Cosmology

Progress in modern cosmology has been remarkable, leading to a detailed understanding of the structure and evolution of the universe. In this approachable and self-contained textbook, the author—a leading theoretical cosmologist—expands on his widely acclaimed lecture notes, presenting the theoretical foundations of cosmology and describing the observations that have turned the subject into a precision science.

Notable features of the book are:

- many worked examples and exercises
- numerous end-of-chapter problems
- high-quality figures to illustrate the concepts introduced in the text
- detailed appendices to provide the necessary background material
- online resources—including an annotated list of important cosmology papers, Mathematica notebooks and updates on observational results—to keep students abreast of the latest research.

The step-by-step approach and explicit presentation of key calculations make this book a definitive resource for advanced undergraduate or beginning graduate students in physics, astronomy and applied mathematics, as well as for self-study.

Daniel Baumann is a world-leading theoretical cosmologist. He obtained his doctorate from Princeton University in 2008, after which he was a postdoctoral researcher at Harvard University and the Institute for Advanced Study in Princeton. From 2011, he was a faculty member at Cambridge University, until he was appointed Professor of Theoretical Cosmology at the University of Amsterdam in 2015. Baumann has received numerous awards, including an ERC Starting Grant, an NWO VIDI Grant and a Yushan Professorship at National Taiwan University. His cosmology lecture notes are used worldwide. He is the author of the book *Inflation and String Theory* (with Liam McAllister).

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For Celia and Kosmo

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Preface

Cosmology is both an old and a new subject. While questions about the origin and structure of the universe have been asked for a long time, concrete answers have emerged only relatively recently. A century ago, we didn't know that there are other galaxies beyond our own, we didn't know why stars shine, and we didn't know that the universe is expanding. Even just twenty-five years ago, we didn't know what most of the universe is made of.

Progress in cosmology has been nothing short of astonishing. We have learned that the light elements were created in the first few minutes of the hot Big Bang and that the heavy elements were made inside of stars. We have taken a picture of the universe when it was just 370 000 years old, discovering small fluctuations in the first light that eventually grew into all of the structures we see around us. We have developed powerful numerical simulations to predict the growth of this large-scale structure and measured its statistical properties in galaxy surveys. We have discovered that the universe is dominated by dark matter and dark energy, although their origin is still a mystery. Finally, we have found evidence that the primordial density perturbations originated from microscopic quantum fluctuations, stretched to cosmic scales during a period of inflationary expansion.

This book is about these developments in modern cosmology. I will present the theoretical foundations of the subject and describe the observations that have turned cosmology into a precision science.

About this book

This book grew out of lectures given in Part III of the Mathematical Tripos at the University of Cambridge. Elements of the book were also developed in courses taught at numerous graduate schools. Although the material has been significantly expanded and reworked, the book retains some of the flavor and tone of the original lecture notes. In particular, many calculations are presented in more detail than is common in the traditional textbooks on the subject. A large number of worked examples and problems has been included and coordinated closely with the material. The book is therefore particularly well-suited for self-study or as a course companion.

Although the material covered in the book has mostly been tested on Masters students, I have tried to be very pedagogical, so that large parts of the book should also be accessible to advanced undergraduates. Familiarity with basic relativity, quantum mechanics and statistical mechanics will be helpful, but the book includes all necessary background material and is therefore self-contained.

Structure of the book

Chapter 1 is a brief invitation to the subject. I describe the enormous range of scales involved in cosmology, introduce the forms of matter and energy that fill the universe and sketch key events in the history of the universe.

The rest of the book is organized in two parts. Part I deals with the dynamics of the homogeneous universe, while Part II discusses the evolution of small fluctuations.

- Chapter 2 introduces the expanding universe. I first derive the spacetime geometry of the universe and then determine its evolution in the different stages of the universe's history. The chapter also includes a summary of the key observations that have helped to establish the ΛCDM concordance model.
- Chapter 3 describes the hot Big Bang. I first explain how the thermal history of the universe is shaped by a competition between the rate of particle interactions and the expansion rate. I discuss the concept of local thermal equilibrium and show how the principles of statistical mechanics are applied in the cosmological context. Finally, I introduce the Boltzmann equation as the key tool to go beyond thermal equilibrium. This is then applied to three examples: dark matter freeze-out, Big Bang nucleosynthesis and recombination.
- Chapter 4 explores cosmological inflation. The chapter begins with a careful examination of the horizon and flatness problems of the hot Big Bang. I then explain how inflation solves these problems and discuss the physics of inflation from a modern perspective. The chapter closes with a discussion of several open problems in the field of inflationary cosmology.
- Chapter 5 introduces structure formation in Newtonian gravity. I derive the linearized equations of motion for a non-relativistic fluid and apply them to the evolution of dark matter fluctuations. I present the key statistical properties of these fluctuations in the linear regime. I then give a brief discussion of the nonlinear clustering of dark matter fluctuations and review current observations of the large-scale structure of the universe.
- Chapter 6 develops cosmological perturbation theory in general relativity. This allows us to extend the Newtonian treatment of the previous chapter to relativistic fluids and superhorizon scales. I derive the linearized evolution equations for the coupled matter and metric perturbations, and show how the initial conditions are imposed on superhorizon scales. I then use the relativistic framework to discuss the evolution of dark matter, photons and baryons. The chapter closes with a heuristic description of the CMB anisotropies.
- Chapter 7 presents the physics of the CMB anisotropies in more detail. I derive the propagation of sound waves in the photon–baryon fluid before recombination and show how it leads to a characteristic pattern of fluctuations at the moment

of photon decoupling. I also describe how these fluctuations are modified as the photons travel through the inhomogeneous universe, compute the angular power spectrum of the CMB anisotropies and show how the result depends on the cosmological parameters. Finally, the chapter ends with a discussion of CMB polarization, including the E/B decomposition of the polarization field. I explain why B-modes are a clean probe of primordial gravitational waves.

