the Nucleus

In the atom, the nucleus provides the Coulomb field in which negatively charged electrons (-e) move independently of each other in single-particle orbitals. The filling of these orbitals explains Mendeleev's periodic table. Thus the valence of the chemical elements as well as the particular stability of the noble gases associated with the closing of shells (2(He), 10(Ne), 18(Ar), 36(Kr), 54(Xe), 86(Ra)). The dimension of the atom is measured in angstroms (Å=10⁻⁸cm) and typical energies in eV, the electron mass being $m_e \approx 0.511$ MeV (MeV=10⁶eV).

The atomic nucleus is made out of positively charged protons (+e) and of (uncharged) neutrons, nucleons, of mass $\approx 10^3$ MeV ($m_p = 938.3$ MeV, $m_n = 939.6$ MeV). Nuclear dimensions are of the order of a few fermis (fm= 10^{-13} cm). The stability of the atom is provided by a source external to the electrons, namely, the atomic nucleus. On the other hand, this system is self-bound as a result of the strong interaction of range $a_0 \approx 0.9$ fm and strength $v_0 \approx -100$ MeV that acts among nucleons.

1.1.1 The Liquid Drop and the Shell Models

While most of the atom is empty space, the density of the atomic nucleus is conspicuous ($\rho = 0.17$ nucleon/fm³). The "closed packed" nature of this system implies, a priori, a short mean free path λ as compared to nuclear dimensions. This can be estimated from classical kinetic theory $\lambda \approx (\rho \sigma)^{-1} \approx 1$ fm, where

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 $\sigma \approx 2\pi a_0^2$ is the nucleon–nucleon cross section. It seems, then, natural to liken the atomic nucleus to a liquid drop.¹ This picture of the nucleus provided the framework to describe the basic features of the fission process.²

The leptodermic properties of the atomic nucleus are closely connected with the semi-empirical mass formula:³

$$m(N, Z) = (Nm_n + Zm_p) - \frac{1}{c^2}B(N, Z), \qquad (1.1.1)$$

the binding energy being

$$B(N,Z) = \left(b_{vol}A - b_{surf}A^{2/3} - \frac{1}{2}b_{sym}\frac{(N-Z)^2}{A} - \frac{3}{5}\frac{Z^2e^2}{R_c}\right),\qquad(1.1.2)$$

where A = N + Z is the mass number, sum of the number of neutrons (N) and of protons (Z). The first term is the volume energy representing the binding energy in the limit of large A, for N = Z and in the absence of the Coulomb interaction ($b_{vol} \approx 15.6 \text{ MeV}$). The second term represents the surface energy, where

$$b_{surf} = 4\pi r_0^2 \gamma. \tag{1.1.3}$$

The nuclear radius is written as $R = r_0 A^{1/3}$, with $r_0 = 1.2$ fm, the surface tension energy being $\gamma \approx 0.95$ MeV/fm². The third term in (1.1.2) is the symmetry term, which reflects the tendency toward stability for N = Z, with $b_{sym} \approx 50$ MeV. The symmetry energy can be divided into a kinetic and a potential energy part. A simple estimate of the kinetic energy part can be obtained by making use of the Fermi gas model, which gives $(b_{sym})_{kin} \approx (2/3)\epsilon_F \approx 25$ MeV ($\epsilon_F \approx 36$ MeV). Consequently,

$$V_1 = (b_{sym})_{pot} = b_{sym} - (b_{sym})_{kin} \approx 25 \text{ MeV}.$$
 (1.1.4)

The last term of (1.1.2) is the Coulomb energy corresponding to a uniformly charged sphere of radius $R_c = 1.25 A^{1/3}$ fm.

When, in a heavy ion reaction, two nuclei come within the range of the nuclear forces, the Coulomb trajectory of relative motion will be changed by the attraction that will act between the nuclear surfaces. This surface interaction is a fundamental quantity in heavy ion reactions. Assuming two spherical nuclei at a relative distance $r_{aA} = R_a + R_A$, where R_a and R_A are the corresponding half-density radii, the (maximum) force acting between the two surfaces is

$$\left(\frac{\partial U_{aA}^{N}}{\partial r}\right)_{r_{aA}} = 4\pi\gamma \frac{R_{a}R_{A}}{R_{a}+R_{A}}.$$
(1.1.5)

¹ Bohr and Kalckar (1937).

² Meitner and Frisch (1939); Bohr and Wheeler (1939).

³ Weizsäcker (1935).

