

## Quantum Mechanics

### A Graduate Course

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Written for a two-semester graduate course in quantum mechanics, this comprehensive text helps the reader to develop the tools and formalism of quantum mechanics and its applications to physical systems. It will suit students who have taken some introductory quantum mechanics and modern physics courses at undergraduate level, but it is self-contained and does not assume any specific background knowledge beyond appropriate fluency in mathematics. The text takes a modern logical approach rather than a historical one and it covers standard material, such as the hydrogen atom and the harmonic oscillator, the WKB approximations, and Bohr–Sommerfeld quantization. Important modern topics and examples are also described, including Berry phase, quantum information, complexity and chaos, decoherence and thermalization, and nonstandard statistics, as well as more advanced material such as path integrals, scattering theory, multiparticles, and Fock space. Readers will gain a broad overview of quantum mechanics as a solid preparation for further study or research.

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“There are a few dozen books on quantum mechanics. Who needs another one? Graduate students. Most of the existing books aim at the undergraduate level. Often, students of graduate courses of QM are all familiar with the motivation and its historical development but have a very varied technical background. This book skips the history of the field and enables all the students to reach a common needed level after going over the first chapters.

This book spans a very wide manifold of QM topics. It provides the tools for further researching any quantum system. An important further value of the book is that it covers the forefront aspects of QM including the Berry phase, anyons, entanglement, quantum information, quantum complexity and chaos, and quantum thermalization. Readers understand a topic only if they are able to solve problems associated with it; the book includes at least 7 exercises for each of its 59 chapters.”

— **Professor Jacob Sonnenschein, Tel Aviv University**

“Năstase’s *Quantum Mechanics* is another marvellous addition to his encyclopaedic collection of graduate-level courses that take both the dedicated student and the hardened researcher on a grand tour of contemporary theoretical physics. It will serve as an ideal bridge between a comprehensive undergraduate course in quantum mechanics and the frontiers of research in quantum systems.”

— **Professor Jeff Murugan, University of Cape Town**

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To the memory of my mother,  
who inspired me to become a physicist

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Contents

<i>Preface</i>	<i>page</i> xix
<i>Acknowledgements</i>	xx
<i>Introduction</i>	xxi
<b>Part I Formalism and Basic Problems</b>	<b>1</b>
<b>Introduction: Historical Background</b>	<b>3</b>
0.1 Experiments Point towards Quantum Mechanics	3
0.2 Quantized States: Matrix Mechanics, and Waves for Particles: Correspondence Principle	6
0.3 Wave Functions Governing Probability, and the Schrödinger Equation	8
0.4 Bohr–Sommerfeld Quantization and the Hydrogen Atom	8
0.5 Review: States of Quantum Mechanical Systems	10
<b>1 The Mathematics of Quantum Mechanics 1: Finite-Dimensional Hilbert Spaces</b>	<b>12</b>
1.1 (Linear) Vector Spaces $\mathbb{V}$	12
1.2 Operators on a Vector Space	15
1.3 Dual Space, Adjoint Operation, and Dirac Notation	17
1.4 Hermitian (Self-Adjoint) Operators and the Eigenvalue Problem	19
1.5 Traces and Tensor Products	21
1.6 Hilbert Spaces	21
<b>2 The Mathematics of Quantum Mechanics 2: Infinite-Dimensional Hilbert Spaces</b>	<b>24</b>
2.1 Hilbert Spaces and Related Notions	24
2.2 Functions as Limits of Discrete Sets of Vectors	25
2.3 Integrals as Limits of Sums	26
2.4 Distributions and the Delta Function	27
2.5 Spaces of Functions	29
2.6 Operators in Infinite Dimensions	30
2.7 Hermitian Operators and Eigenvalue Problems	30
2.8 The Operator $D_{xx'}$	31
<b>3 The Postulates of Quantum Mechanics and the Schrödinger Equation</b>	<b>35</b>
3.1 The Postulates	35
3.2 The First Postulate	37
3.3 The Second Postulate	37
3.4 The Third Postulate	38

