# Contents

**Preface**  
page xvii

1  **Some Preliminaries**  
1.1 Introduction to the Linear Theory of Sound Wave Motion  
1.1.1 Linearization Hypothesis  
1.1.2 Partitioning Turbulent Fluctuations  
1.1.3 Linearization of Inviscid Fluid Flow  
1.1.4 Evolution of Non-Linear Waves  
1.2 Representation of Acoustic Waves in the Frequency Domain  
1.2.1 Fourier Transform  
1.2.2 Periodic Functions  
1.2.3 Impulse Sampling  
1.2.4 Power Spectral Density  
1.3 Representation of Waves in the Wavenumber Domain  
1.3.1 Spatial Fourier Transform  
1.3.2 Briggs’ Criterion  
1.4 Intensity and Power of Sound Waves  
1.5 Introduction to the Linear System View of Duct Acoustics  
References  

2  **Introduction to Acoustic Block Diagrams**  
2.1 Introduction  
2.2 Classification of Acoustic Models of Ducts  
2.2.1 Classification by Number of Ports  
2.2.2 Classification by Type of Port  
2.2.2.1 One-Dimensional Elements  
2.2.2.2 Modal Elements  
2.3 Mathematical Models of Acoustic Elements  
2.3.1 One-Port Elements  
2.3.2 Two-Port Elements  
2.3.3 Multi-Port Elements  
2.4 Assembly of Blocks  
2.4.1 Assembly of Two-Ports  
2.4.2 Assembly of Multi-Ports
2.4.3 Optimization of Global Matrix Size 41
2.4.4 Contraction of Assembled Modal Two-Ports 42
2.5 Acoustic Elements Based on Numerical Methods 44
  2.5.1 The Finite Element Method 45
  2.5.2 The Boundary Element Method 49
2.6 Programming Considerations 51

References 58

3 Transmission of Low-Frequency Sound Waves in Ducts 59
  3.1 Introduction 59
  3.2 One-Dimensional Theory of Sound Propagation in Ducts 60
    3.2.1 Unsteady Flow Equations 60
    3.2.2 Equations Governing Acoustic Wave Motion 63
  3.3 Solution of Linearized Acoustic Equations 65
    3.3.1 Homogeneous Ducts ($\varepsilon' = 0$) 66
      3.3.1.1 Uniform Ducts 67
      3.3.1.2 Homogeneous Non-Uniform Ducts 68
    3.3.2 Inhomogeneous Ducts ($\varepsilon' \neq 0$) 68
    3.3.3 Numerical Matrizant Method 69
  3.4 Time-Averaged Power of One-Dimensional Acoustic Waves 72
  3.5 Hard-Walled Uniform Ducts 73
    3.5.1 Wave Transfer Matrix 74
    3.5.2 Traveling Waves and Direction of Propagation 75
    3.5.3 Reflection Coefficient and Standing Waves 77
    3.5.4 Lumped Acoustic Elements 79
  3.6 Hard-Walled Homogeneous Ducts with Non-Uniform Cross Section 80
    3.6.1 Wave Transfer Matrix 81
    3.6.2 High Frequency Approximation 84
  3.7 Ducts Packed with Porous Material 85
  3.8 Acoustic Boundary Conditions on Duct Walls 87
    3.8.1 Impermeable Walls 87
      3.8.1.1 No-Slip Model 87
      3.8.1.2 Full-Slip Model 88
      3.8.1.3 Partial-Slip Model 88
      3.8.1.4 Rough-Wall Model 89
      3.8.1.5 Unified Boundary Condition 90
    3.8.2 Permeable Walls 91
  3.9 Homogeneous Ducts with Impermeable Finite Impedance Walls 92
    3.9.1 Non-Uniform Duct 93
    3.9.2 Uniform Duct 95
      3.9.2.1 Direction of Propagation 97
      3.9.2.2 Wave Equation 98
      3.9.2.3 Impedance Eduction Formula 98
      3.9.2.4 Peripherally Non-Uniform Wall Impedance 98
### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.9.3 Finite Wall Impedance Models</td>
<td>99</td>
</tr>
<tr>
<td>3.9.3.1 Lined Impermeable Walls</td>
<td>99</td>
</tr>
<tr>
<td>3.9.3.2 Permeable Rigid Porous Walls</td>
<td>102</td>
</tr>
<tr>
<td>3.9.3.3 Perforated Rigid Walls</td>
<td>103</td>
</tr>
<tr>
<td>3.9.3.4 Turbulent Boundary Layer Over Rigid Walls</td>
<td>105</td>
</tr>
<tr>
<td>3.9.3.5 Elastic Walls</td>
<td>105</td>
</tr>
<tr>
<td>3.10 Inhomogeneous Ducts</td>
<td>106</td>
</tr>
<tr>
<td>3.10.1 Linearized Energy Equation</td>
<td>106</td>
</tr>
<tr>
<td>3.10.2 Matrizant of a Duct with Finite Impedance Walls</td>
<td>107</td>
</tr>
<tr>
<td>3.10.3 Hard-Walled Ducts with Mean Temperature Gradient</td>
<td>109</td>
</tr>
<tr>
<td>3.11 Ducts with Two-Phase Flow</td>
<td>112</td>
</tr>
<tr>
<td>3.12 Ducts with Time-Variant Mean Temperature</td>
<td>116</td>
</tr>
<tr>
<td>References</td>
<td>122</td>
</tr>
</tbody>
</table>

