

### **Cosmic Messengers**

Martin Harwit, author of the influential book *Cosmic Discovery*, asks key questions about the scope of observational astronomy. Humans have long sought to understand the world we inhabit. Recent realization of how our unruly Universe distorts information before it ever reaches us reveals distinct limits on how well we will ultimately understand the Cosmos. Even the best instruments we might conceive will inevitably be thwarted by ever-more complex distortions and will never untangle the data completely. Observational astronomy, and the cost of pursuing it, will then have reached an inherent end. Only some totally different lines of approach, as yet unknown and potentially far more costly, might then need to emerge if we wish to learn more. This accessible book is written for all astronomers, astrophysicists, and those curious about how well we will ever understand the Universe and the potential costs of pushing those limits.

Martin Harwit is Professor Emeritus of Astronomy at Cornell University, New York. For many years he also served as Director of the National Air and Space Museum in Washington, DC. For much of his astrophysical career he built instruments and made pioneering observations in infrared astronomy. His advanced textbook, *Astrophysical Concepts* (1973), has taught several generations of astronomers through its four editions. Harwit has had an abiding interest in questions first raised in *Cosmic Discovery* on how science advances or is constrained by factors beyond the control of scientists. His subsequent book, *In Search of the True Universe* (Cambridge University Press, 2014), explores how philosophical outlook, historical precedents, industrial progress, economic factors, and national priorities have affected our understanding of the Cosmos. This new book rounds out his informal trilogy on the themes of cosmic discovery. Harwit is a recipient of the Astronomical Society of the Pacific's highest honor, the Bruce Medal, which commends "his original ideas, scholarship, and thoughtful advocacy."





# Cosmic Messengers

The Limits of Astronomy in an Unruly Universe

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## Preface

Astronomy is largely a curiosity-driven science. We wonder how our home, the Universe, came into being; how myriad galaxies like our Milky Way may have originated; and how the billions of stars in those galaxies, many of them resembling our Sun, were formed. How many stars have planets similar to those in our Solar System? How many of them harbor life? Does life quite different from any found on Earth exist elsewhere? And could it be intelligent?

Answers to such questions come at considerable cost with no guarantees that our curiosity will ever be fully satisfied.

Certainly, astronomy is not solely driven by curiosity. Astronomers take pride in recalling that our forebears produced charts of the heavens to help sailors navigate the high seas, explore our planet Earth, and set up new trade routes. Today, we are similarly charting the Solar System to map the orbits of kilometer-sized asteroids, piles of rock traversing the spaces between the planets. We need to know whether any asteroid might foreseeably collide with Earth any time soon so we might avoid calamities threatening the survival of life on our planet.

Such practical concerns, nevertheless, are not the main reason many of us chose to become astronomers. Rather, it is curiosity about the origin of the world we inhabit and hope that we might someday discover a guiding principle driving the workings of the Cosmos so we could convince ourselves that "Yes, all this now makes good sense!"

By itself, this wish for clarity is not all we should consider. The public supporting our efforts through the taxes it pays has every right to ask, "Could your curiosity-led search turn out to be endless? Will you then endlessly ask for further support for your quest? Will you astronomers ever be satisfied you have gone as far as possible even though you fall short of meeting your ultimate goals?"

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Such questions deserve answers, not only because our fellow citizens have every right to know, but because the clarity we'd gain in formulating a reply could also help us find better ways to pursue our quest.

My purpose in writing this book thus was partly to answer these questions, at least to my own satisfaction, though when I started I didn't know whether I'd succeed: I found myself straying from traditional questions about the origins and workings of the world of planets, stars, supernovae, quasars, and ultimately the entire Cosmos.

Cosmic Discovery, my first attempt to provide a broader look, had described how entirely new astronomical phenomena had frequently been unveiled by unanticipated novel instruments. The invention of telescopes, and the improved designs Galileo had introduced, had revealed moons orbiting Jupiter, mountains broaching the lunar landscape, and a Milky Way resolving into countless individual stars. Similarly, the introduction of radio techniques, three to four centuries later, had led to a string of baffling discoveries of quasars, pulsars, gamma-ray bursts, and other phenomena initially merely given descriptive names because the basic mechanisms at work in them were not yet understood.<sup>1</sup>

In Search of the True Universe, my second effort devoted to similar questions, investigated the extent to which novel approaches of theoretical physics had led to major new astronomical insights. Hans Bethe at Cornell University in Ithaca, New York, had shown how nuclear physics could explain the enormous amounts of energy ordinary stars like the Sun radiate in their lifetimes. And Albert Einstein's general relativity had predicted the potential existence of gravitational waves first indirectly detected more than twenty years after his death.<sup>2</sup>

Cosmic Messengers, the concluding volume in this trilogy, now investigates how information reaches us from across the Cosmos. It focuses on how the messengers – the radio waves, X-rays, visible light, infrared radiation, neutrinos, gravitational waves, cosmic rays, and occasionally odd mixtures of atomic isotopes or stray dust grains – bear witness to remote astronomical events, often after surviving long journeys across vast regions of space.