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Figure 1.1.1 (a) Nucleon-nucleon (NN) interaction in a scattering experiment; emergent properties (collective nuclear modes). (b) Assembly of nucleons condensing into drops of nuclear matter displaying emergent properties, examples of which are shown in (c) and (e). (c) Anelastic heavy ion reaction $a + A \rightarrow a + A^*$ setting the nucleus A into an octupole surface oscillations (d). In inset (I) the time-dependent nuclear plus Coulomb field associated with the reaction (c) is represented by a cross followed by a dashed line, while the wavy line labeled λ describes the propagation of the $\lambda^{\pi} = 3^{-}$ surface vibration schematically shown in (d), time running upward. (e) The (weakly) quadrupole deformed nucleus ²²³Ra can rotate as a whole with a moment of inertia considerably smaller than the rigid moment of inertia, a fact intimately connected with the role played by pairing in nuclei. The role becomes overwhelming in the phenomenon of exotic decay displayed in (f) in which the nuclear surface zero-point fluctuations (quadrupole $(\lambda = 2)$, octupole $(\lambda = 3)$, etc.) can get, with a small but finite probability $(P \approx 10^{-10})$, spontaneously in phase and produce a neck-in (saddle conformation), leading eventually to the (exotic) decay mode ${}^{223}Ra \rightarrow {}^{209}Pb + {}^{14}C$, as experimentally observed (g) (Rose and Jones (1984); see Brink and Broglia (2005), chapter 7 and refs. therein). As correctly explained in Matsuyanagi et al. (2013) for vibrations in general, and valid also in the case of the ZPF leading to the saddle (neck-in) configuration, such fluctuations are associated with genuine quantum vibrations (where superfluidity and shell structure play a central role), and thus are essentially different in character from surface oscillations of a classical liquid drop. The intimate connection between pairing and collective vibrations reveals itself through the inertial masses governing the collective kinetic energies.

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This result allows for the calculation of the ion–ion (proximity) potential, which, supplemented with a position-dependent absorption, can be used to accurately describe heavy ion reactions.⁴ In such reactions, not only elastic processes are observed, but also anelastic reactions in which one or both surfaces of the interacting nuclei are set into vibration (Fig. 1.1.1).

The restoring force parameter of the leptodermous system associated with surface oscillations of multipolarity λ is

$$C_{\lambda} = (\lambda - 1)(\lambda + 2)R_0^2\gamma - \frac{3}{2\pi}\frac{\lambda - 1}{2\lambda + 1}\frac{Z^2e^2}{R_c},$$
 (1.1.6)

where the second term corresponds to the contribution of the Coulomb energy to C_{λ} . Assuming the flow associated with surface vibrations to be irrotational, the associated inertia for small amplitude oscillations is

$$D_{\lambda} = \frac{3}{4\pi} \frac{1}{\lambda} AmR^2, \qquad (1.1.7)$$

the energy of the corresponding mode being

$$\hbar\omega_{\lambda} = \hbar \sqrt{\frac{C_{\lambda}}{D_{\lambda}}}.$$
(1.1.8)

The label λ stands for the angular momentum of the vibrational mode. Furthermore, the vibrations can be characterized by the parity quantum number $\pi = (-1)^{\lambda}$ and the third component of λ , denoted μ . Aside from λ , μ , surface vibrations can also be characterized by an integer n (= 1, 2, ...), an ordering number indicating increasing energy. For simplicity, a single common label α will also be used.

A picture apparently antithetic to that of the liquid drop, the shell model, emerged from the study of experimental data, plotting them against either the number of protons (atomic number) or the number of neutrons in nuclei, rather than against the mass number. One of the main nuclear features that led to the development of the shell model was the study of the stability and abundance of nuclear species and the discovery of what are usually called magic numbers.⁵ What makes a number magic is that a configuration of a magic number of neutrons, or of protons, is unusually stable whatever the associated number of other nucleons is.⁶

The strong binding of a magic number of nucleons and weak binding for one more reminds one of the results concerning the atomic stability of rare gases. In the nuclear case, the spin–orbit coupling plays an important role, as can be seen

⁴ See, e.g., Broglia and Winther (2004) p. 110, and refs. therein.

⁵ Elsasser (1933); Mayer (1948); Haxel et al. (1949).

⁶ Mayer (1949); Mayer and Teller (1949).