3.5	The Fourth Postulate	38
3.6	The Fifth Postulate	41
3.7	The Sixth Postulate	41
3.8	Generalization of States to Ensembles: the Density Matrix	42
<b>4</b>	<b>Two-Level Systems and Spin-1/2, Entanglement, and Computation</b>	<b>45</b>
4.1	Two-Level Systems and Time Dependence	45
4.2	General Stationary Two-State System	47
4.3	Oscillations of States	50
4.4	Unitary Evolution Operator	52
4.5	Entanglement	52
4.6	Quantum Computation	54
<b>5</b>	<b>Position and Momentum and Their Bases; Canonical Quantization, and Free Particles</b>	<b>57</b>
5.1	Translation Operator	57
5.2	Momentum in Classical Mechanics as a Generator of Translations	58
5.3	Canonical Quantization	60
5.4	Operators in Coordinate and Momentum Spaces	61
5.5	The Free Nonrelativistic Particle	63
<b>6</b>	<b>The Heisenberg Uncertainty Principle and Relations, and Gaussian Wave Packets</b>	<b>66</b>
6.1	Gaussian Wave Packets	66
6.2	Time Evolution of Gaussian Wave Packet	68
6.3	Heisenberg Uncertainty Relations	69
6.4	Minimum Uncertainty Wave Packet	71
6.5	Energy–Time Uncertainty Relation	72
<b>7</b>	<b>One-Dimensional Problems in a Potential <math>V(x)</math></b>	<b>74</b>
7.1	Set-Up of the Problem	74
7.2	General Properties of the Solutions	75
7.3	Infinitely Deep Square Well (Particle in a Box)	78
7.4	Potential Step and Reflection and Transmission of Modes	81
7.5	Continuity Equation for Probabilities	83
7.6	Finite Square Well Potential	84
7.7	Penetration of a Potential Barrier and the Tunneling Effect	87
<b>8</b>	<b>The Harmonic Oscillator</b>	<b>91</b>
8.1	Classical Set-Up and Generalizations	91
8.2	Quantization in the Creation and Annihilation Operator Formalism	92
8.3	Generalization	95
8.4	Coherent States	96
8.5	Solution in the Coordinate, $ x\rangle$ , Representation (Basis)	96
8.6	Alternative to $ x\rangle$ Representation: Basis Change from $ n\rangle$ Representation	99
8.7	Properties of Hermite Polynomials	100
8.8	Mathematical Digression (Appendix): Classical Orthogonal Polynomials	101



<b>9 The Heisenberg Picture and General Picture; Evolution Operator</b>	<b>106</b>
9.1 The Evolution Operator	106
9.2 The Heisenberg Picture	108
9.3 Application to the Harmonic Oscillator	109
9.4 General Quantum Mechanical Pictures	110
9.5 The Dirac (Interaction) Picture	112
<b>10 The Feynman Path Integral and Propagators</b>	<b>116</b>
10.1 Path Integral in Phase Space	116
10.2 Gaussian Integration	119
10.3 Path Integral in Configuration Space	120
10.4 Path Integral over Coherent States (in “Harmonic Phase Space”)	121
10.5 Correlation Functions and Their Generating Functional	123
<b>11 The Classical Limit and Hamilton–Jacobi (WKB Method), the Ehrenfest Theorem</b>	<b>127</b>
11.1 Ehrenfest Theorem	127
11.2 Continuity Equation for Probability	129
11.3 Review of the Hamilton–Jacobi Formalism	130
11.4 The Classical Limit and the Geometrical Optics Approximation	132
11.5 The WKB Method	133
<b>12 Symmetries in Quantum Mechanics I: Continuous Symmetries</b>	<b>137</b>
12.1 Symmetries in Classical Mechanics	137
12.2 Symmetries in Quantum Mechanics: General Formalism	139
12.3 Example 1. Translations	141
12.4 Example 2. Time Translation Invariance	142
12.5 Mathematical Background: Review of Basics of Group Theory	143
<b>13 Symmetries in Quantum Mechanics II: Discrete Symmetries and Internal Symmetries</b>	<b>149</b>
13.1 Discrete Symmetries: Symmetries under Discrete Groups	149
13.2 Parity Symmetry	150
13.3 Time Reversal Invariance, $T$	152
13.4 Internal Symmetries	154
13.5 Continuous Symmetry	155
13.6 Lie Groups and Algebras and Their Representations	155
<b>14 Theory of Angular Momentum I: Operators, Algebras, Representations</b>	<b>159</b>
14.1 Rotational Invariance and $SO(n)$	159
14.2 The Lie Groups $SO(2)$ and $SO(3)$	160
14.3 The Group $SU(2)$ and Its Isomorphism with $SO(3) \bmod \mathbb{Z}_2$	162
14.4 Generators and Lie Algebras	164
14.5 Quantum Mechanical Version	165
14.6 Representations	166