#### 4 Transmission of One-Dimensional Waves in Coupled Ducts

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Introduction</td>
<td>125</td>
</tr>
<tr>
<td>4.2 Quasi-Static Theory of Wave Transmission at Compact Junctions</td>
<td>125</td>
</tr>
<tr>
<td>4.2.1 Quasi-Static Conservation Laws</td>
<td>125</td>
</tr>
<tr>
<td>4.2.2 Transformation to Pressure Wave Components</td>
<td>128</td>
</tr>
<tr>
<td>4.3 Two Ducts Coupled by Forming Sudden Area Change</td>
<td>129</td>
</tr>
<tr>
<td>4.3.1 Open Area Change</td>
<td>131</td>
</tr>
<tr>
<td>4.3.1.1 The Case of Zero Mean Flow</td>
<td>134</td>
</tr>
<tr>
<td>4.3.1.2 Core-Flow Model</td>
<td>134</td>
</tr>
<tr>
<td>4.3.1.3 Effect of Inner Duct Wall Thickness</td>
<td>139</td>
</tr>
<tr>
<td>4.3.2 Closed Area Changes</td>
<td>140</td>
</tr>
<tr>
<td>4.3.3 End-Correction</td>
<td>141</td>
</tr>
<tr>
<td>4.4 Sudden Area Changes Formed by Multiple Ducts</td>
<td>143</td>
</tr>
<tr>
<td>4.4.1 Identical Inner Ducts</td>
<td>144</td>
</tr>
<tr>
<td>4.4.2 Staggered Inner Duct Extensions</td>
<td>145</td>
</tr>
<tr>
<td>4.4.3 Duct Splits</td>
<td>146</td>
</tr>
<tr>
<td>4.5 Wave Transmission Through a Perforated Rigid Baffle</td>
<td>147</td>
</tr>
<tr>
<td>4.5.1 Area-Change Model</td>
<td>148</td>
</tr>
<tr>
<td>4.5.2 Lumped Impedance Model</td>
<td>149</td>
</tr>
<tr>
<td>4.6 Wave Transmission in Junction Cavities</td>
<td>150</td>
</tr>
<tr>
<td>4.6.1 Multi-Duct Junction</td>
<td>151</td>
</tr>
<tr>
<td>4.6.2 Two-Duct Junction</td>
<td>152</td>
</tr>
<tr>
<td>4.7 Continuously Coupled Perforated Ducts</td>
<td>153</td>
</tr>
<tr>
<td>4.7.1 Single-Coupled Perforated Ducts</td>
<td>156</td>
</tr>
<tr>
<td>4.7.1.1 Identical Perforated Ducts</td>
<td>159</td>
</tr>
<tr>
<td>4.7.2 Double-Coupled Perforated Ducts</td>
<td>160</td>
</tr>
<tr>
<td>4.8 Row-Wise Coupled Perforated Ducts</td>
<td>162</td>
</tr>
<tr>
<td>4.8.1 Wave Transfer Across a Row of Apertures</td>
<td>163</td>
</tr>
<tr>
<td>4.8.2 Single-Coupled n-Duct Section</td>
<td>165</td>
</tr>
<tr>
<td>4.8.3 Double-Coupled n-Duct Section</td>
<td>166</td>
</tr>
<tr>
<td>4.8.4 Wave Transfer Matrix of n-Duct Element</td>
<td>167</td>
</tr>
</tbody>
</table>
## Table of Contents

