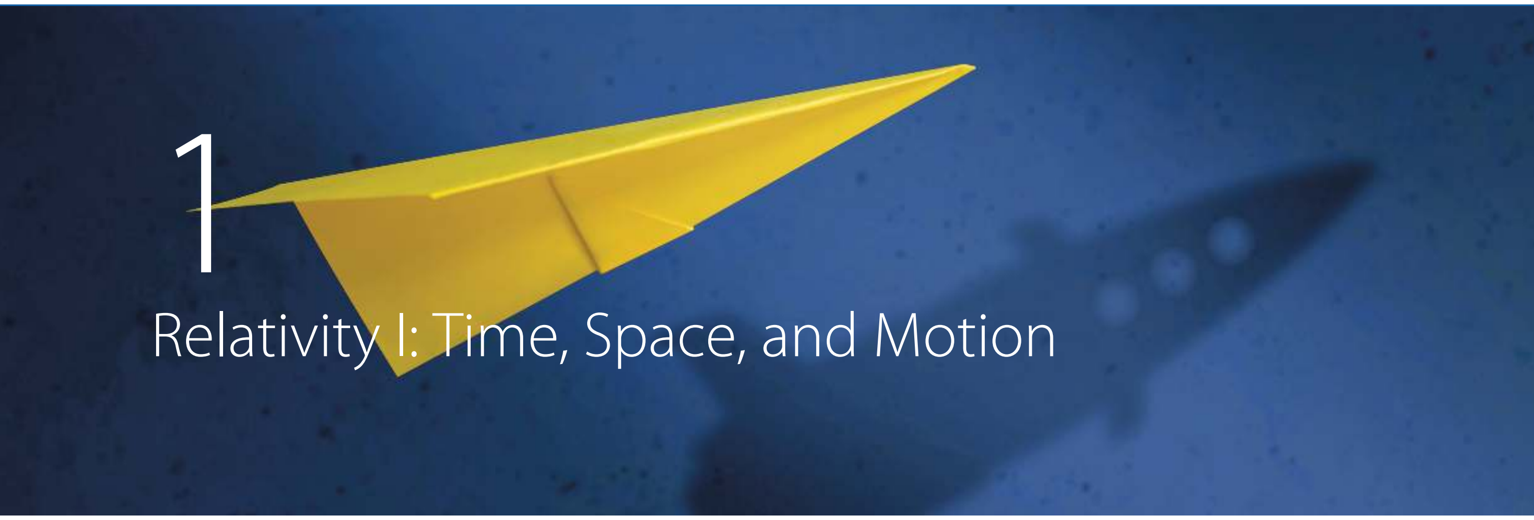
CAMBRIDGE

Cambridge University Press & Assessment
978-1-108-84289-1 — Modern Physics
Gary N. Felder , Kenny M. Felder
Excerpt
More Information



1 Relativity I: Time, Space, and Motion

Einstein's theories have become part of popular culture. The fact that time passes differently for different observers ("time dilation") is a staple of science fiction, from *Planet of the Apes* (1968) to *Interstellar* (2014). If someone can only name one physics equation, it's probably $E = mc^2$. You have probably heard people use the word "spacetime," assure you that "science tells us everything is relative," and mention that Einstein's work somehow helped Oppenheimer invent the atomic bomb.

But how many of these people actually understand what the theory says?

As so often happens, the real physics is more exciting than the popular version. More subtle. More infuriating. More *weird.* And it is this last characteristic, perhaps – relativity's stubborn refusal to conform to our physical intuition – that marks the definitive break between "classical" and "modern" physics.

1.1 Galilean Relativity

Galilean relativity (or "Galilean invariance") is one of the cornerstones of Newtonian physics. If your mechanics professor ever said "Let's look at this problem from another reference frame" then you were learning about Galilean relativity, even if you never heard the name.