<b>15 Theory of Angular Momentum II: Addition of Angular Momenta and Representations; Oscillator Model</b>	172
15.1 The Spinor Representation, $j = 1/2$	172
15.2 Composition of Angular Momenta	174
15.3 Finding the Clebsch–Gordan Coefficients	177
15.4 Sums of Three Angular Momenta, $\vec{J}_1 + \vec{J}_2 + \vec{J}_3$ : Racah Coefficients	178
15.5 Schwinger’s Oscillator Model	179
<b>16 Applications of Angular Momentum Theory: Tensor Operators, Wave Functions and the Schrödinger Equation, Free Particles</b>	183
16.1 Tensor Operators	183
16.2 Wigner–Eckhart Theorem	185
16.3 Rotations and Wave Functions	186
16.4 Wave Function Transformations under Rotations	188
16.5 Free Particle in Spherical Coordinates	190
<b>17 Spin and <math>\vec{L} + \vec{S}</math></b>	194
17.1 Motivation for Spin and Interaction with Magnetic Field	194
17.2 Spin Properties	196
17.3 Particle with Spin $1/2$	197
17.4 Rotation of Spinors with $s = 1/2$	199
17.5 Sum of Orbital Angular Momentum and Spin, $\vec{L} + \vec{S}$	200
17.6 Time-Reversal Operator on States with Spin	201
<b>18 The Hydrogen Atom</b>	204
18.1 Two-Body Problem: Reducing to Central Potential	204
18.2 Hydrogenoid Atom: Set-Up of Problem	205
18.3 Solution: Sommerfeld Polynomial Method	207
18.4 Confluent Hypergeometric Function and Quantization of Energy	210
18.5 Orthogonal Polynomials and Standard Averages over Wave Functions	211
<b>19 General Central Potential and Three-Dimensional (Isotropic) Harmonic Oscillator</b>	214
19.1 General Set-Up	214
19.2 Types of Potentials	215
19.3 Diatomic Molecule	220
19.4 Free Particle	221
19.5 Spherical Square Well	221
19.6 Three-Dimensional Isotropic Harmonic Oscillator: Set-Up	222
19.7 Isotropic Three-Dimensional Harmonic Oscillator in Spherical Coordinates	223
19.8 Isotropic Three-Dimensional Harmonic Oscillator in Cylindrical Coordinates	225
<b>20 Systems of Identical Particles</b>	229
20.1 Identical Particles: Bosons and Fermions	229
20.2 Observables under Permutation	231
20.3 Generalization to $N$ Particles	232
20.4 Canonical Commutation Relations	233

