Introduction

Why Study Mathematical Commentaries?

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Strictly speaking, this book is not about the history of mathematics, except indirectly. Rather, it is devoted to commentaries, a specific textual form that many practitioners of mathematics, like other scholars active in a variety of other domains, have sometimes chosen in order to carry out their inquiries. To the modern reader, commentaries might look like a rather odd phenomenon: but until early modern times, for an author to write a commentary on a base text of any sort was quite a common choice of genre, and the practice has certainly not disappeared in our own age. By “commentary,” here, we do not mean merely the activity of jotting down one’s thoughts for oneself while reading a text or afterwards, but, instead, deciding to compose not only for oneself but above all for other readers (known and unknown students, colleagues, and others) a kind of text that complies with a base text but remains ancillary to it.

Doubtless, when many modern readers hear the word “commentaries,” they are likely to think in the first instance of exegetical texts composed about writings like Aristotle’s or Homer’s or the Bible. But in fact the genre of commentaries was not limited to philosophy or literature or theology. On the contrary, even in the mathematical and medical sciences, for example, writing a commentary on someone else’s text was an activity that was also frequently performed by practitioners. This was not only because these scientific disciplines were tightly connected with other fields of scholarship, but also because writing in the genres of commentaries was a practice shared in common by most forms of institutionalized intellectual pursuits.

The geographic distribution of this genre of writing is another striking phenomenon, since commentaries have actually represented a worldwide...
phenomenon, or almost so. They are found among cuneiform writings, at least from the first millennium BCE. They are also encountered among Chinese sources, at least after the establishment of the Empire in the third century CE. They likewise exist in ancient Greek, Arabic, Latin, and Sanskrit, until at least early modern times.

What motivated the writing of such kinds of texts? Why did actors opt for this genre of writing at so many times, and in which contexts? Why did this genre decline, or has it indeed really done so? These are some of the obvious questions one might ask.

However, in taking mathematical commentaries as the topic of this book, we also aim to address a number of deeper questions that might well be of interest for the history of science at large as well as for classical studies considered globally.

For the history of science, commentaries constitute a kind of source that is extraordinarily precious because they can provide a rare glimpse into the concrete practices in terms of which actors read and dealt with the scientific documents that they used. This is manifestly an issue of major importance for the field; however, there has not yet been a consistent effort to make use of this kind of evidence, in part because of a dearth of published sources. Herein precisely lies one of the major benefits that we suggest could be derived from doing research on commentaries. They document precisely how actors of the past interpreted the ancient writings they relied on – or at least how their authors wished to present themselves to their readers in their engagement with those writings. They thus offer us the opportunity to develop systematically a source-based history of the reading of those mathematical and other scientific texts that were the object of commentaries at different time periods, as well as a history of the use of mathematical and other scientific knowledge and practices in interpreting ancient texts that were not fundamentally mathematical or scientific in character. Commentaries further yield evidence that allows us to enter into ancient attitudes and expectations towards earlier mathematical and other scientific texts and into the modes of self-presentation of commentators on such texts.

For classical studies, on the other hand, the issue of comparing across a range of disciplinary and geographical contexts the practices to which we refer in using the same term "commentaries" is an important one – especially so, if one pursues, as we do, the project of breaking down the barriers that have traditionally separated the various antiquities and instead considers them globally. In this respect, mathematical commentaries represent a unique opportunity, since they offer a specific outlook on commentaries worldwide involving a highly determinate, sophisticated, and rule-bound discipline: mathematics.
1.1 Status quaestionis

This book is premised upon two observations: that in general commentaries *qua* commentaries have long been a relatively neglected type of writing; and that commentaries that deal in particular with scientific matters have tended to seem especially uninteresting to many modern scholars. The editors and authors of this volume are animated instead by the conviction that this neglect and lack of interest constitute a serious deficiency in the study of scientific sources, one that represents an important loss both for the study of commentaries *qua* commentaries and for the history of science.

