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Fabien Durand , Dominique Perrin
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This book is the first self-contained exposition of the fascinating link between dynamical systems and dimension groups. The authors explore the rich interplay between topological properties of dynamical systems and the algebraic structures associated with them, with an emphasis on symbolic systems, particularly substitution systems. It is recommended for anybody with an interest in topological and symbolic dynamics, automata theory or combinatorics on words.

Intended to serve as an introduction for graduate students and other newcomers to the field as well as a reference for established researchers, the book includes a thorough account of the background notions as well as detailed exposition – with full proofs – of the major results of the subject. A wealth of examples and exercises, with solutions, serve to build intuition, while the many open problems collected at the end provide jumping-off points for future research.

Fabien Durand is Full Professor in Mathematics at University of Picardie Jules Verne. His interests include topological dynamical systems and the relations with theoretical computer science. He is currently the president of the Société Mathématique de France.

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Dimension Groups and Dynamical Systems

Substitutions, Bratteli Diagrams and Cantor Systems

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