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Figure 1.1.2 Sequence of levels of the harmonic oscillator potential labeled with the principal oscillator quantum number $(N(\hbar\omega) = 0(\hbar\omega), 1(\hbar\omega), 2(\hbar\omega), \ldots$, the parity being $\pi = (-1)^N$). The next column shows the splitting of major shell degeneracies obtained using a more realistic potential (Woods–Saxon), the quantum number being the number of radial nodes of the associated single-particle s, p, d, etc. states. The levels shown at the center result when a spin-orbit term is also considered, the quantum numbers nlj characterizing the states of degeneracy (2j + 1) $(j = l \pm 1/2)$. To the left we schematically (in particular in the case of Li, which displays non-Meyer and Jensen sequence) indicate the Fermi energy associated with a light (exotic), medium, and heavy nucleus, namely, ${}_{3}^{11}$ Li, ${}_{50}^{120}$ Sn, and ${}_{82}^{208}$ Pb. In the inset, a schematic graphical representation of the reaction 208 Pb(d, p)²⁰⁹Pb(gs) is shown. A cross followed by a horizontal dashed line represents, in the present case, the (d, p) field, while a single arrowed line describes the odd nucleon moving in the $g_{9/2}$ orbital above the N = 126 shell closure drawn as a bold line labeled 0⁺. After Mayer and Jensen (1955).

from the level scheme shown in Fig. 1.1.2, obtained by assuming that nucleons move independently of each other in an average potential of spherical symmetry.

A closed shell, or a filled level, has angular momentum zero, in keeping with the fact that, in such a case, there is a single way to arrange the fermions. Thus, nuclei with one nucleon outside (missing from the) closed shell should have the spin and parity of the orbital associated with the odd nucleon (nucleon hole), a prediction confirmed by the data (available at that time) throughout the mass table.

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Such a picture implies that the nucleon mean free path is large compared to nuclear dimensions.

Systematic studies of the binding energies leading to the shell model found also that the relation (1.1.2) had to be supplemented to take into account the fact that nuclei with A odd, that is, with an odd number of either protons or neutrons, are energetically unfavored compared with the neighboring even-even ones by a quantity of the order of $\delta \approx 33 \text{ MeV}/A^{3/4}$, called the pairing energy⁷ and found at the basis of the odd-even staggering effect.

1.1.2 Nuclear Excitations

In addition to the quantum numbers λ , μ , and π , one can characterize nuclear excitations by additional quantum numbers, such as isospin τ and spin σ . Furthermore, one can assign a particle (baryon or transfer) quantum number⁸ β . For a nucleon moving above the Fermi surface, one has $\beta = +1$, while for a hole in the Fermi sea, $\beta = -1$. For (quasi-) bosonic excitations, $\beta = 0$ for a mode associated with, for example, surface oscillations, which can also be viewed as a correlated particle-hole (*p*-*h*) excitation (within this context, see Fig. 1.2.3). In particular, the low-lying quadrupole and octupole vibrations of even-even nuclei (see Fig. 1.1.3) have quantum numbers $\beta = 0$, $\lambda^{\pi} = 2^+$, 3^- , $\tau = 0$ (protons and neutrons oscillate in phase), and $\sigma = 0$ (no spin-flip in the excitation).

For modes that involve the addition or substraction of two correlated nucleons to the nucleus, $\beta = +2$ (Fig. 1.3.1) and $\beta = -2$, respectively. The excitation that, around closed shells, connects the ground state of an even nucleus to the ground state of the next even nucleus, which is a monopole pairing vibration ($\lambda^{\pi} = 0^+, \beta = +2$), is of this type (Fig. 1.3.2). Multipole pairing vibrations with quantum numbers $\beta = \pm 2$ and⁹ $\lambda^{\pi} = 2^+, 4^+ \dots$, have also been observed throughout the mass table.¹⁰

The low-lying excited state of closed shell nuclei can be interpreted as a rule, as a harmonic quadrupole, or as an octupole collective surface vibration (Fig. 1.1.3) described by the collective Hamiltonian¹¹

$$H_{coll} = \sum_{\lambda\mu} \left(\frac{1}{2D_{\lambda}} |\hat{\Pi}_{\lambda\mu}|^2 + \frac{C_{\lambda}}{2} |\hat{\alpha}_{\lambda\mu}|^2 \right).$$
(1.1.9)

⁹ It is of note that the quantum numbers of pairing vibrations are $\beta = \pm 2$ and $\pi = (-1)^{\lambda}$ (see App. 7.D).

¹⁰ See, e.g., Flynn et al. (1971, 1972a); Brink and Broglia (2005) chapter 5. See also footnotes 42, 43, and 44 of chapter 7, and references therein.

¹¹ Classically, $\Pi_{\lambda\mu} = D_{\lambda}\dot{\alpha}_{\lambda\mu}$.