Here is how Galileo himself expressed this principle in his *Dialogue Concerning the Two Chief World Systems* (1632):

> *Motion exists relatively to things that lack it; and among things which all share equally in any motion, it is as if it did not exist.*

Galileo explained this principle with a thought experiment:

> *The goods with which a ship is laden leaving Venice pass by Corfu, by Crete, by Cyprus and go to Aleppo. Venice, Corfu, Crete, etc. stand still and do not move with the ship; but as to the sacks, boxes, and bundles with which the boat is laden and with respect to the ship itself, the motion from Verflice to Syria is as nothing, and in no way alters their relation among themselves. This is so because it is common to all of them and all share equally in it. If, from the cargo in the ship, a sack were shifted from a chest one single inch, this alone would be more of a movement for it than the two-thousand-mile journey made by all of them together.*

> *It is obvious, then, that motion which is common to many moving things is idle and inconsequential to the relation of these movables among themselves, and that it is operative only in the relation that they have with other bodies lacking that motion.*

Einstein did not overthrow or revise this principle. On the contrary, he enthusiastically reaffirmed it. A review of this classical idea will give you an important first step toward Einstein's modern theory.

1.1.1 Discovery Exercise: Galilean Relativity

Spaceman Spiff is hurtling through the solar system at 3 kajillion miles per hour. His enemy on a nearby planet shoots a deadly missile straight toward him at 4 kajillion miles per hour. Spiff's rocket can withstand an impact of 4.5 kajillion mph without harm; anything above that will puncture the hull.

1. Suppose the enemy is ahead of Spiff, so the missile and the rocket crash head-on. Does the missile penetrate the hull?

   See Check Yourself #1 at www.cambridge.org/felder-modernphysics/checkyourself
2. Suppose the enemy is behind Spiff, so the missile catches up and rear-ends the rocket. Does it penetrate the hull?
3. Suppose the missile is coming in from the side, and hits perpendicular to the rocket's direction of travel. Does it penetrate the hull?

1.1.2 Explanation: Galilean Relativity

Imagine a small girl in the back of a blue car, heading straight down the road at 60 mph. Her father is behind the steering wheel in the front. Outside her left window, a green car going 65 mph passes her car. Outside the right window, a row of trees lines the side of the road. Across the highway she sees cars going at 60 mph in the opposite direction (see Figure 1.1).

What does all this look like from the girl's perspective? She sees her own car, and her father, perfectly at rest. The green car drifts by slowly – at only 5 mph, it takes a while to pass her. The trees are rushing backward at 60 mph, and the cars across the highway are zooming by at a breakneck 120 mph.

You could have figured all that out yourself. But you may be tempted to think about it this way:

> *The girl sees all those incorrect velocities because she's moving. If she knew she was moving forward at 60 mph, she could use that information to calculate the true velocities of all those objects.*

Our goal is to talk you out of the preceding paragraph and into the following one instead:

> *The girl's perspective is perfectly correct. Her description of events is just as valid as the first description we gave (from the perspective of the road). "The green car is*

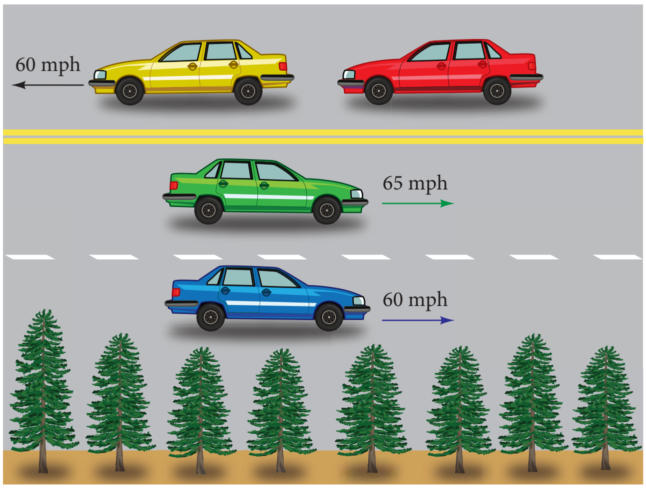


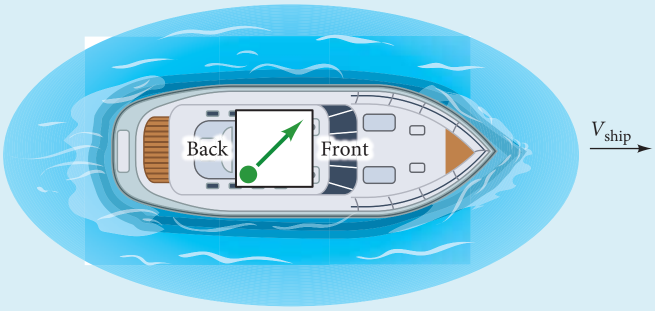
Figure 1.1 A blue car sees a green car passing it and red and yellow cars moving the other way.

going 65 mph from the road perspective" and "The green car is going 5 mph from the girl's perspective" are two equivalent ways of expressing the same fact, and neither one is more correct than the other.