1.1.1 Commentaries as Tools to Interpret the Base Text or as Writings Standing on Their Own

Traditionally, commentaries in all fields have tended to be studied above all for the sake of the light they could be thought capable of casting on the meaning of the base texts they commented on. This practice amounted to tacitly (and sometimes involved explicitly) denying the significance of the contribution to knowledge that commentaries themselves might be able to make, and attributing such a contribution exclusively to the base text: if the commentary helped us to understand the base text better, then it was worth studying, but only to the extent that it performed this function well; if it did not help us to understand the base text better, it could be safely ignored. Accordingly, commentaries were most often edited primarily for the use of scholars interested in the base texts, but only rarely were they translated or commented upon themselves so that they could thereby be made more widely available as documents worthy of attention in their own right.

These assertions hold true in general, and they hold true in particular for mathematical sources. The Sanskrit treatise *Āryabhaṭīya*, which Āryabhaṭa is considered to have completed in 499, is a case in point. The oldest commentary on this treatise, composed in the seventh century by Bhāskara I, has been used exclusively in order to interpret the base text, but until recently it was never studied in its own right, *qua* commentary. The first critical edition of the commentary was not published until 1976 (Shukla 1976), in contrast with Bhāskara I’s non-commentarial writings, which began to be published thirty years earlier, in 1946 (Keller 2006, vol. 2: 228). It was only in 2006 that part of Bhāskara I’s commentary was translated for the first time into any European language at all (Keller 2006). Likewise, one could argue that the strong interest in the ancient commentaries on Aristotle’s writings, which is manifested in the massive editorial work on them that was carried out at the end of the nineteenth
and the beginning of the twentieth centuries, followed a similar pattern. Interest in these commentaries was motivated primarily by the help they were thought to be able to provide for the interpretation of Aristotle’s works and by the evidence they yielded about authors whose works had not been handed down in direct transmission (Most (ed.) 1997). And while the Greek texts of the extant commentaries on Aristotle were all published in critical editions between 1882 and 1909, they only began to appear in English translation a century later, in 1989; as of this date, the series of English translations is still unfinished. Moreover, to this day, as far as we know, not one of the ancient commentaries on Aristotle has received a full-scale modern commentary in any language. And it is only very recently indeed that a project to edit the medieval Greek commentaries on Aristotle’s writings has been initiated.1

More generally, base texts have almost invariably been translated in modern times without accompanying translations of the commentaries with which for centuries the actors themselves had studied and learned to use them. This is for example the case for the Chinese mathematical canonical text entitled The Nine Chapters on Mathematical Procedures (Jiuzhang suanshu 九章算術), whose completion we date to the first century (see Chapter 6 by Chemla and Zhu in this volume). The first translation of this Chinese classic was into Russian by Elvira I. Biërëzkina, published in 1957, on the basis of which Kurt Vogel prepared his German translation (Vogel 1968). Although all extant ancient Chinese editions of The Nine Chapters included Liu Hui’s 刘徽 commentary (completed in 263) and Li Chunfeng 李淳風 et al.’s subcommentary (presented to the throne in 656), these two European translations only gave the main text and did not include any of the commentaries, as a whole or even in part.

It is noteworthy that, in certain exceptional cases in which commentaries were indeed translated, as for example in Henry Thomas Colebrooke’s early translation of Sanskrit mathematical sources (1817), their textual features and their integrity were significantly altered. Such translations excerpted from the commentaries only those bits that were thought to be useful and, most tellingly, they printed these as sporadic footnotes to the main text, omitting the rest.2 Whether the commentaries were omitted or presented only as marginal nuggets, the modern reader was not provided


any access to the base text in ways that reflected the experience of ancient readers.

When commentaries did prove to be innovative, and their contribution was not limited to a mere elucidation of the base text, they were often treated as writings subsequent to the base text and essentially independent of it, but not as commentaries on it – in a sense, scholars behaved as though the fact that these writings had assumed the form of a commentary on a text were immaterial or arbitrary. Proclus’ commentary on Book 1 of Euclid’s Elements, for example, which is the object of Harari’s Chapter 2 in this book, has until recently mostly been read as if it were a stand-alone treatise on the philosophy of mathematics, not as a reliable or even interesting exegesis of the text of Euclid.