⁷ Mayer and Jensen (1955) p. 9. Connecting with further developments associated with the BCS theory of superconductivity (Bardeen et al. (1957a,b)) and its extension to the atomic nucleus (Bohr et al. (1958)), the quantity δ is identified with the pairing gap Δ parameterized according to $\Delta = 12 \text{MeV}/\sqrt{A}$ (Bohr and Mottelson (1969)). It is of note that for typical superfluid nuclei like ¹²⁰Sn, the expression of δ leads to a numerical value that can be parameterized as $\delta \approx 33 \text{ MeV}/(A^{1/4} \times A^{1/2}) \approx 10 \text{ MeV}/\sqrt{A}$.

⁸ Bohr (1964).

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Figure 1.1.3 Schematic representation of harmonic quadrupole and octupole liquid drop collective surface vibrational modes.

Following Dirac (1930), one can describe the oscillatory motion introducing boson creation (annihilation) operator $\Gamma^{\dagger}_{\lambda\mu}$ ($\Gamma_{\lambda\mu}$) obeying the commutation relation

$$\left[\Gamma_{\alpha}, \Gamma_{\alpha'}^{\dagger}\right] = \delta(\alpha, \alpha') \tag{1.1.10}$$

and leading to

$$\hat{\alpha}_{\lambda\mu} = \sqrt{\frac{\hbar\omega_{\lambda}}{2C_{\lambda}}} \left(\Gamma^{\dagger}_{\lambda\mu} + (-1)^{\mu} \Gamma_{\lambda-\mu} \right).$$
(1.1.11)

A similar expression is valid for the conjugate momentum variable $\hat{\Pi}_{\lambda\mu}$, resulting in

$$\hat{H}_{coll} = \sum_{\lambda\mu} \hbar \omega_{\lambda} \left((-1)^{\mu} \Gamma^{\dagger}_{\lambda\mu} \Gamma_{\lambda-\mu} + 1/2 \right).$$
(1.1.12)

The frequency of the mode is $\omega_{\lambda} = (C_{\lambda}/D_{\lambda})^{1/2}$, while $(\hbar\omega_{\lambda}/2C_{\lambda})^{1/2}$ is the amplitude of the zero-point fluctuation of the bosonic vacuum state $|0\rangle_{B}$, $|n_{\lambda\mu} = 1\rangle = \Gamma_{\lambda\mu}^{\dagger}|0\rangle_{B}$ being the one-phonon state. To simplify the notation, in many cases, one writes $|n_{\alpha} = 1\rangle$.

The ground and low-lying states of nuclei with one nucleon outside a closed shell can be described by the single-particle Hamiltonian

$$H_{sp} = \sum_{\nu} \epsilon_{\nu} a_{\nu}^{\dagger} a_{\nu}, \qquad (1.1.13)$$

where $a_{\nu}^{\dagger}(a_{\nu})$ is the single-particle creation (annihilation) operator,

$$|\nu\rangle = a_{\nu}^{\dagger}|0\rangle_{F} \tag{1.1.14}$$

being the single-particle state of quantum numbers $\nu (\equiv nljm)$, namely, number of nodes, orbital and total angular momentum, and its third component) and energy ϵ_{ν} , $|0\rangle_F$ being the Fermion vacuum. It is of note that

$$\left[H_{coll},\Gamma^{\dagger}_{\lambda'\mu'}\right] = \hbar\omega_{\lambda'}\Gamma^{\dagger}_{\lambda'\mu'} \qquad (1.1.15)$$

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and

 $\left[H_{sp}, a_{\nu'}^{\dagger}\right] = \epsilon_{\nu'} a_{\nu'}^{\dagger}. \tag{1.1.16}$

This outcome results from the bosonic

$$\left[\Gamma_{\alpha}, \Gamma_{\alpha'}^{\dagger}\right] = \delta(\alpha, \alpha') \tag{1.1.17}$$

and fermionic

$$\left\{a_{\nu}, a_{\nu'}^{\dagger}\right\} = \delta(\nu, \nu') \tag{1.1.18}$$

commutation (anti-commutation) relations.

The existence of drops of nuclear matter displaying both collective surface vibrations and independent-particle motion are emergent properties not contained in the particles forming the system, nor in the forces acting among them.

Expressed differently, generalized rigidity closely connected to the inertial parameter D_{λ} implies that acting on a nucleus with an external $\beta = 0$, time-dependent (nuclear/Coulomb) field, the system reacts as a whole (collective vibrations; also rotations see Sect. 1.4), while acting with fields that change particle number by one ($\beta = \pm 1$; e.g. (d, p) and (p, d) reactions), the system reacts in terms of independent particle motion, feeling the pushings and pullings of the other nucleons only when trying to leave the nucleus. Such a behavior can hardly be inferred from the study of the *NN*-forces in free space, being truly emergent properties of the finite, quantum many-body nuclear system.