Are you skeptical? Do you find yourself believing that, in some absolute sense, the road is "really" still and the car is "really" moving? Then consider this: the road is actually spinning around the Earth's center at 1000 mph. And the Earth itself is hurtling around the Sun at over 60,000 mph. How far do you back up to find that "really still" point? The answer is, don't try. Choose your non-moving perspective based on whatever makes the problem you're working easiest.

Example: Change of Perspective

An ocean liner is moving to the right (across the page) at 10 m/s. Elijah is standing in the back corner of a 10 m by 10 m cabin and throws a ball at 5 m/s directly toward the opposite corner. Does the ball hit the front wall, the side wall, or the corner?



Answer: This is a standard vectors problem, right? The ball's velocity vector is the sum of the ship's velocity vector (10 m/s to the right) and the ball's thrown vector (5 m/s at 45°). Then you take into account the fact that the ship (and therefore the walls) are also moving to the right to see where the walls are when the ball hits . . .

That's how many beginning students would approach this problem. It's all perfectly correct, but it's also a lot of work. The more experienced student would work in the reference frame of the ship rather than the ocean. The cabin isn't moving. The ball moves in a straight line toward the opposite corner, so that's where it hits.

Formal Statement of the Principle

When Galileo first put forth his principle of relativity (p. 1), he took the viewpoint of the cargo in a ship on a 2000-mile voyage from Venice to Aleppo. Let's update his example.

Active Reading Exercise: Galileo's Spaceship

Imagine yourself on a spaceship far outside our solar system. It's a roomy ship with all the comforts of home, but it has no windows and no communication of any kind with the outside world. You hold your hand out in the center of the room with a ball in it – not on a table or the floor, just in mid-air – and then you open your hand and let go of the ball.

1. What does the ball do if the ship suddenly speeds up?
2. What does the ball do if the ship suddenly slows down?
3. What does the ball do if the ship makes a hard right turn?
4. What does the ball do if the ship stays the course, moving incredibly fast without change?

Jot down all four answers before you continue reading!

The answers to the first three questions are "It starts moving backward," "It starts moving forward," and "It starts moving left," respectively. But the answer to the fourth question is "nothing." An external observer would say that the ball is hurtling through space with the exact same velocity as the ship. From inside the ship, it looks to you like a non-moving ball.

So if you see the ball hovering, you can't tell if the ship is at rest or moving. If you really want to know which is happening, you might start running experiments. Throw the ball around the room and measure how it moves through the air and bounces off the walls. Bounce beams of light off mirrors and measure their reflection angles. It is all to no avail; you still have no idea if the ship is moving fast or slowly, forward or backward, or not moving at all.

We can summarize all that in one 12-word sentence.

The Principle of Galilean Relativity

The laws of physics are the same in any inertial reference frame.

(The word "inertial" in that sentence means "non-accelerating.")

In an *accelerating* reference frame, the laws of physics are quite different. Billiard balls start flying through the air of their own accord. Enough acceleration (for instance, about 10*g*, or 10 times

Earth-normal gravity, for over a minute) will kill you. Observers in all reference frames will agree that you are dead.

But the world inside a not-moving-at-all spaceship, and the world inside a zooming-steadily-at-half-a-billion-miles-per-hour spaceship, are completely indistinguishable. That's why you are perfectly within your rights to assert that the spaceship is *not* moving – even if you must then conclude that nearby stars are rushing by, backward.

To put it another way, acceleration is an intrinsic property that every object in the universe has at any given moment. But the velocity of an object can only be meaningfully defined in relation to a particular reference frame; other reference frames will record different velocities for the same object, and all are equally correct.

The Math of Galilean Relativity

You are standing by the side of the road as the little girl's blue car zooms by at speed u. You and the little girl are both measuring the positions of cars, trees, and other objects.