At times any of these messengers may arrive unscathed; at others irrevocably transformed. Physical processes by now well understood determine which messengers can be trusted to reach us intact, and which cannot. Most messengers can convey information reliably only within limited energy ranges. The information they transmit is necessarily bounded and finite.

Because of these finite bounds, a finite range of instruments may suffice to teach us all we will ever be able to learn through remote observation of the Cosmos. Even though instruments able to detect wider ranges of messenger traits could in principle be constructed, they might serve few astrophysical purposes if the information the messengers transmitted could not be trusted.



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We still are uncertain about the full set of messengers the Universe actually conveys. Successive generations of astronomers will surely explore this question further and reassess how much additional information on the Cosmos such messengers could reliably yield.

Two trends have surfaced in the past few years. The first has shown technological advances steadily introducing powerful new messengers conveying new insights on the remote Universe. X-rays and ever-more energetic gamma rays are providing new panoramas of supermassive galactic black holes; gravitational waves and equally elusive neutrinos document vast distant explosions; and the most energetic cosmic-ray particles the Cosmos may ever generate have by now been captured, their energy bounds clearly revealed. A new way of studying the world has arisen, a *multi-messenger science*, in which each of these powerful new cosmic tracers complements and reinforces the messages delivered by all the others. If traditional ways of studying the Universe fail us, any of the newer means of probing the world around us should find paths to break through.

Less well known is another realization just sinking in. It is the recognition that not only massive stars can readily deflect light beams from more remote regions, as Albert Einstein's general relativity first predicted. In a universe like ours, now seen to be filled with masses large and small, light reaching us from great distances has to slalom through crowded fields of intervening stars and galaxies before ever arriving at Earth. After so many gravitational twists and turns, how can we be sure about just where that light originated? How can we be certain about the sequence in which light reaching us today was originally emitted? This too depends on the winding path followed. Passing a massive body close by slows the light beam more than passage at greater distances; and often a light beam splits and does both.

Within our own Milky Way galaxy, we now realize, most stars no longer appear isolated. Surrounded by planets and asteroids similar to ours, they too provide more complex gravitational terrains that passing radiation has to navigate to reach us. Making matters even more complex, planets and asteroids are now found wandering on their own throughout the Galaxy, having broken away from their parent stars. They emit virtually no light of their own; we only know they are there because they occasionally act like gravitational lenses, bending the trajectories and magnifying the light from more distant stars as its rays pass close by them.

With the increasingly powerful telescopes we have by now built, we can detect ever-finer deflections such small bodies induce. But rather than yielding more accurate information on the location and timing of remote astronomical events, we are beginning to realize that we are, instead, more desperately trying to untangle just where those sources might be located and



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when the light reaching us was emitted, and finding that unseen intervening masses, ranging from dark matter to planets and asteroids, are interfering with that quest.

These are not problems that multi-messenger science will resolve. Einstein's work taught us that not just light, but all other messengers as well, follow trajectories identically bent and delayed by gravitation. If we cannot trace the trajectories gravity induces on light, we will not find complementing paths followed by other messengers. They all are captives of the same gravity.

The next few years and their multi-messenger approaches are thus likely to teach us a variety of new ways of studying the Universe, but only until the telescopes now being built for detecting these newer messengers will reach the limiting capabilities already being approached by optical and radio astronomy – capabilities ultimately hostage to gravitational effects that have already become evident. As we approach the bounds at which the reliability of any and all the messengers reaching us wanes, further observations with instruments already in hand could continue to yield occasional new insights; but building more powerful instruments and observatories with superior sensitivity or resolving powers is likely to yield diminishing returns. A finite set of observational tools could then suffice to take us as far as conceivable instrumentation ever will.