1.1.2 The Recent Increase in Commentary Studies and Their Lacunae

Only in recent decades has a sustained attention to commentaries themselves, considered in their own right, begun to develop, and one might reflect on the reasons for this development. Scholars have recently started to direct their attention to various aspects, such as commentarial procedures, language, didactic techniques, institutional contexts, implicit disciplinary presuppositions, and so on. This phenomenon has affected commentaries written in a large number of languages, in Greek and Latin, in Sanskrit, Tamil, Chinese, cuneiform, and Arabic, to mention only these. One encouraging sign of this development is the recent appearance of special issues of two very different scholarly journals, Philological Encounters and Historia Mathematica, both devoted to commentaries.3

Some of these studies have been devoted to specific commentaries, like Rudolf Wagner’s books on Wang Bi’s 王弼 third-century commentary on the Dao De Jing 道德經.4 Some have compared different commentaries on the same base text, for example, Wang Bi’s and Heshang Gong’s commentaries on the Laozi (Chan 1991). Other studies have borne on commentarial traditions within a whole canon, for example, the Confucian canon (Henderson 1991, Makeham 2003). Yet others have embraced commentaries in the context of an entire discipline (e.g., Greek philosophical commentaries in (Sorabji (ed.) 1990, 2004a, 2004b, 2004c), or specific parts of commentaries within a discipline, like the isagogical parts of

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3 See (Dayeh (ed.) 2018) and (Sialaros, Christianidis, and Megremi (eds.) 2019b).
mathematical commentaries among other Greek isagogical writings (Mansfeld 1998) or the Arabic commentaries on the Hippocratic aphorisms.5

In the wake of this developing interest in commentaries, some books have adopted a more global approach, for example, for cuneiform sources (Frahm 2011) and for sources from the Indian subcontinent (Wilden ed.) 2009, Angot 2017). However, even in this context, there have been only very few attempts to address the issues raised by commentaries in a broadly comparative perspective. Exceptions include Assmann and Gladigow (eds.) (1995), Most (ed.) (1999), and Dayeh (ed.) (2018).

Moreover – and this is a key point for this book – scientific and mathematical commentaries, in the sense both of commentaries commenting upon a scientific and mathematical base text and of commentaries using scientific and mathematical knowledge in order to comment upon a fundamentally non-scientific and non-mathematical base text, have tended to be almost completely absent even from such general reflections on commentaries. Here, too, however, there have been a few exceptions, such as Most (ed.) (1999) and Gibson and Shuttleworth Kraus (eds.) (2002).

Accordingly, when modern historians have worked on commentaries that were deploying scientific and mathematical knowledge so as to comment upon certain parts of a fundamentally non-scientific and non-mathematical base text, most of the time they have omitted these parts. For instance, the parts of the ancient commentaries on the Confucian canon in which ancient commentators used mathematical knowledge and procedures so as to comment upon the base text do not seem to have received any attention from scholars until Zhu Yiwen pointed out their significance (see Zhu Yiwen 2016, forthcoming, and Chemla and Zhu, Chapter 6, and Morgan, Chapter 4, in this volume). Scholars in the Greek and Roman traditions have indeed studied the commentaries on the scientific and especially mathematical sections of the works of such authors as Plato and Aristotle, but the general lack of attention accorded to scientific and mathematical commentaries on fundamentally non-scientific and non-mathematical base texts outside the Greek and Roman traditions has had the unfortunate effect of obscuring the centrality of mathematical arguments in those other traditions. This is one of the ways in which the historiography of commentaries has contributed to buttressing the widespread, and in our view false, impression of a grand divide between science and

5 The latter commentaries were the object of the ERC project “Arabic Commentaries on the Hippocratic Aphorisms,” whose Principal Investigator was Peter Pormann (https://cordis.europa.eu/project/id/283626/fr, accessed on April 5, 2021). See also (Pormann and Karimullah 2017).
scholarship in Europe and North America on the one hand and in the rest of the world on the other. We will see below that this was not the only way.

