Quantum Mechanics

A Graduate Course

Written for a two-semester graduate course in quantum mechanics, this comprehensive text helps the reader to develop the tools and formalism of quantum mechanics and its applications to physical systems. It will suit students who have taken some introductory quantum mechanics and modern physics courses at undergraduate level, but it is self-contained and does not assume any specific background knowledge beyond appropriate fluency in mathematics. The text takes a modern logical approach rather than a historical one and it covers standard material, such as the hydrogen atom and the harmonic oscillator, the WKB approximations, and Bohr–Sommerfeld quantization. Important modern topics and examples are also described, including Berry phase, quantum information, complexity and chaos, decoherence and thermalization, and nonstandard statistics, as well as more advanced material such as path integrals, scattering theory, multiparticles, and Fock space. Readers will gain a broad overview of quantum mechanics as a solid preparation for further study or research.

Horaţiu Năstase is a Researcher at the Institute for Theoretical Physics, State University of São Paulo. He completed his PhD at Stony Brook with Peter van Nieuwenhuizen, co-discoverer of supergravity. While in Princeton as a postdoctoral fellow, in a 2002 paper with David Berenstein and Juan Maldacena he started the pp-wave correspondence, a sub-area of the AdS/CFT correspondence. He has written more than 100 scientific articles and five other books, including *Introduction to the AdS/CFT Correspondence* (2015), *String Theory Methods for Condensed Matter Physics* (2017), *Classical Field Theory* (2019), *Introduction to Quantum Field Theory* (2019), and *Cosmology and String Theory* (2019).

"There are a few dozen books on quantum mechanics. Who needs another one? Graduate students. Most of the existing books aim at the undergraduate level. Often, students of graduate courses of QM are all familiar with the motivation and its historical development but have a very varied technical background. This book skips the history of the field and enables all the students to reach a common needed level after going over the first chapters.

This book spans a very wide manifold of QM topics. It provides the tools for further researching any quantum system. An important further value of the book is that it covers the forefront aspects of QM including the Berry phase, anyons, entanglement, quantum information, quantum complexity and chaos, and quantum thermalization. Readers understand a topic only if they are able to solve problems associated with it; the book includes at least 7 exercises for each of its 59 chapters."

— **Professor Jacob Sonnenschein, Tel Aviv University**

"Năstase's *Quantum Mechanics* is another marvellous addition to his encyclopaedic collection of graduate-level courses that take both the dedicated student and the hardened researcher on a grand tour of contemporary theoretical physics. It will serve as an ideal bridge between a comprehensive undergraduate course in quantum mechanics and the frontiers of research in quantum systems."

— **Professor Jeff Murugan, University of Cape Town**

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To the memory of my mother,
who inspired me to become a physicist

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Preface

There are so many books in quantum mechanics, that one has to ask: why write another one? When teaching graduate quantum mechanics in either the US or Brazil (countries I am familiar with), as well as in other places, one is faced with a conundrum: how to address all possible backgrounds, while keeping the course both interesting and also comprehensive enough to offer the graduate student a chance to follow competitively in any area? Indeed, while graduate students have usually very different backgrounds, there is usually a compulsory graduate quantum mechanics course, usually a two-semester one. The students will certainly have some introductory undergraduate quantum mechanics and some modern physics, but some have studied these topics in detail, while others less so.

To that end, I believe there is little need for a long historical introduction, or a detailed explanation of why we need to define quantum mechanics in the way we do. In order to challenge students, we cannot simply repeat what they heard in the undergraduate course. So one needs a tighter presentation, with more emphasis on the building up of the formalism of quantum mechanics rather than its motivation, as well as a presentation that contains more interesting new developments besides the standard advanced concepts. On the other hand, I personally found that many (even very bright) students are caught up between the two systems and miss on important information: they have had an introductory quantum mechanics course that did not treat all the standard classical material (examples: the hydrogen atom in all detail, the harmonic oscillator in various treatments, the WKB and Bohr–Sommerfeld formalisms), yet the graduate course assumes that students have covered all these topics and so they struggle in their subsequent research as a result.

Since I have found no graduate book that adheres to these conditions of comprehensiveness to my satisfaction, I decided to write one, and this is the result. The book is intended as a two-semester course, corresponding to the two parts, each chapter corresponding to one two-hour lecture, though sometimes an extended version of a lecture.

Acknowledgements

I should first thank all the people who have helped and guided me on my journey as a physicist, starting with my mother Ligia, who was the first example I had of a physicist, and from whom I first learned to love physics. My high school physics teacher, Iosif Sever Georgescu, helped to start me on the career of a physicist, and made me realize that it was something that I could do very well. My student exchange advisor at the Niels Bohr Institute, Poul Olesen, first showed me what string theory is, and started me in a career in this area, and my PhD advisor, Peter van Nieuwenhuizen, taught me the beauty of theoretical physics, of rigor and of long calculations, and an approach to teaching graduate physics that I still try to follow.

With respect to teaching quantum mechanics, most of the credit goes to my teachers in the Physics Department of Bucharest University, since during my four undergraduate years there (1990–1994), many of the courses there taught me about various aspects of quantum mechanics. Some recent developments described here I learned from my research, so I thank all my collaborators, students, and postdocs for their help with understanding them. My wife Antonia I thank for her patience and understanding while I wrote this book, as well as for help with an accident during the writing. I would like to thank my present editor at Cambridge University Press, Vince Higgs, who helped me get this book published, as well as my outgoing (now retired) editor Simon Capelin, for his encouragement in starting me on the path to publishing books. To all the staff at CUP, thanks for making sure this book, as well as my previous ones, is as good as it can be.

Introduction

As described in the Preface, this book is intended for graduate students, and so I assume that readers will have had both an introductory undergraduate course in quantum mechanics, which familiarized them with the historical motivation as well as the basic formalism. For completeness, however, I have included a brief historical background as an optional introductory chapter. I do assume a certain level of mathematical proficiency, as might be expected from a graduate student. Nevertheless, the first two chapters are dedicated to the mathematics of quantum mechanics; they are divided into finite- and infinite-dimensional Hilbert spaces, since these concepts are necessary to a smooth introduction.

I start the discussion of quantum mechanics with its postulates and the Schrödinger equation, after which I start developing the formalism. I mostly use the bra-ket notation for abstract states, as it is often cleaner, though for most of the standard systems I use wave functions. I introduce early on the notion of the path integral, since it is an important alternative way to describe quantization, through the sum over all paths, classical or otherwise, though I mostly use the operatorial formalism. I also introduce early on the notion of pictures, including the Heisenberg and interaction pictures, as alternatives for the description of time evolution. The important notion of angular momentum theory is presented within the larger context of symmetries in quantum mechanics, since this is the modern viewpoint, especially within the relativistic quantum field theory that extends quantum mechanics.

With respect to topics, in Part I, on basic concepts, I consider both the older, but still very useful, topics such as WKB and Bohr–Sommerfeld quantization as well as more recent ones such as the Berry phase and Dirac quantization condition. In Part II I consider scattering theory, variational methods, occupation number space, quantum entanglement and information, quantum complexity, quantum chaos, and thermalization; these are the more advanced topics. The advanced foundations of Quantum Mechanics (Part II_a) are discussed in Part II since they are newer and harder to understand even though they could be said to belong to Part I for being foundational issues.

After each chapter, I summarize a set of “Important Concepts to Remember”, and present seven exercises whose solution is meant to clarify the concepts in the chapter.