Collective surface vibrations and independent particle motion are examples of what are called elementary modes of excitation in finite many-body physics and collective variables in soft-matter physics.

1.2 The Particle–Vibration Coupling

The oscillation of the nucleus under the influence of the surface tension implies that the potential U(r, R) in which nucleons move independently of each other changes with time. For low-energy collective vibrations, this change is slow as compared with single–particle motion. Within this scenario the nuclear radius can be written as

$$R = R_0 \left(1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\hat{r}) \right).$$
(1.2.1)

Assuming small-amplitude motion,

$$U(r, R) = U(r, R_0) + \delta U(r), \qquad (1.2.2)$$

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Figure 1.2.1 Graphical representation of a process by which a nucleon, bouncing inelastically off the nuclear surface, sets it into vibration. Particles are represented by an arrowed line pointing upward, which is also the direction of time, while the vibration is represented by a wavy line. In the cartoon to the right, the black dot represents a nucleon moving in a spherical mean field of which it excites, through the PVC vertex, an octupole vibration after bouncing inelastically off the surface.

where

$$\delta U = \kappa \hat{\alpha} \hat{F} = \Lambda_{\alpha} \left(\Gamma^{\dagger}_{\lambda\mu} + (-1)^{\mu} \Gamma_{\lambda-\mu} \right) \hat{F} = H_c, \qquad (1.2.3)$$

with

$$\Lambda_{\alpha} = \kappa \sqrt{\frac{\hbar\omega_{\lambda}}{2C_{\lambda}}},\tag{1.2.4}$$

is the particle-vibration coupling (PVC) strength (Fig. 1.2.1), product of the dynamic deformation

$$\beta_{\lambda} = \sqrt{2\lambda + 1} \sqrt{\frac{\hbar\omega_{\lambda}}{2C_{\lambda}}}$$
(1.2.5)

and of the strength κ , while

$$\hat{F} = \sum_{\nu_1 \nu_2} \langle \nu_1 | F | \nu_2 \rangle a_{\nu_1}^{\dagger} a_{\nu_2}$$
(1.2.6)

is a single-particle field with (dimensionless) formfactor

$$F = -\frac{R_0}{\kappa} \frac{\partial U}{\partial r} Y^*_{\lambda\mu}(\hat{r}).$$
(1.2.7)

An estimate of κ is given below (Eq. (1.2.13)).

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Figure 1.2.2 Schematic representation of the processes characterizing the Hartree-Fock ground state (single-particle vacuum), in terms of Feynman diagrams. (a) Nucleon-nucleon interaction through the bare (instantaneous) NNpotential. (b) Hartree mean field contribution. (c) Fock mean field contribution. (d,e) ground state correlations (ZPF) associated with the Hartree and Fock fields. (f) There is, in HF (mean field) theory, a complete decoupling between occupied and empty states, labeled i and k, respectively, and thus a sharp discontinuity at the Fermi energy of the occupation probability, from the value of 1 to 0. (g) This decoupling allows for the definition of two annihilation operators: $a_k(b_i)$ particle (hole) annihilation operators, implying the existence of hole (antiparticle) states $(b_i^{\dagger}|HF\rangle)$ with quantum numbers time reversed to that of particle states (for details, see, e.g., Brink and Broglia (2005), App. A). In other words, the $|HF\rangle$ ground (vacuum) state is filled up to the Fermi energy (ϵ_F) with N nucleons. The system with (N-1) nucleons can, within the language of (Feynman's) field theory, be described in terms of the degrees of freedom of that of the missing nucleon (hole-, antiparticle state). Such a description is considerably more economic than that corresponding to an antisymmetric wavefunction with (N-1) spatial and spin coordinates (r_i, σ_i) . Within the above scenario, a stripping reaction N(d, p)(N + 1) can be viewed as the creation of a particle state $(a_k^{\dagger}|HF\rangle = |k\rangle)$ and that of a pickup reaction N(p, d)(N-1) as that of a hole state $(b_i^{\dagger} | HF \rangle \equiv |\tilde{i}\rangle)$. (h) Hartree, mean field contribution to the nuclear density, the density operator being represented by a cross followed by a dashed horizontal line (see also Fig. 1.8.1).

Diagonalizing δU making use of the graphical (Feynman) rules of nuclear field theory (NFT) to be discussed in the following chapter, one obtains structure results that can be used in the calculation of absolute transition probabilities and differential reaction cross sections, quantities that can be compared with the experimental findings.