Historians have thereby tended to disregard the mathematical competences in particular, but also more generally the set of competences, that ancient actors brought to the study of their base texts. They have further tended to overlook the concrete ways in which actors acquired and refined these competences. This is precisely one of the issues that, for example, Morgan addresses in Chapter 4 in this book, with reference to early commentaries on the Confucian canon.

Modern historians have likewise often failed to take adequate advantage of the study of commentaries on mathematical base texts when dealing with the genres of commentary production. This is again all the more true when one deals with East Asia or South Asia, since, outside the field of Greek and Roman Classical studies, Keller’s remark, in Chapter 3 in this book, holds quite generally: “This omission [scil. the failure to take commentaries in the field of mathematical sciences into account for the general study of commentaries] no doubt is in part due to the difficult reciprocal integration of the history of the ‘exact sciences’ and South Asian Studies.” This bias has contributed towards distorting scholars’ sense of the vital part played by mathematics in various regions of the globe.

1.1.3 The Difficulty of Integrating Mathematical Commentaries into the History of Science

It is also true, and particularly so, that, until quite recently, commentaries have largely tended to be ignored in the discipline of the History of Science. In addition to the general factors indicated above that help account for the broad neglect of commentarial literature, the specific lack of interest that as a rule historians of science have shown for commentaries might also be explained by specific values that have been associated with commentaries in this discipline. This issue would be worth a separate historiographic study. In the context of a kind of historiography of science redolent of Romanticism, which was obsessed by the ideals of novelty and progress, commentaries have instead been considered a priori to be “pedantic,” attached to trifling matters such as “explaining the obvious” and “delving” on the form” – and certainly to be anything but innovative.6

6 The identification of the latter features as characterizing historians’ perception of commentators’ activity is offered in (Netz 1998: 263, 264, 268), respectively. (De Young 2019: 70) likewise argues that this perception of commentaries explains why mathematical commentaries in Arabic have been understudied.
Commentaries seem to have appeared to some scholars as if they were to petrify the base text, thereby turning it, by the mere fact of commenting upon it, into something inalterable, official, orthodox, authoritative, and indeed authoritarian—all values that many people since the eighteenth century have learned to scorn as being antagonistic to any properly scientific spirit. A clear symptom of this attitude towards commentaries is the fact that certain historiographies of ancient science point to the very practice of commentaries on scientific texts as a manifest sign of decline. Histories of ancient Greek science offer a case in point here, though of course there are some exceptions to this generalization (for example, Galen’s commentaries on Hippocratic texts). Such an interpretation of the composition of commentaries by actors as one of their preferred genres of writing has in this case lent apparent support to a periodization in terms of golden age and decline. The same type of reasoning has also been used to argue for a divide between different “cultures.” In particular, the composition of many commentaries in non-European parts of the world has been interpreted as proof of a privilege granted there to orthodoxy, authority, and the like, and hence, once again, as evidence for attitudes opposed to the scientific spirit that many European scholars have been proud to identify as being primarily, indeed essentially, European.

Seen from another angle, commentaries have regularly, and often rightly, been associated with some kind of more or less formal educational context. Such an institutional context is something that a history of science obsessed with innovation has tended to perceive as the domain of mere repetition par excellence—at least until recent historiography exposed the flaws in this view.\footnote{See, for instance, a theoretical reflection on the topic in (Belhoste 1998), and case studies devoted to ancient mathematics in (Volkov 2012) for China, and (Proust 2015) for Mesopotamia. The importance of teaching for the training of future generations and the historical insights that might be gained from exploring this issue are the topics of (Ehrhardt 2008; Kaiser (ed.) 2005; Olesko 1991; Warwick 2003), which are all devoted to modern science. Part of the new importance that this issue has taken on in the History of Science can probably be traced back to the influence of Thomas Kuhn’s emphasis on the concept of “paradigm.” However, an approach of this kind does not necessarily place enough stress on the impulse to innovate that actors receive from the very activities attached to teaching contexts.}

The neglect of commentaries \emph{qua} commentaries in the History of Science may be interpreted as only one symptom of a larger neglect of all forms of textuality in this field. Indeed, despite the fact that for most historians texts remain the very basis of their work, all too often the material dimensions of these sources (in the broad sense that this expression might take) and the clues that they offer are left unattended. As a rule,
most historians of science seem to have been interested above all in ideas, devices, institutions, and practices, and to have neglected the degree to which textual approaches to their sources could provide invaluable insights into all four domains and beyond. But some recent developments suggest to us that the tide is turning. Diagrams, and more generally the visual dimensions of scientific writings, genres of writings, and means of publication, the genesis of texts as well as the history of archives are only some of the topics that are now beginning to be actively pursued in the textual approaches that are beginning to proliferate in the History of Science.  