### 4.9 Dissipative Units and Lined Ducts

### 4.10 Wave Transfer Across Adiabatic Pressure Loss Devices

### References

### 5 Resonators, Expansion Chambers and Silencers

#### 5.1 Introduction

- 5.1.1 Mufflers and Silencers
- 5.1.2 The System and Its Environment

#### 5.2 Transmission Loss

- 5.2.1 Single-Frequency Analysis
- 5.2.2 Overall Transmission Loss

#### 5.3 Duct Resonances

- 5.3.1 Resonance and Anti-Resonance Frequencies
- 5.3.2 Resonators
- 5.3.3 Single-Duct Resonator
- 5.3.4 Resonators with Open Outlet
- 5.3.5 Helmholtz Resonator
- 5.3.6 Transmission Loss of Resonators
- 5.3.7 Interferential Resonator
  - 5.3.7.1 Wave Transfer Matrix of Parallel Two-Ports
  - 5.3.7.2 Wave Transfer Matrix of the HQ Tube
  - 5.3.7.3 Herschel–Quincke Tube Resonator
- 5.3.8 Straight-Through Resonator

#### 5.4 Expansion Chambers

- 5.4.1 Through-Flow Expansion Chambers
- 5.4.2 Transmission Loss of Expansion Chambers
- 5.4.3 Pure Expansion Chambers
- 5.4.4 The Strongest Pure Expansion Chamber
- 5.4.5 Tuning Inlet and Outlet Duct Extensions
- 5.4.6 Effect of Inlet and Outlet Duct Configurations
- 5.4.7 Division of a Pure Expansion Chamber
- 5.4.8 Low-Frequency Response of Chambers
- 5.4.9 Chambers with a Perforated Duct Bridge
- 5.4.10 Packed Chambers

#### 5.5 Reciprocal Two-Ports

#### 5.6 Some Practical Issues

- 5.6.1 Irregular Geometry
- 5.6.2 Variable Mean Flow Conditions
- 5.6.3 Multiple Outlet Ducts
- 5.6.4 Flow Excited Resonators and Chambers

#### 5.7 Flow Rate and Back-Pressure Calculation

- 5.7.1 Calculation of Mean Temperature Drop

#### 5.8 Shell Noise

#### References
6 Multi-Modal Sound Propagation in Ducts

6.1 Introduction 238

6.2 Uniform Ducts with Axial Mean Flow 239

6.3 Boundary Condition on Impermeable Walls 243

6.3.1 No-Slip Model 243

6.3.2 Partial-Slip Model 244

6.3.3 Ingard–Myers Model 244

6.3.4 Modified Ingard–Myers Models 245

6.4 Wave Transmission in a Uniform Duct with Uniform Mean Flow 247

6.4.1 General Solution of the Convected Wave Equation 247

6.4.2 Modal Wave Transfer Matrix 249

6.5 Hard-Walled Ducts with Uniform Mean Flow 250

6.5.1 Eigenvalues and the Orthogonality of Eigenfunctions 251

6.5.2 Propagating and Evanescent Modes 252

6.5.3 Modal Propagation Angles 253

6.5.4 Transverse Modes of Common Duct Sections 255

6.5.4.1 Rectangular Ducts 255

6.5.4.2 Hollow Circular Ducts 257

6.5.4.3 Annular Circular Ducts 260

6.5.4.4 Spinning Modes 261

6.5.5 Numerical Determination of Transverse Duct Modes 263

6.5.6 Time-Averaged Acoustic Power 265

6.6 Hard-Walled Uniform Ducts Packed with Porous Material 268

6.7 Lined Uniform Ducts with Uniform Mean Flow 269

6.7.1 Dispersion Equation for Uniformly Lined Circular Ducts 270

6.7.1.1 Hard-Liner Solution for Hollow Ducts 270

6.7.1.2 Iterative Graphical Solution 272

6.7.2 Dispersion Equations for Uniformly Lined Rectangular Ducts 273

6.7.3 Discussion of Transverse Modes 276

6.7.3.1 Surface Modes 278

6.7.3.2 Orthogonality of Modes 279

6.7.4 Liner Optimization 280

6.7.5 Multi-Modal Attenuation Characteristics 282

6.7.6 Non-Uniformly Lined Ducts 285

6.8 Uniform Ducts with Sheared Mean Flow 287

6.8.1 Solution of the Pridmore-Brown Equation 288

6.8.2 Effect of the Mean Boundary Layer Thickness 290

6.9 Ducts with Axially Non-Uniform Cross-Sectional Area 294

6.10 Circularly Curved Ducts 299

6.10.1 Rectangular Ducts 303

6.10.2 Numerical Determination of Angular Wavenumbers 309

6.10.3 Fundamental-Mode Approximation 310

6.11 Uniform Ducts with Mean Swirl 312
Table of Contents

6.12 Ducts with Mean Temperature Gradient
  6.12.1 Ducts without Mean Flow 317
  6.12.2 Effect of Mean Flow 321
References 322