We will use primes to indicate measurements in the girl's reference frame. For instance, if x designates the position of a particular tree, both of the following might be true at once (Figure 1.2):

- $x = 30$ m (You see the tree 30 m ahead of you.)
- $x' = -10$ m (The girl sees the same tree 10 m behind her.)

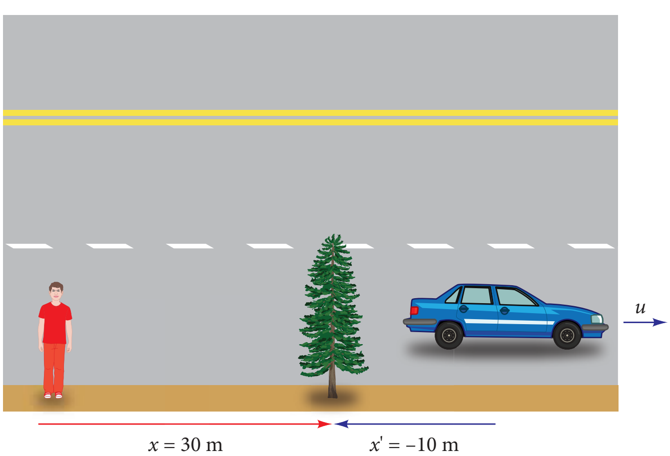


Figure 1.2 **A tree is 30 m ahead of you, but 10 m behind the girl in the car.**

These primes do *not* indicate derivatives; throughout this chapter and Chapter 2, primed variables will designate the perspective of an alternative reference frame.

At the moment the girl's car passes you, you both set your stopwatches to zero. Hence, that particular place and time is $t = 0$ and $x = 0$ by your measurements. It is also $t' = 0$ and $x' = 0$ by hers.

Keep in mind that for you, $x = 0$ will always mean the spot where you are standing. The girl's x-value increases over time ($x_{\text{girl}} = ut$). For her, on the other hand, $x' = 0$ will always mean the spot where she is sitting. The x'-value she measures for your position is steadily receding ($x'_{\text{you}} = -ut'$).

Cambridge University Press & Assessment
978-1-108-84289-1 — Modern Physics
Gary N. Felder , Kenny M. Felder
Excerpt
More Information

More generally, here are the conversions from your system to hers; the variable v represents the velocity of any arbitrary object as measured by you, and v' the velocity of the same object as measured by the girl; accelerations are similarly a and a':

$$\left.\begin{aligned} t' &= t \\ a' &= a \\ v' &= v - u \\ x' &= x - ut \end{aligned}\right\} \tag{1.1}$$

For instance, in our earlier example ($u = 60$ mph), the green car zoomed by at $v = 65$ mph. The girl saw it creeping ahead at $v' = v - u = 5$ mph.

In 3D we might add two more equations to that list: $y' = y$ and $z' = z$. Because your relative motion is along the x axis, you will both agree entirely about the y and z coordinates. This important principle will remain unchanged as we graduate from Galileo to Einstein.

All those equations demonstrate an important property of classical physics: they may look confusing at first, but if you are willing to put in the time you will eventually conclude that they make perfect sense. Enjoy that feeling while you can, because in the next section we're going to undermine it.

1.1.3 Questions and Problems: Galilean Relativity

Conceptual Questions and ConcepTests

1. You are standing at rest and Mary is moving by you with constant velocity in the positive y direction. Ben is moving nearby in some way that we haven't specified (see Figure 1.3). Each of the following questions refers to a measurement of Ben taken some time after Mary has passed you, and asks you to compare that measurement in your frame (unprimed) with Mary's (primed). For each one, say whether the unprimed variable is greater, the primed variable is greater, they are equal, or there isn't enough information to tell. *Remember that all the variables in this problem refer to measurements of Ben, as measured by you (unprimed) or by Mary (primed).*

(a) x and x'

(b) y and y'