20.5	Spin–Statistics Theorem	234
20.6	Particles with Spin	236
<b>21</b>	<b>Application of Identical Particles: He Atom (Two-Electron System) and H<sub>2</sub> Molecule</b>	<b>240</b>
21.1	Helium-Like Atoms	240
21.2	Ground State of the Helium (or Helium-Like) Atom	243
21.3	Approximation 3: Variational Method, “Light Version”	245
21.4	H <sub>2</sub> Molecule and Its Ground State	246
<b>22</b>	<b>Quantum Mechanics Interacting with Classical Electromagnetism</b>	<b>250</b>
22.1	Classical Electromagnetism plus Particle	250
22.2	Quantum Particle plus Classical Electromagnetism	251
22.3	Application to Superconductors	254
22.4	Interaction with a Plane Wave	256
22.5	Spin–Magnetic-Field and Spin–Orbit Interaction	256
<b>23</b>	<b>Aharonov–Bohm Effect and Berry Phase in Quantum Mechanics</b>	<b>260</b>
23.1	Gauge Transformation in Electromagnetism	260
23.2	The Aharonov–Bohm Phase $\delta$	262
23.3	Berry Phase	264
23.4	Example: Atoms, Nuclei plus Electrons	265
23.5	Spin–Magnetic Field Interaction, Berry Curvature, and Berry Phase as Geometric Phase	266
23.6	Nonabelian Generalization	269
23.7	Aharonov–Bohm Phase in Berry Form	269
<b>24</b>	<b>Motion in a Magnetic Field, Hall Effect and Landau Levels</b>	<b>272</b>
24.1	Spin in a Magnetic Field	272
24.2	Particle with Spin 1/2 in a Time-Dependent Magnetic Field	272
24.3	Particle with or without Spin in a Magnetic Field: Landau Levels	273
24.4	The Integer Quantum Hall Effect (IQHE)	275
24.5	Alternative Derivation of the IQHE	278
24.6	An Atom in a Magnetic Field and the Landé g-Factor	278
<b>25</b>	<b>The WKB; a Semiclassical Approximation</b>	<b>282</b>
25.1	Review and Generalization	282
25.2	Approximation and Connection Formulas at Turning Points	283
25.3	Application: Potential Barrier	285
25.4	The WKB Approximation in the Path Integral	287
<b>26</b>	<b>Bohr–Sommerfeld Quantization</b>	<b>290</b>
26.1	Bohr–Sommerfeld Quantization Condition	290
26.2	Example 1: Parity-Even Linear Potential	291
26.3	Example 2: Harmonic Oscillator	292
26.4	Example 3: Motion in a Central Potential	294
26.5	Example: Coulomb Potential (Hydrogenoid Atom)	298

<b>27 Dirac Quantization Condition and Magnetic Monopoles</b>	<b>301</b>
27.1 Dirac Monopoles from Maxwell Duality	301
27.2 Dirac Quantization Condition from Semiclassical Nonrelativistic Considerations	303
27.3 Contradiction with the Gauge Field	304
27.4 Patches and Magnetic Charge from Transition Functions	305
27.5 Dirac Quantization from Topology and Wave Functions	307
27.6 Dirac String Singularity and Obtaining the Dirac Quantization Condition from It	308
<b>28 Path Integrals II: Imaginary Time and Fermionic Path Integral</b>	<b>311</b>
28.1 The Forced Harmonic Oscillator	311
28.2 Wick Rotation to Euclidean Time and Connection with Statistical Mechanics Partition Function	315
28.3 Fermionic Path Integral	318
28.4 Gaussian Integration over the Grassmann Algebra	320
28.5 Path Integral for the Fermionic Harmonic Oscillator	321
<b>29 General Theory of Quantization of Classical Mechanics and (Dirac) Quantization of Constrained Systems</b>	<b>325</b>
29.1 Hamiltonian Formalism	325
29.2 Constraints in the Hamiltonian Formalism: Primary and Secondary Constraints, and First and Second Class Constraints	326
29.3 Quantization and Dirac Brackets	329
29.4 Example: Electromagnetic Field	332
<b>Part II<sub>a</sub> Advanced Foundations</b>	<b>337</b>
<b>30 Quantum Entanglement and the EPR Paradox</b>	<b>339</b>
30.1 Entanglement: Spin 1/2 System	339
30.2 Entanglement: The General Case	341
30.3 Entanglement: Careful Definition	342
30.4 Entanglement Entropy	343
30.5 The EPR Paradox and Hidden Variables	345
<b>31 The Interpretation of Quantum Mechanics and Bell's Inequalities</b>	<b>350</b>
31.1 Bell's Original Inequality	350
31.2 Bell–Wigner Inequalities	353
31.3 CHSH Inequality (or Bell–CHSH Inequality)	356
31.4 Interpretations of Quantum Mechanics	357
<b>32 Quantum Statistical Mechanics and “Tracing” over a Subspace</b>	<b>361</b>
32.1 Density Matrix and Statistical Operator	361
32.2 Review of Classical Statistics	363
32.3 Defining Quantum Statistics	365
32.4 Bose–Einstein and Fermi–Dirac Distributions	370
32.5 Entanglement Entropy	371