Added insight might then be gained only through extensive searches for events occurring so seldom that their very existence had never been suspected. Searches for these rare occurrences could potentially require century-long surveys conducted with combinations of optical telescopes, radio arrays, cosmicray observatories and other major facilities. To make these affordable they would have to be thoroughly automated. For cost will inevitably determine how well we may eventually come to understand the Universe!

Ultimately, we may find that we already possess whatever instruments could usefully teach us more about the Universe. We may then reach the realization that major cosmic phenomena could exist that simply do not generate messengers able to reach us unscathed and intact, and therefore might never be directly observed. Circumstantial evidence might still convince us that some of these, among them a little-understood *dark matter*, or an equally obscure *dark energy* do indeed generate messengers. But doubt of their utility could long linger.

Two considerations, utility and cost, will then force themselves on us. "Will we astronomers at this stage still be asking for further support for our curiosity-driven cosmic quest?" and "How long will it take us to be satisfied that we have pursued our searches to a fruitful conclusion?"

The simple answer to these questions is that astronomy as conducted at that stage will no longer be particularly interesting or worth pursuing purely by means of remote observations.



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If society, for whatever pressing reason, wished astronomers to pursue their search further than observations from afar permit, the alternative of launching exploratory voyages beyond the bounds of the Solar System might need to be seriously considered, though recognizing that the expenditures entailed in such ventures would, at least as far as we can judge today, vastly exceed those ever spent on observational astronomy.

A third alternative may then still prevail. An unanticipated class of messengers has recently emerged, delivering information previously thought to be well beyond observational reach. They are the asteroids, comets, or similar bodies ejected from exoplanetary systems. Two such interplanetary stragglers have recently been discovered crossing the inner Solar System along distinct trajectories, making it likely that countless similar intruders may follow.

If we can intercept these stragglers with spacecraft to probe their nature up close, sampling their chemical and structural, or even biological properties before they wander on, we could gather valuable evidence, over time, on planetary processes occurring elsewhere in our Galaxy, the Milky Way. This could avoid costly voyages lasting centuries if not millennia because even the nearest stars are so distant and the spacecraft we could readily build are likely to travel at speeds far below the speed of light.

We should, by all means, derive as much information as possible by studying these intruders, though recognizing that the knowledge potentially gained will almost certainly be restricted solely to an improved understanding of nearby exoplanetary systems. Extragalactic astronomy would, most likely, not be significantly advanced.

Given where we now stand today, the present book aims to explore the range of information accessible to us by purely observational means from within the Solar System, and to compile a list of instruments we may ultimately require to observationally derive most of, if not all, the useful information the Universe can convey. For want of a better name, we may call this assembly of instruments the *Cosmic Toolkit*.

Five considerations may illustrate the usefulness of envisioning such a toolkit.

• First, and foremost: We understand the physical processes governing the transmission of information across the Universe rather better, today, than we grasp most of the complex astrophysical phenomena we have by now observed. This includes the formation of stars and planetary systems, the explosions of supernovae, the mergers of galaxies, or any number of other readily observed events. Without our reliance on the properties of transmitted messengers, we would not even be able to speak with confidence about the nature of those more intricate astrophysical processes.



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- Second: Defining the size and complexity of the cosmic toolkit will
  provide us with estimates of the efforts, time, and funds we may need to
  invest on designing and assembling a toolkit containing at least fully
  working prototypes of all required tools. Experience shows that once a
  satisfactory prototype tool is in hand copies of it may generally be
  fabricated at appreciably lower cost.
- Third: Over the past 75 years, ever since the end of World War II, new astronomical tools have been adopted with surprising speed. Given our recognition of the primary messengers transmitting information our way, and knowing how rapidly we have assembled the fraction of the prototypical toolkit already in hand today, it seems quite likely that, at current annual expenditures on astronomy world-wide, the entire toolkit could be in place within another century or two. We may then have to provide many clones of this kit for use in observatories across the globe to discover all we may ultimately come to know. The magnitude of those added expenditures would then depend on how rapidly we want to complete our search.
- Fourth: In the course of investigations to date, we have found that major astronomical discoveries generally follow within less than a century of the first inkling or prediction that some new messenger might exist. On this basis it might take no longer, at current expenditure rates on astronomy internationally, than the one or two centuries just cited, before we have all the tools at hand to complete the toolkit and make the major discoveries they would enable. If so, it would resolve one of the two questions this book seeks to address: "How much will it cost and how long will it take to discover the major astronomical phenomena available cosmic messengers may ultimately reveal?"
- Finally: Once the complete toolkit is firmly in place, the future of observational astronomy, as currently practiced, will gradually phase out, as we successively reach saturation levels on information gathered by available means.