(c) v_x and v'_x

(d) v_y and v'_y

(e) a_x and a'_x

(f) a_y and a'_y

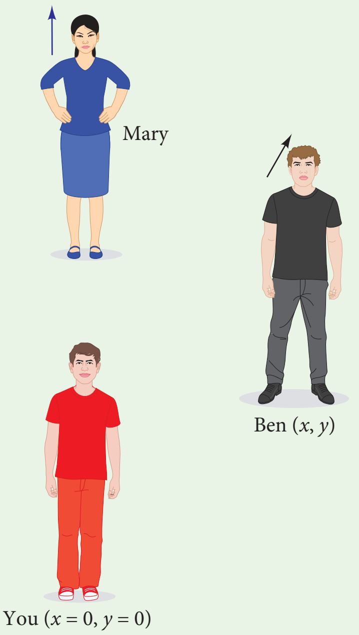


Figure 1.3

2. Two observers in different reference frames are measuring everything they can measure about the same moving object. Based on Galilean relativity, list three quantities that these two observers must agree on, and three that they might disagree about. (Some of the answers might change in Einstein's theory, but don't worry about that for now.)

3. Explain the meaning of the equation $t' = t$ in Equations (1.1).

4. If you are driving at 35 mph when another car passes in the same direction as you, going at 50 mph, we say "the other car is going at 15 mph in your reference frame." What does that mean? Your answer should contain the number 15 and should explain what it means in this context. It should not include the phrase "reference frame."

5. A cart has a vertical spring with a ball on top of it. When you push the spring down and release it with the cart at rest, the ball goes straight up and down and falls back into the cart. For each of the scenarios below, will the ball fall (A) into the cart, (B) behind the cart, or (C) in front of the cart?
   (a) You give the cart a push and release the spring as it is moving at steady velocity along the floor. (Ignore friction.)
   (b) The same as Part (a), but this time assume the cart does experience significant friction as it slides.

6. You are biking at 20 mph when a frisbee moving perpendicularly to you at 10 mph slams into the side of your head. How much will the frisbee hurt your head? Explain your answer. (This might involve a bit of math.)
   A. It will cause the same injury that a 10 mph frisbee would cause if you were not riding.
   B. It will cause the same injury that a 20 mph frisbee would cause if you were not riding.
   C. It will cause the same injury that a 30 mph frisbee would cause if you were not riding.
   D. None of the above.

7. Superman is sitting calmly in outer space, not moving, when your spaceship passes by at constant velocity. Inside the spaceship you are still looking for an experiment that will tell you whether the ship is moving or not. You grab a pool ball and throw it up into the air. It rises, bounces off the ceiling, and comes back into your hand.
   (a) Describe the motion of the pool ball as it appears to you. What shape does its path trace out?
   (b) Superman is watching the whole thing with his X-ray vision. Describe the motion of the ball as it appears to him. What shape does its path trace out?

8. In Equations (1.1) we wrote conversions from your side-of-the-road reference frame to a little girl's zooming-by-in-a-car frame. Write equations to convert the other way. What's the only difference between the two sets of conversions?

9. Aristotle believed that there is a fundamental difference between the states "in motion" and "at rest." But Galileo said that this distinction depends on your reference frame: what one observer calls "at rest" another calls "in motion" and vice versa. How, then, to explain Aristotle's observation that moving objects tend to return to the state of rest? We see that all the time, don't we?

For Deeper Discussion

10. We said that every object in the universe has a well-defined acceleration at any given moment. If we want the velocity of a given object, we can just integrate its acceleration with respect to time. Given that the acceleration must be the same in every reference frame, how can the velocity come out differently?

11. Part of Galileo's motivation for proposing his relativity was to dispute a common objection to the Copernican solar system. On a galloping horse you feel the wind. If the Earth is hurtling rapidly through space, why aren't we buffeted by gale-force winds?

Problems

12. Your friend Al is driving at 60 mph when another car passes him at 70 mph. "In your reference frame," you explain to Al, "The other car is going at 10 mph." But Al doesn't believe you. Al somehow (annoyingly) always wins these debates, so you have

to hit him with an argument that is irrefutable at every step. "Let's suppose you and the other car both start together, at $t = 0$ hours and $x = 0$ miles," you say. Al has no objection so far.

(a) Find the positions of Al's car and the other car one hour after it passes him, in the reference frame of the road. Don't just answer with two numbers; justify those numbers beyond any possible doubt from Al.

(b) Now find the position of the other car *relative to Al* at that same moment, one hour after the other car passed him. Don't use any equations from this section; just use a common sense argument that Al can't possibly refute.