<b>33 Elements of Quantum Information and Quantum Computing</b>	<b>375</b>
33.1 Classical Computation and Shannon Theory	375
33.2 Quantum Information and Computation, and von Neumann Entropy	377
33.3 Quantum Computation	378
33.4 Quantum Cryptography, No-Cloning, and Teleportation	380
<b>34 Quantum Complexity and Quantum Chaos</b>	<b>384</b>
34.1 Classical Computation Complexity	384
34.2 Quantum Computation and Complexity	385
34.3 The Nielsen Approach to Quantum Complexity	386
34.4 Quantum Chaos	388
<b>35 Quantum Decoherence and Quantum Thermalization</b>	<b>393</b>
35.1 Decoherence	393
35.2 Schrödinger's Cat	393
35.3 Qualitative Decoherence	394
35.4 Quantitative Decoherence	395
35.5 Qualitative Thermalization	397
35.6 Quantitative Thermalization	398
35.7 Bogoliubov Transformation and Appearance of Temperature	400
<b>Part II<sub>b</sub> Approximation Methods</b>	<b>405</b>
<b>36 Time-Independent (Stationary) Perturbation Theory: Nondegenerate, Degenerate, and Formal Cases</b>	<b>407</b>
36.1 Set-Up of the Problem: Time-Independent Perturbation Theory	407
36.2 The Nondegenerate Case	408
36.3 The Degenerate Case	410
36.4 General Form of Solution (to All Orders)	412
36.5 Example: Stark Effect in Hydrogenoid Atom	414
<b>37 Time-Dependent Perturbation Theory: First Order</b>	<b>418</b>
37.1 Evolution Operator	418
37.2 Method of Variation of Constants	419
37.3 A Time-Independent Perturbation Being Turned On	421
37.4 Continuous Spectrum and Fermi's Golden Rule	421
37.5 Application to Scattering in a Collision	423
37.6 Sudden versus Adiabatic Approximation	425
<b>38 Time-Dependent Perturbation Theory: Second and All Orders</b>	<b>429</b>
38.1 Second-Order Perturbation Theory and Breit–Wigner Distribution (Energy Shift and Decay Width)	429
38.2 General Perturbation	432
38.3 Finding the Probability Coefficients $b_n(t)$	433