The tools required to capture all reliably available information will define the scope of the entire observational approach. Embarking on cosmic voyages to gather additional information directly would then almost certainly escalate costs unaffordably. Such voyages might conceivably be justified if life on our planet were seriously threatened and finding a new habitat for future generations was the only hope for survival of life as we know it.



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A new realization then arises. With the finite number and scope of the messengers anticipated to complete our quest, we should expect that the number of major cosmic phenomena we may ultimately discover observationally will remain finite as well – as anticipated in my earlier book *Cosmic Discovery*.<sup>3</sup> The larger body of observational data available today permits an update not only of the list of major cosmic phenomena recognized by now, four decades after *Cosmic Discovery*'s original compilation. It also enables a new estimate of the total number we may ultimately discover, which still appears to be of the order of 100. Other phenomena might then still exist, but the finite scope of the messengers reliably transmitted might not suffice to reveal them.

Some readers may be troubled by the thought that the scope of astronomy may ever become bounded – that someday we may find ourself blocked from learning more. We have become so used to taking for granted that new vistas will always be revealed by ever-improved instruments, that it now seems difficult to accept that technological advances will no longer serve a purpose once further information transmitted by cosmic messengers arrives corrupted.

With these introductions on the ways in which the Universe transmits information, the costs entailed in the capture and reading of that information, and the ultimate limits these two factors impose on how much we may learn about the Universe, we now can outline four questions the book addresses in its four distinct parts.

Part I *Instruments, Messengers, and Cosmic Messages* sketches the role that instrumentation and observations have played in advancing the scope of astrophysics and cosmochemistry.

Part II *The Bounded Energies of Nature's Messengers* dwells on the nature of the particles and radiations conveying all the information reaching us today. It explains how basic physical processes inevitably restrict the energies of different types of messengers to distinct, bounded energy ranges.

Part III *Parameters Specifying Individual Messengers* historically traces instrumental achievements to date, and lists fundamental limits on the information that any known messengers will ever be able to transmit.

Finally, Part IV *The Pace of Progress* lists the most powerful astronomical gains to date, and cites the expense of implementing the observational means and the sociological changes inevitably entailed. It envisions projections into the future based on the history of discoveries to date.

Astronomy as pursued today has been enabled by fundamentally changed social structures, largely reflected in rapidly expanding research groups and the progressive compartmentalization of skills. In turn these changes have raised



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economic and societal concerns we still have not satisfactorily resolved, but urgently should. They directly affect the lives of the very people on whose efforts the future of astronomy will ultimately depend.

If our research has been sound, it will provide alternatives for society to consider once we arrive at the edge of knowledge, look out into our surroundings as far as observations alone will ultimately permit, and humanity's isolation and confinement to our Solar System begins to fully sink in.

Martin Harwit

#### **Notes**

- 1 Cosmic Discovery: The Search, Scope and Heritage of Astronomy, M. Harwit, Basic Books, New York, 1981; reissued by Cambridge University Press, 2019
- 2 In Search of the True Universe: The Tools, Shaping, and Cost of Cosmological Thought, M. Harwit, Cambridge University Press, 2013
- 3 Ibid., Harwit, 1981



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James R. Johnson, a dedicated amateur scientist and author of a recent book *Comprehending the Cosmos: An Analysis*, envisions the world around us almost entirely in terms of visual displays – symmetries, diagrams, groupings, flow charts – as well as thought-provoking epithets. I greatly appreciated his unique perspective and recommendations.

Three anonymous referees provided useful assessments solicited by Cambridge University Press. I am indebted to them for their forthright advice.

Rachel Ivie at the American Institute of Physics pointed out the existence of a substantive literature on the prospects of young researchers currently entering astronomy and astrophysics as graduate students or as post-docs. And Drew Brisbin, a former student at Cornell and now a member of the post-doc community, commented on the topic from a personal perspective.

Help with finding some of the early photographs appearing in the book, and possibly finding who might own their copyrights, was provided by a number of colleagues. Jean E. Mueller provided advice on the holdings of the Palomar and Caltech archives, from which the portrait of Fritz Zwicky in Figure 5.1 was obtained. Ruth Isaacson of the Genetics Society of America secured permission for me to publish the likeness of Ronald Aylmer Fisher, in Figure 8.3. Thijs de Graauw, now working on a space project with colleagues in Russia, indefatigably searched for the copyright ownership of the expressive portrait



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of Yakov Borisovich Zel'dovich reproduced in Figure 2.3. Haruyuki Okuda and Takenori Nakano in Japan similarly were of great help in documenting Chushiro Hayashi's portrait dating back to the early 1940s, and now appearing as Figure 2.4. Nakano, who had carefully preserved this photo since his own student days at that time, later worked closely with Hayashi on a theory of how stars first form. I am deeply indebted to all of these colleagues for taking the time to help me locate the ownership of these sources.