(c) "By the definition of velocity," you say, "if a car moves x miles in an hour then that car is moving (on average) x miles per hour." Al agrees with that. Using that definition and your result from Part (b), convince Al that the other car is going at 10 mph in his reference frame.

Postscript to Problem 12: You made a sound argument, but Al's going to come back to this one in the next section. Al somehow (annoyingly) always wins these debates.

13. A swallow is flying at exactly 25 mph directly northward. Let north be the y direction and east the x direction. Write the swallow's velocity from the perspective of . . .

(a) a tree that it flies over.

(b) another swallow, also flying 25 mph directly northward.

(c) a third swallow, flying 25 mph directly southward.

(d) a fourth swallow, flying 25 mph directly eastward.

14. Equations (1.1) give the Galilean conversions between two reference frames, one moving at speed u with respect to the other, for four quantities: time, position, velocity, and acceleration. Give the analogous conversions for mass, momentum, and kinetic energy.

15. María José runs by you at 20 m/s, moving along a straight line 30° above the positive x axis. At the moment she passes the origin (you), you both synchronize your stopwatches to time 0. At that same moment, a car goes by with velocity $\vec{v}$ in your frame. At some later time t you both measure its position, velocity, and acceleration. Write the conversions from your t, x, y, v_x, v_y, a_x, and a_y measurements to María José's corresponding primed coordinates. (Your answer will consist of seven equations, one for each variable.)

16. You are standing still in a field with some x and y axes drawn on it. (Of course you are standing at the origin.) James runs by at a constant velocity of $(3\,\text{m/s})\hat{i} + (4\,\text{m/s})\hat{j}$, and at that same moment Cecelia bikes by at 10 m/s in the positive y direction. She has her brakes on and is accelerating at $-(2\,\text{m/s}^2)\hat{j}$. At the moment they both pass you all of your stopwatches are set to 0.

(a) Two seconds after she passes you, what are Cecelia's position, velocity, and acceleration in your reference frame?

(b) At that same moment, what are Cecelia's position, velocity, and acceleration in James' reference frame?

17. Equations (1.1) give the conversions for measurements of any object from one reference frame to another, assuming the primed frame is moving at speed u in the x direction relative to the unprimed frame. Rewrite those conversions assuming the primed frame is moving at speed u at an angle ϕ counterclockwise from the x axis. Your answer should consist of seven equations, one for time and one each for the x and y components of position, velocity, and acceleration.

18. If you are in a train that is going at a fast (but constant!) speed around a small circle, you don't need to look out the window to know you're moving; you will perceive a pull toward the outside of the circle.

(a) Why does this *not* violate Galileo's principle that the laws of physics are the same in all inertial reference frames? (You can answer this using only words.)

(b) So why don't we feel the fact that we are in constant circular motion around the Earth's rotational axis? (This one will require some calculation.)

19. A piece of gum is stuck to the edge of a bicycle tire. The tire has a radius of $1/3$ m and makes a full rotation every $\pi/12$ seconds as the bicycle moves forward 8 m/s. At time $t=0$ the gum is at the highest point on the tire.

(a) Write the function $x'(t)$ giving the horizontal position of the gum, relative to the center of the tire. Assume t is measured in seconds and give your answer in meters.

(b) Write the function $x(t)$ giving the horizontal position of the gum relative to the road. Assume that $x(0)=0$. Assume t is measured in seconds and give your answer in meters.

20. Dudley Do-Right is floating safely downward toward the ground, parachute open, at a constant 10 m/s. His arch-enemy Snidely Whiplash, 500 m above him, drops a brick, which begins at rest but accelerates at 9.8 m/s^2 downward. How long does it take the brick to reach Dudley?

(a) Solve this problem from the perspective of the ground. (The numbers given above were all from this perspective.)

(b) Start over and solve the same problem from Dudley's perspective. You should find that although many of the individual measurements (positions and velocities) are completely different, the equation you end up solving for time is identical.

(c) Why didn't we ask you to solve the problem from the brick's perspective?

21. Remember the little girl in the 60 mph car? Well, suddenly her father spots an emergency: a trailer 2 miles ahead is rolling along the road at only 20 mph, neither speeding up nor slowing down. Dad slams on the brakes! What acceleration must his brakes apply if his car is to avoid hitting the trailer? First you're going to solve this in the reference frame of the road.