This change of focus can also be perceived with regard to our own topic, that is, mathematical commentaries. To begin with, ancient commentaries have finally begun to be translated into modern European languages. In addition to the example of Bhāskara I’s commentary on the mathematical chapter of the Āryabhaṭīya (Keller 2006) mentioned above, we can refer to the translation of the first two chapters, devoted to mathematics, of the astronomical treatise entitled Siddhāntaśiromani and composed by the twelfth-century scholar Bhāskara II (Patte 2004). The first one, Līlāvatī, has been translated together with the early fifteenth-century commentary composed by Gaṅgādhara, while the second one, Bījaganita, is given together with its early sixteenth-century commentary, ascribed to Sūryadāsa.  

More recently, Jean-Michel Delire has published the translation of another Sanskrit mathematical commentary that bears on a completely different kind of writing, since the base text, the Baudhāyana Śulbasūtra, considered to be much more ancient, belongs to the environment of Vedic scriptures and is devoted to the construction of appropriate altars. Delire (2016) translates it in the context of a commentary composed prior to the fifteenth century by Dvārakānātha.  

These publications thus allow the modern reader to read a base text in the context of a textual presentation similar to the one in which the ancient reader approached it. The same holds true for ancient Chinese canonical texts. We have mentioned above the older publications, in which the main
text of the Chinese mathematical canonical text, *The Nine Chapters*, was translated as a stand-alone work. In the last several decades the commentaries too have gradually started to be translated together with *The Nine Chapters*: Kawahara (1980) translated the base text with the commentary attributed to the third-century scholar Liu Hui, while more recently Shen, Crossley, and Liu (1999) and Chemla and Guo (2004) have translated the base text with the two layers of commentaries from the third and the seventh centuries with which *The Nine Chapters* was transmitted. In Greek studies as well, the large-scale translation project devoted to the ancient commentaries on Aristotle has already been mentioned; and in the last few years translations of other bodies of ancient scholia and commentaries have started to appear in English, French, and Italian.

In parallel with these translations, scholars have begun to study mathematical commentaries, in the two senses of this term that we distinguished above. This appears to us to be all the more important in view of the fact that commentaries have had a manifest importance in the acquisition, growth, and dissemination of scientific knowledge. However, we will argue that much more is at stake in the historical study of mathematical commentaries that we are advocating. In any event, the time might be ripe for a general and comparative approach to mathematical commentaries in the ancient world, for which this book endeavors to offer, in its detailed case studies and this general introduction, at least a model and a stimulus. In this introductory chapter, we explore how we can rely on earlier general studies of commentaries and on studies of commentaries in other fields in order to see to what degree they can provide some benefit for the study of mathematical commentaries; in doing so, we hope along the way to highlight the connections and the differences among various types of commentaries as well as thereby to locate more concretely the practice of mathematics among other scholarly practices. More generally, we aim to explore what is at stake for the history of mathematics, and, further, for the understanding of commentaries in general, in the study of the very specific type of commentary represented by mathematical commentaries.

In brief, our question is: what is the contribution that the study of mathematical commentaries in particular can make to the study of commentaries in general and to the history of mathematics in general? And, to establish on firmer grounds the arguments that follow, we have chosen to...

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9 Interestingly, in (Shen et al. 1999), the commentaries are fully translated, but they are laid out as footnotes and put on a par with the comments of modern historians.

10 And see now the recent special number of *Historia Mathematica* on commentaries (Sialaros, Christianidis, and Megremi (eds.) 2019b).