<b>39 Application: Interaction with (Classical) Electromagnetic Field, Absorption, Photoelectric and Zeeman Effects</b>	<b>436</b>
39.1 Particles and Atoms in an Electromagnetic Field	436
39.2 Zeeman and Paschen–Back Effects	437
39.3 Electromagnetic Radiation and Selection Rules	438
39.4 Absorption of Photon Energy by Atom	442
39.5 Photoelectric Effect	444
<b>40 WKB Methods and Extensions: State Transitions and Euclidean Path Integrals (Instantons)</b>	<b>447</b>
40.1 Review of the WKB Method	447
40.2 WKB Matrix Elements	448
40.3 Application to Transition Probabilities	451
40.4 Instantons for Transitions between Two Minima	452
<b>41 Variational Methods</b>	<b>457</b>
41.1 First Form of the Variational Method	457
41.2 Ritz Variational Method	458
41.3 Practical Variational Method	459
41.4 General Method	460
41.5 Applications	462
<b>Part II, Atomic and Nuclear Quantum Mechanics</b>	<b>467</b>
<b>42 Atoms and Molecules, Orbitals and Chemical Bonds: Quantum Chemistry</b>	<b>469</b>
42.1 Hydrogenoid Atoms (Ions)	469
42.2 Multi-Electron Atoms and Shells	470
42.3 Couplings of Angular Momenta	471
42.4 Methods of Quantitative Approximation for Energy Levels	472
42.5 Molecules and Chemical Bonds	473
42.6 Adiabatic Approximation and Hierarchy of Scales	474
42.7 Details of the Adiabatic Approximation	475
42.8 Method of Molecular Orbitals	476
42.9 The LCAO Method	477
42.10 Application: The LCAO Method for the Diatomic Molecule	478
42.11 Chemical Bonds	478
<b>43 Nuclear Liquid Droplet and Shell Models</b>	<b>483</b>
43.1 Nuclear Data and Droplet Model	483
43.2 Shell Models 1: Single-Particle Shell Models	484
43.3 Spin–Orbit Interaction Correction	487
43.4 Many-Particle Shell Models	489
<b>44 Interaction of Atoms with Electromagnetic Radiation: Transitions and Lasers</b>	<b>492</b>
44.1 Two-Level System for Time-Dependent Transitions	492
44.2 First-Order Perturbation Theory for Harmonic Potential	494

44.3	The Case of Quantized Radiation	495
44.4	Planck Formula	497
<b>Part II<sub>d</sub> Scattering Theory</b>		501
<b>45</b>	<b>One-Dimensional Scattering, Transfer and S-Matrices</b>	503
45.1	Asymptotics and Integral Equations	504
45.2	Green's Functions	506
45.3	Relations between Abstract States and Lippmann–Schwinger Equation	507
45.4	Physical Interpretation of Scattering Solution	509
45.5	S-Matrix and T-Matrix	511
45.6	Application: Spin Chains	512
<b>46</b>	<b>Three-Dimensional Lippmann–Schwinger Equation, Scattering Amplitudes and Cross Sections</b>	516
46.1	Potential for Relative Motion	516
46.2	Behavior at Infinity	517
46.3	Scattering Solution, Cross Sections, and S-Matrix	518
46.4	S-Matrix and Optical Theorem	521
46.5	Green's Functions and Lippmann–Schwinger Equation	522
<b>47</b>	<b>Born Approximation and Series, S-Matrix and T-Matrix</b>	528
47.1	Born Approximation and Series	528
47.2	Time-Dependent Scattering Point of View	530
47.3	Higher-Order Terms and Abstract States, S- and T-Matrices	530
47.4	Validity of the Born Approximation	532
<b>48</b>	<b>Partial Wave Expansion, Phase Shift Method, and Scattering Length</b>	535
48.1	The Partial Wave Expansion	535
48.2	Phase Shifts	537
48.3	T-Matrix Element	540
48.4	Scattering Length	541
48.5	Jost Functions, Wronskians, and the Levinson Theorem	542
<b>49</b>	<b>Unitarity, Optics, and the Optical Theorem</b>	547
49.1	Unitarity: Review and Reanalysis	547
49.2	Application to Cross Sections	548
49.3	Radial Green's Functions	550
49.4	Optical Theorem	551
49.5	Scattering on a Potential with a Finite Range	553
49.6	Hard Sphere Scattering	554
49.7	Low-Energy Limit	554
<b>50</b>	<b>Low-Energy and Bound States, Analytical Properties of Scattering Amplitudes</b>	557
50.1	Low-Energy Scattering	557
50.2	Relation to Bound States	558