7 Transmission of Wave Modes in Coupled Ducts
  7.1 Introduction 326
  7.2 Weak Form of the Convected Wave Equation 327
  7.3 Ducts with Identical Sections 328
  7.4 Sudden Area Changes 332
    7.4.1 Open Sudden Expansion 333
    7.4.1.1 Closed Through-Flow Expansion 335
    7.4.1.2 Closed Flow-Reversing Expansion 336
    7.4.2 Sudden Area Contraction 337
    7.4.3 Open Area Change with Multiple Ducts 337
  7.5 Perforated Baffles 340
  7.6 Cavity Coupled with Multiple Ducts 342
    7.6.1 Closed Cavity Modes 343
    7.6.2 The Green Function of the Cavity 344
    7.6.3 Coupling the Cavity with Ducts 345
  7.7 Coupled Perforated Ducts 347
    7.7.1 Acoustic Field in a Duct with a Single Aperture 349
    7.7.2 Wave Transfer Across a Row of Apertures 351
    7.7.3 Dissipative Silencers 355
    7.7.4 Lined Ducts 358
  7.8 Contracted Models of Silencers 358
    7.8.1 Expansion Chamber with Offset Inlet and Outlet Ducts 359
    7.8.2 Expansion Chamber with Double Outlet 361
    7.8.3 Flow-Reversing Chamber 362
    7.8.4 Through-Flow Resonator and Muffler 363
    7.8.5 Three-Pass Muffler 365
References 367

8 Effects of Viscosity and Thermal Conductivity
  8.1 Introduction 369
  8.2 Convected Wave Equation for a Viscothermal Fluid 370
  8.3 Low Reduced Frequency Theory 372
    8.3.1 Circular Hollow Ducts 373
      8.3.1.1 Hard-Walled Ducts 375
      8.3.1.2 Wide-Duct Approximation 377
      8.3.1.3 Effect of Parabolic Mean Flow Velocity Profile 379
      8.3.1.4 Effect of Turbulent Boundary Layer 380
      8.3.2 Circular Annular Ducts 381
    8.3.3 Rectangular Ducts 383

References
# Table of Contents

8.4 Time-Averaged Acoustic Power 386  
8.5 Sudden Area Changes and Junctions 388  
8.6 Coupled Narrow Ducts with Porous Walls 392  
References 397

9 Reflection and Radiation at Open Duct Terminations 399  
9.1 Introduction 399  
9.2 Reflection Matrix and End-Correction 400  
9.3 Flanged and Unflanged Open Terminations without Mean Flow 401  
\hspace{1em} 9.3.1 Exterior Surface Helmholtz Equation 401  
\hspace{1em} 9.3.2 Flanged Open End 402  
\hspace{2em} 9.3.2.1 Circular Ducts 404  
\hspace{1em} 9.3.3 Unflanged Open End 406  
9.4 Reflection Matrix at an Unflanged Open End with Mean Flow 408  
\hspace{1em} 9.4.1 The Exhaust Problem 408  
\hspace{2em} 9.4.2 Circular Duct 411  
\hspace{3em} 9.4.2.1 Plane-Wave Reflection Coefficient 414  
\hspace{3em} 9.4.2.2 Reflection of Higher-Order Incident Modes 416  
\hspace{1em} 9.4.3 Reflection at Flow Intakes 417  
\hspace{2em} 9.4.3.1 Plane-Wave Reflection Coefficient 418  
9.5 Acoustic Radiation from Open Ends of Ducts 419  
\hspace{1em} 9.5.1 Modal Radiation Transfer Function 419  
\hspace{1em} 9.5.2 Radiated Acoustic Power 419  
\hspace{2em} 9.5.3 Flanged Open End without Mean Flow 421  
\hspace{3em} 9.5.3.1 Circular Ducts 422  
\hspace{3em} 9.5.3.2 Rectangular Ducts 424  
\hspace{1em} 9.5.4 Unflanged Circular Open End without Mean Flow 426  
\hspace{2em} 9.5.5 Radiation from Unflanged Circular Open End with Mean Flow 427  
\hspace{1em} 9.5.6 Simple-Source Approximation 429  
\hspace{2em} 9.5.6.1 Effect of Vorticity 431  
\hspace{1em} 9.5.7 Power Source Model 432  
\hspace{1em} 9.5.8 Effect of Reflecting Surfaces 433  
References 435