(a) Call the car's position $x=0$ at the moment Dad hits the brakes. Write an equation for the car's position as a function of time and another for the trailer's position. Set the two equal to represent the moment they collide. The result should be one equation with two unknowns: t and a.

To finish solving for a you would need to solve that quadratic equation for t and then find an inequality involving a for when that solution for t has no real solutions, meaning they never collide. But you might prefer a different approach.

(b) Start over and solve the problem in the reference frame of the trailer. If you do this correctly it's a much easier calculation. *Hint:* You do not need to use time at any point in the solution.

22. [*This problem depends on Problem 21.*] Finish doing the calculation from Problem 21 in the car's reference frame. Make sure you get the same answer you did in the trailer's reference frame.

23. In the Example on p. 3 we determined what would happen to a ball on a ship using the ship's reference frame. Now re-do that calculation in the ocean's reference frame. Take the x axis to point in the direction of the ship's motion.

(a) What are the ball's x- and y- velocities in the ship's reference frame?

(b) What are the ball's x- and y- velocities in the ocean's reference frame? (The rest of the problem will be in the ocean's reference frame.)

(c) Write equations for the ball's positions $x_b(t)$ and $y_b(t)$. Take its initial position to be $(0,0)$.

(d) Write equations for the position of the opposite corner as a function of time, $x_c(t)$ and $y_c(t)$.

(e) Solve for the time at which $y_b = y_c$.

(f) Find x_b and x_c at that time. If they are equal then the ball hits the corner. If not then it hits one of the walls. If you didn't get the same answer as we got in the Example, go back and find your mistake.

24. Two particles with masses m_a and m_b and initial velocities v_{a_0} and v_{b_0} collide and move off with velocities v_{a_f} and v_{b_f}. All the motion is in the x direction, and the velocities are given in the lab reference frame. Show that if momentum is conserved in the lab reference frame then it is conserved in any inertial reference frame moving with velocity u in the x direction relative to the lab.

25. Non-Inertial Reference Frames

Galilean relativity says that all inertial reference frames are equally valid. It's certainly possible to calculate things like position and velocity in a non-inertial reference frame, but the physics we are familiar with doesn't work. To see why that is, suppose you are in your ship floating at rest in space with nothing exerting a force on you. Asma is at rest next to you. At a time that you both call $t = 0$, she starts accelerating at a rate a_C in the x direction.

(a) Write your position, velocity, and acceleration in Asma's reference frame.

(b) Explain how you can tell from your answers that Newton's laws are not valid in Asma's reference frame.

1.2 Einstein's Postulates and Time Dilation

Newtonian physics can be exciting. You can model and predict the behavior of familiar systems like pendulums and ocean waves. You can notice possibly unfamiliar phenomena like the way a top wobbles as it slows down, and then explain them. It gives you the sense that the world around you follows a comprehensible set of rules.

Modern physics is exciting in a very different way, because its effects do *not* obey the laws that we see in our daily lives. In special relativity, time passes differently for different observers. Objects change length as they move. Different people can disagree about the order in which things happened, and they can all be right! (And that's the least confusing of the modern physics theories.)

So when you study modern physics, some of the fun is finding out that the universe is stranger and more complex than we normally think. You will need to set aside some of your lifelong intuitions and slowly start to build up new ones. We're going to start all that with Einstein's Special Theory of Relativity.

1.2.1 Discovery Exercise: Einstein's Postulates and Time Dilation

The speed of light, generally represented by the letter c, is approximately 3×10^8 m/s.

Spaceman Spiff is floating motionless in space when a spaceship zooms past at speed $c/3$ (one third the speed of light). At the instant the ship passes him Spiff turns on his flashlight, pointing the same direction that the ship is traveling. The beam leaves the flashlight at c, the speed of light, in Spiff's reference frame.

1. After one second, the beam of light has traveled how far in front of Spiff?
2. After one second, the spaceship has traveled how far in front of Spiff?
3. So after one second, the beam of light has traveled how far in front of the spaceship?

   See Check Yourself #2 at www.cambridge.org/felder-modernphysics/checkyourself
4. Use your previous responses to answer the question: how fast is the light beam traveling in the reference frame of the spaceship?