50.3	Bound State from Complex Poles of $S_l(k)$	560
50.4	Analytical Properties of the Scattering Amplitudes	561
50.5	Jost Functions Revisited	563
<b>51</b>	<b>Resonances and Scattering, Complex <math>k</math> and <math>l</math></b>	<b>566</b>
51.1	Poles and Zeroes in the Partial Wave S-Matrix $S_l(k)$	566
51.2	Breit–Wigner Resonance	568
51.3	Physical Interpretation and Its Proof	570
51.4	Review of Levinson Theorem	573
51.5	Complex Angular Momentum	574
<b>52</b>	<b>The Semiclassical WKB and Eikonal Approximations for Scattering</b>	<b>577</b>
52.1	WKB Review for One-Dimensional Systems	577
52.2	Three-Dimensional Scattering in the WKB Approximation	578
52.3	The Eikonal Approximation	581
52.4	Coulomb Scattering and the Semiclassical Approximation	584
<b>53</b>	<b>Inelastic Scattering</b>	<b>588</b>
53.1	Generalizing Elastic Scattering from Unitarity Loss	588
53.2	Inelastic Scattering Due to Target Structure	590
53.3	General Theory of Collisions: Inelastic Case and Multi-Channel Scattering	592
53.4	General Theory of Collisions	593
53.5	Multi-Channel Analysis	595
53.6	Scattering of Identical Particles	597
	<b>Part II<sub>e</sub> Many Particles</b>	<b>601</b>
<b>54</b>	<b>The Dirac Equation</b>	<b>603</b>
54.1	Naive Treatment	603
54.2	Relativistic Dirac Equation	606
54.3	Interaction with Electromagnetic Field	607
54.4	Weakly Relativistic Limit	607
54.5	Correction to the Energy of Hydrogenoid Atoms	611
<b>55</b>	<b>Multiparticle States in Atoms and Condensed Matter: Schrödinger versus Occupation Number</b>	<b>614</b>
55.1	Schrödinger Picture Multiparticle Review	614
55.2	Approximation Methods	616
55.3	Transition to Occupation Number	616
55.4	Schrödinger Equation in Occupation Number Space	617
55.5	Analysis for Fermions	621
<b>56</b>	<b>Fock Space Calculation with Field Operators</b>	<b>623</b>
56.1	Creation and Annihilation Operators	623
56.2	Occupation Number Representation for Fermions	624
56.3	Schrödinger Equation on Occupation Number States	625



56.4 Fock Space	626
56.5 Fock Space for Fermions	627
56.6 Schrödinger Equations for Fermions, and Generalizations	628
56.7 Field Operators	628
56.8 Example of Interaction: Coulomb Potential, as a Limit of the Yukawa Potential, for Spin 1/2 Fermions	630
<b>57 The Hartree–Fock Approximation and Other Occupation Number Approximations</b>	<b>632</b>
57.1 Set-Up of the Problem	632
57.2 Derivation of the Hartree–Fock Equation	633
57.3 Hartree and Fock Terms	635
57.4 Other Approximations in the Occupation Number Picture	637
<b>58 Nonstandard Statistics: Anyons and Nonabelions</b>	<b>641</b>
58.1 Review of Statistics and Berry Phase	641
58.2 Anyons in 2+1 Dimensions: Explicit Construction	642
58.3 Chern–Simons Action	644
58.4 Example: Fractional Quantum Hall Effect (FQHE)	646
58.5 Nonabelian Statistics	647
<i>References</i>	650
<i>Index</i>	652

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[More Information](#)

Preface

There are so many books in quantum mechanics, that one has to ask: why write another one? When teaching graduate quantum mechanics in either the US or Brazil (countries I am familiar with), as well as in other places, one is faced with a conundrum: how to address all possible backgrounds, while keeping the course both interesting and also comprehensive enough to offer the graduate student a chance to follow competitively in any area? Indeed, while graduate students have usually very different backgrounds, there is usually a compulsory graduate quantum mechanics course, usually a two-semester one. The students will certainly have some introductory undergraduate quantum mechanics and some modern physics, but some have studied these topics in detail, while others less so.