10 Modeling of Ducted Acoustic Sources 438  
10.1 Introduction 438  
10.2 One-Port Sources Characterized by Unsteady Mass Injection 440  
10.3 Moving the Active Plane of One-Port Sources 446  
10.4 Two-Port Sources Characterized by Fluctuating Force Application 451  
\hspace{1em} 10.4.1 Flow Noise 457  
10.5 Two-Port Sources Characterized by Ducted Combustion 459  
\hspace{1em} 10.5.1 Combustion Oscillations and Instability 463  
10.6 Moving Source Planes of Two-Port Sources 469
10.7 Ducted Loudspeakers 470
References 472

11 Radiated Sound Pressure Prediction 474
11.1 Introduction 474
11.2 Calculation of Sound Pressure Field of Ducted Sources 474
   11.2.1 Ducted One-Port Sources 476
      11.2.1.1 One-Dimensional Sources 478
   11.2.2 Ducted Two-Port Sources 480
   11.2.3 Multiple Radiating Outlets 482
11.3 Analysis of Sound Pressure 483
11.4 Insertion Loss 487
   11.4.1 Noise Reduction 488
   11.4.2 Attenuation 490
11.5 Multi-Modal Transmission Loss Calculations 491
11.6 In-Duct Sources Characterized by Acoustic Power 494
   11.6.1 The ASHRAE Method 497
References 500

12 Measurement Methods 501
12.1 Introduction 501
12.2 Measurement of In-Duct Acoustic Field 501
   12.2.1 Multi-Modal Wave Field Decomposition 502
   12.2.2 The Two-Microphone Method 503
      12.2.2.1 Calibration of Microphones 505
      12.2.2.2 Signal Enhancement 507
   12.2.3 Measurement of the Plane-Wave Reflection Coefficient 507
   12.2.4 Measurement of Wavenumbers 508
12.3 Measurement of Passive Acoustic Two-Ports 510
   12.3.1 Basics of the Four Microphone Method 510
   12.3.2 Measurement of Attenuation 511
   12.3.3 Measurement of Transmission Loss 512
   12.3.4 Measurement of the Wave Transfer Matrix 512
12.4 Measurement of One-Port Source Characteristics 514
   12.4.1 The Two-Load Method 516
      12.4.1.1 Implementation with Non-Calibrated Loads 516
      12.4.1.2 Implementation with Calibrated Loads 517
   12.4.2 Geometrical Interpretation of the Two-load Method 517
   12.4.3 The Apollonian Circle of Two Loads 518
      12.4.3.1 Upper and Lower Bounds for Source Pressure Strength 520
   12.4.4 Calculation Bounds to Sound Pressure 522
   12.4.5 The Three-Load Method 523
12.4.6 Over-Determined Methods
  12.4.6.1 Over-Determined Two-Load Method 525
  12.4.6.2 Over-Determined Three-Load Method 525
12.4.7 The Fuzzy Two-Load Method 526
12.4.8 The Explicit N-Load Method 527
12.5 Measurement of Two-Port Source Characteristics 530
References 530

13 System Search and Optimization 533
13.1 Introduction 533
13.2 Direct Random Search 534
13.3 Interval Analysis 539
13.4 The Inverse Method 540
  13.4.1 Acoustic Path Space 540
  13.4.2 Acoustic Path Space on the Attenuation Plane 543
  13.4.3 Signature of Acoustic Paths 546
  13.4.4 System Search in Acoustic Path Space 548
  13.4.5 Acoustic Path Spaces for Different Targets 549
    13.4.5.1 Noise Reduction 549
    13.4.5.2 Insertion Loss 550
References 551

Appendix A Basic Equations of Fluid Motion 552
A.1 Integral Forms of Conservation Laws 552
  A.1.1 Conservation of Mass 553
  A.1.2 Conservation of Momentum 553
  A.1.3 Conservation of Energy 554
A.2 State Equations and the Speed of Sound 555
A.3 Equations of Motion of Ideal Fluids 556
  A.3.1 Continuity Equation 556
  A.3.2 Momentum Equation 557
  A.3.3 Energy Equation 557
A.4 Equation of Motion of Newtonian Fluids 558
  A.4.1 Momentum Equation 558
  A.4.2 Energy Equation 559
References 560

Appendix B Acoustic Properties of Rigid-Frame Fibrous Materials 561
References 565

Appendix C Impedance of Compact Apertures 567
C.1 Empirical and Semi-Empirical Models 567
C.2 Theoretical Models 571
References 577

Index 579