

## Quantum Atom Optics

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The rapid development of quantum technologies has driven a revolution in related research areas such as quantum computation and communication, and quantum materials. The first prototypes of functional quantum devices are beginning to appear. One of the leading platforms to realize such devices are neutral atoms, which allow for the observation of sensitive quantum effects, and have important applications in quantum simulation and matter wave interferometry. This modern text offers a self-contained introduction to the fundamentals of quantum atom optics and atomic many-body matter wave systems. Assuming a familiarity with undergraduate quantum mechanics, this book will be accessible for graduate students and early career researchers moving into this important new field. A detailed description of the underlying theory of quantum atom optics is given, before development of the key quantum technological applications, such as atom interferometry, quantum simulation, quantum metrology, and quantum computing.

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Tim Byrnes , Ebubechukwu O. Ilo-Okeke  
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## Theory and Applications to Quantum Technology

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**To our parents, loved ones, children, friends, the colleagues  
who encouraged and supported us, and the visionaries who dared  
to pursue their curiosity**

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Foreword

Having never written a book foreword beforeward, I did the logical thing and Googled “How to write a book foreword.” I found the following information: 1. Be honest. 2. Use your unique voice. 3. Discuss your connection to the story and authors. 4. Mimic the style of the book. 5. Sign off. From these tidbits of insight, it sounds more like I am conducting an hour-long radio show and not writing a foreword for a book. Nevertheless, I shall endeavor to address each point in turn.

*1. Be honest:* When my friends and colleagues Tim Byrnes and Ebubechukwu Ilo-Okeke asked me to write the foreword to *Quantum Atom Optics*, I was initially delighted because I have been an active researcher in this field for over 30 years and I have many tens of publications in the field. Hence, I was a bit flabbergasted when they sent me a draft of the book upon which perusing that I discovered that they had not cited a single paper written by me. Thus, my original draft of this foreword was, “Not too shabby book by what’s his face and colleague.” Since then I have calmed down a bit, but could not let that one slide. Aside from this unintentional slight, the book is actually very nice. I attributed not referencing my papers to the foolishness of youth and the fact that my papers were published before the authors were born. The field of quantum atom optics is hence quite mature by now, although I’m not dead yet, and such a book is timely and I can’t say enough nice things about the authors. The book is an exhaustive primer of a field that has a plethora of applications to a host of quantum technologies — particularly quantum information processing and quantum sensing. It would make a very nice supplementary book to any course on this topic. In truth the book is a compendium of just about everything that is known in quantum atom optics and as such it will be a much-used reference for many years.

*2. Use your unique voice:* As you can see from the above, my voice is so unique that I often have to translate my anonymous referee reports into Russian and then back into English in order to disguise my writing style. I can assure you that *Quantum Atom Optics* is a much better read than that. It is in fact very well written and delightfully organized in a clear and enticing fashion. Each chapter builds on the previous chapters such that the book builds to a true crescendo at the end, much like Ravel’s orchestration of *Bolero*.

*3. Discuss your connection to the story and authors:* As I mentioned above, apparently unbeknownst to the authors, I have been working in the field of quantum atom optics for much of my career, and this book does an excellent job of covering

the field. I have been collaborating with Tim and Ebube on this topic for five years now and we have a series of papers together, which gives me a unique insight into the writing of their book.

4. *Mimic the style of the book:* Well I can’t use my unique voice and mimic the style of the book, now can I? The book was written by two sane and distinguished scientists. I’m more of an extinguished scientist myself.

5. *Sign off:*

Good bye and good luck,

Jonathan P. Dowling

October 13, 2019  
St. George, Louisiana

Authors’ comment: We made the unfortunate mistake of sending a preliminary version of the manuscript with incomplete referencing. We hope the current version better captures the relevant literature. Among the many new references added, Prof. Dowling’s seminal paper (“Quantum Technology: The Second Quantum Revolution,” co-authored by Gerard J. Milburn), where the much quoted term “Second Quantum Revolution” was first coined is now included. This, along with several other brilliantly named papers (e.g., “Quantum Optical Metrology – The Lowdown on High-N00N States”), will live on forever in the field. Rest in peace Jon, you were an absolute inspiration and will be missed by us all.

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## Preface

This book is an introduction to the field of quantum atom optics, the study of atomic many-body matter wave systems. In many ways, the field mirrors the field of quantum optics – the study of light at its fundamental quantum level. In quantum atom optics, many analogous concepts to quantum optics are encountered. The phenomenon of Bose–Einstein condensation (BEC) can be thought of as the atomic counterpart of lasing. Coherent states of light have analogues with spin coherent states for atoms, which can be visualized with Wigner and  $Q$ -functions. In this book, we start from the basic concepts of many-body atomic systems, learn methods for controlling the quantum state of atoms, and understand how such atoms can be used to store quantum information and be used for various quantum technological purposes.

Despite the many analogous concepts, there are also many differences of atoms and photons when treated at the quantum level. Most fundamentally, atoms can be bosons or fermions and possess mass, in contrast to photons. While lasing and BECs both have a macroscopic occupation of bosons, the mechanism that forms the BECs is ultimately at thermal equilibrium, whereas lasing is a nonequilibrium process. Coherent states of light do not conserve photon number, whereas generally atom numbers do not change shot to shot. Another stark difference is that atoms tend to possess much stronger interactions, whereas it is generally difficult to make photons interact strongly with each other. This makes the details of many-body atomic systems often quite different, which often leads to rather different approaches conceptually and theoretically.

One of the recent major developments has been the explosion of interest in the field of quantum information and technologies. Within a span of 20 years, it has turned from a niche field studied by a small community of physicists with various backgrounds in quantum optics, computer science, and foundations of quantum mechanics to a major research field in its own right. Much of the way of thinking in the quantum information community originates from the field of quantum optics. Now, there is great excitement in how quantum systems can be utilized toward new technologies. In this book, we cover several of the promising applications that atomic systems offer, including spin and matter wave interferometry, quantum simulation, and quantum computing.

Several excellent texts already exist in the field. Notable are Pierre Meystre's *Atom Optics* and Daniel Steck's *Quantum and Atom Optics*. These are both excellent comprehensive resources that cover the theory of atom optics at the fundamental level. For BECs, we refer the reader to excellent texts such as those by Pitaevskii and Stringari and by Pethick and Smith. Rather than duplicate these works, we wished to provide a text at the senior undergraduate to junior graduate levels that covers the basic principles that are necessary so that one can get up to speed with the current literature in a simple and straightforward way. While the topics that we cover

inevitably are only a brief selection of the extensive achievements in the field, we hope our choices of topics reflect the current interest toward applications of such systems. This book would suit students who wish to obtain the necessary skills for working with many-body atomic systems and have an interest toward quantum technology applications.