To that end, I believe there is little need for a long historical introduction, or a detailed explanation of why we need to define quantum mechanics in the way we do. In order to challenge students, we cannot simply repeat what they heard in the undergraduate course. So one needs a tighter presentation, with more emphasis on the building up of the formalism of quantum mechanics rather than its motivation, as well as a presentation that contains more interesting new developments besides the standard advanced concepts. On the other hand, I personally found that many (even very bright) students are caught up between the two systems and miss on important information: they have had an introductory quantum mechanics course that did not treat all the standard classical material (examples: the hydrogen atom in all detail, the harmonic oscillator in various treatments, the WKB and Bohr–Sommerfeld formalisms), yet the graduate course assumes that students have covered all these topics and so they struggle in their subsequent research as a result.

Since I have found no graduate book that adheres to these conditions of comprehensiveness to my satisfaction, I decided to write one, and this is the result. The book is intended as a two-semester course, corresponding to the two parts, each chapter corresponding to one two-hour lecture, though sometimes an extended version of a lecture.

## Acknowledgements

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With respect to teaching quantum mechanics, most of the credit goes to my teachers in the Physics Department of Bucharest University, since during my four undergraduate years there (1990–1994), many of the courses there taught me about various aspects of quantum mechanics. Some recent developments described here I learned from my research, so I thank all my collaborators, students, and postdocs for their help with understanding them. My wife Antonia I thank for her patience and understanding while I wrote this book, as well as for help with an accident during the writing. I would like to thank my present editor at Cambridge University Press, Vince Higgs, who helped me get this book published, as well as my outgoing (now retired) editor Simon Capelin, for his encouragement in starting me on the path to publishing books. To all the staff at CUP, thanks for making sure this book, as well as my previous ones, is as good as it can be.

# Introduction

As described in the Preface, this book is intended for graduate students, and so I assume that readers will have had both an introductory undergraduate course in quantum mechanics, which familiarized them with the historical motivation as well as the basic formalism. For completeness, however, I have included a brief historical background as an optional introductory chapter. I do assume a certain level of mathematical proficiency, as might be expected from a graduate student. Nevertheless, the first two chapters are dedicated to the mathematics of quantum mechanics; they are divided into finite- and infinite-dimensional Hilbert spaces, since these concepts are necessary to a smooth introduction.

I start the discussion of quantum mechanics with its postulates and the Schrödinger equation, after which I start developing the formalism. I mostly use the bra-ket notation for abstract states, as it is often cleaner, though for most of the standard systems I use wave functions. I introduce early on the notion of the path integral, since it is an important alternative way to describe quantization, through the sum over all paths, classical or otherwise, though I mostly use the operatorial formalism. I also introduce early on the notion of pictures, including the Heisenberg and interaction pictures, as alternatives for the description of time evolution. The important notion of angular momentum theory is presented within the larger context of symmetries in quantum mechanics, since this is the modern viewpoint, especially within the relativistic quantum field theory that extends quantum mechanics.

With respect to topics, in Part I, on basic concepts, I consider both the older, but still very useful, topics such as WKB and Bohr–Sommerfeld quantization as well as more recent ones such as the Berry phase and Dirac quantization condition. In Part II I consider scattering theory, variational methods, occupation number space, quantum entanglement and information, quantum complexity, quantum chaos, and thermalization; these are the more advanced topics. The advanced foundations of Quantum Mechanics (Part II<sub>a</sub>) are discussed in Part II since they are newer and harder to understand even though they could be said to belong to Part I for being foundational issues.

After each chapter, I summarize a set of “Important Concepts to Remember”, and present seven exercises whose solution is meant to clarify the concepts in the chapter.