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Our children, Alex Harwit, Eric Harwit, and Emily Harwit, provided unique perspectives, largely shaped by their respective professional insights. Alex is a physicist and engineer in the US aerospace industry. Eric has written about recent industrial revolutions in Asia, primarily in China. Emily has worked on aid to post-war nations and other fragile economies. In one way or another all these communities are seeking to adjust to an ever-evolving modern technological world – as is the astrophysics research community world-wide.

Finally, I want to thank my wife Marianne who, over the years, was always first to read a new book I was drafting. She'd comment on style and substance, and often could sense uncertainties reflected in awkward sentences. Questions she had asked on reading earlier books seemed to her to have accumulated, largely unanswered in this new draft, though by now so obvious that they no longer should or could be ignored. If I was writing about the scope of astronomy and how far we might still have to go, would it not also make sense to ask who was going to pay the bills for all that? And why should they want to, given so many other urgent priorities crying out for support? Did astronomers have a right to ask for public support without at least attempting to assess



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how many more years, decades, centuries, perhaps even millennia, it might take to satisfactorily understand our Universe? And even if we could answer that question, would the cost of that enterprise demand ever greater public expenditures? And then, what could society reasonably expect to gain in return. Not everybody, after all, was interested in what was going on out there in the Universe, when school lunches for their children were a more immediate concern.

It seemed to me that perhaps, but only perhaps, answers to some of those questions might by now lie within reach, emerging from a perspective of astronomy gained not by immediately focusing on major phenomena, such as supernova explosions, newly discovered planetary systems, or the nature of the microwave background radiation, but by asking how the information gained on these phenomena was reaching us. Which were the messengers, the light, the radio waves, the cosmic rays, the occasional meteorites falling on Earth, conveying all this information? How much more information than we already had in hand by now, could we, in principle, extract from them? And if that information was finite, how could we best assess its scope and how much its extraction could cost?

Initially, I wasn't sure I could succeed in answering all these questions. I thought we still knew too little. Later, it occurred to me that we might know enough, by now, to at least partially assess what observational astronomy and astrophysics might eventually offer.

This book describes the perspective that largely guided me. Finding how earlier generations of astronomers advanced our understanding also inspired me throughout. Readers will find me often referring to their work as I provide my best estimate of how far astronomical observations will ultimately take us; where the end of the current journey may lie; how long it may take us to get there; what we may have learned by the time we arrive; and what all this will cost.

I hope readers will find the book useful.





## How to Read This Book

I wrote this book for people interested in the Universe around us. This includes astronomers and astrophysicists, of course. But physicists, chemists, biologists, mathematicians, public officials, governmental representatives, and others may also wish to read it. Many among them will be unfamiliar with astronomical usage – definitions, symbols, various units of energy, time, length, energy, and how these are mutually related.

The book's Appendix provides much of this information, defining vocabulary terms, supplying lists of commonly used symbols, and tabulating units. It begins with an alphabetically ordered list of symbols and then provides a dictionary of commonly used astronomical and physical terms. A number of tables and sketches clarify relationships.

The Appendix is self-contained. It does not list pages in the text on which a given subject is further discussed.

The Index at the book's end is similarly self-contained. It excludes material listed in the Appendix, but serves as a guide to the main pages in Chapters 1 to 9, on which an indexed topic, or the contributions made by a listed scientist may be found. The Index and Appendix thus serve parallel purposes, the Appendix largely acting as a dictionary for non-astronomers, the index guiding readers to subjects of especial interest.

For astronomers, astrophysicists, and other scholars wishing to pursue presented topics to greater depths, the text interweaves references to original papers in which a subject being discussed may be further pursued. Within a chapter these references are marked by numbered superscripts and then spelled out in numerical order in the Notes ending the chapter. Because the text is intended to be self-contained, most readers may rarely need to turn to these added sources.

Occasional footnotes within the text provide supplementary information tagged by alphabetic superscripts. The supplement then appears in smaller print at the bottom of the tagged page.

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