- Chapter 8 derives the initial conditions predicted by inflation. I first show that each Fourier mode of the inflationary perturbations satisfies the equation of a harmonic oscillator. This implies that the perturbations experience the same zero-point fluctuations as a harmonic oscillator in quantum mechanics. I devote some time to explaining the natural choice of the vacuum state. Finally, I derive the two-point functions of scalar and tensor fluctuations created by inflation. The chapter also includes a discussion of the most promising observational tests of the inflationary paradigm.
- Chapter 9 provides a brief outlook on the bright future of modern cosmology.

The book contains four appendices: Appendix A is a review of the fundamentals of general relativity. Appendix B presents a detailed analysis of the CMB anisotropies based on the Boltzmann equation. This complements the more approximate hydrodynamic treatment of Chapter 7. Appendix C collects parameters and relations that are frequently used in cosmological calculations. Finally, Appendix D reviews the properties of the special functions that make an appearance in the book.

Sections marked with an asterisk (*) contain more advanced material that can be skipped without loss of continuity.

Teaching from this book

My goal in writing this book has been to produce a text that is ideally suited for teaching a class on cosmology (or for self-study). I have therefore limited my choice of topics to those that I think a first introduction to the subject should include. I tried to resist the temptation to include more advanced and specialized topics. In my experience, it takes about one and a half semesters to teach all of the material covered in this book. However, it is also possible to use the book for a one-semester course if some of the details in Chapters 6, 7 and 8 are sacrificed.

Although the book presents the material in the order that I find most logical, it is possible to change that order in some places and I have experimented with this in my own teaching. For example, in my lectures, I have sometimes interchanged the order of Chapters 3 and 4. My students at Cambridge were rather mathematically minded and easily put off by the more dirty aspects of physics. It therefore helped to first seduce them to the subject, before revealing to them the real complexities of the hot Big Bang. Physics and astronomy students might be less put off by the use of nuclear and atomic physics in Chapter 3 and so don't mind seeing it earlier in the course. I have opted to start with the Newtonian treatment of structure formation in the book because it provides a quick and intuitive path to the relevant equations of motion. However, in my lectures, I have often cut down the discussion of Newtonian structure formation in Chapter 5 and merged it with the treatment of relativistic perturbation theory in Chapter 6. Getting to the relativistic treatment directly works for students with a strong math background, but sacrifices the physically intuitive aspects of the Newtonian analysis. In fact, if the book is used in an undergraduate course then presenting only the Newtonian analysis is probably the best option. Finally, I have often interchanged the order of Chapters 7 and 8. Since Chapter 7 is quite detailed, I suggest teaching this material separately in a second part of the course. The first part of the course could instead contain a more heuristic description of the CMB anisotropies, like that in Chapter 6.

Web page for the book

Together with Cambridge University Press, I will maintain a web page for the book (www.cambridge.org/baumann). This web page includes the following content:

- Color versions of selected figures.
- Updates on observational results.
- Notes on historically important cosmology papers.
- Mathematica notebooks for calculations in cosmological perturbation theory.
- For instructors: solutions to the problems and exercises.

Notation and Conventions

This book uses the $(-+++)$ signature of the metric, so that the line element in Minkowski space is $\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2$. We will often employ natural units with $c = \hbar \equiv 1$ and define the reduced Planck mass as $M_{\rm Pl} \equiv 1/\sqrt{8\pi G}$. The conversion between units is explained in Appendix C.

We use Greek letters for spacetime indices, $\mu = 0, 1, 2, 3$, and Latin letters for spatial indices, $i = 1, 2, 3$. Spatial vectors are denoted by $\mathbf{x}$ and their components by x^i. The corresponding three-dimensional wavevectors are $\mathbf{k}$, with magnitudes $k \equiv |\mathbf{k}|$ and unit vectors written as $\hat{\mathbf{k}} \equiv \mathbf{k}/|\mathbf{k}|$. Our convention for the Fourier transform of a function $f(\mathbf{x})$ is

$$f(\mathbf{k}) = \int \mathrm{d}^3x\, f(\mathbf{x})\, e^{-i\mathbf{k}\cdot\mathbf{x}}\,.$$

Derivatives with respect to physical time t are denoted by overdots, while those with respect to conformal time η are given by primes, $f' = a\dot{f}$, where $a(t)$ is the scale factor.

Most of the notation in this book will be introduced as we go along. Here, I just list some of the most commonly used variables, especially if they do not have a uniform notation in the literature:

η	conformal time (or baryon-to-photon ratio!)
τ	proper time (or optical depth!)
H	Hubble parameter, $H \equiv \dot{a}/a$
$\mathcal{H}$	conformal Hubble parameter, $\mathcal{H} \equiv a'/a = Ha$
q	deceleration parameter, $q \equiv -\ddot{a}/(\dot{a}H)$
ρ	energy density
P	pressure
w	equation of state, $w \equiv P/\rho$
δ	density contrast, $\delta \equiv \delta\rho/\rho$
Δ	density contrast in comoving gauge
$\delta_{\rm D}$	Dirac delta function
v_i	bulk velocity, $v_i \equiv \partial_i v + \hat{v}_i$
θ	velocity divergence, $\theta \equiv \partial_i v^i$
q_i	momentum density, $q_i \equiv (\bar{\rho} + \bar{P})v_i$
P^μ	four-momentum
U^μ	four-velocity, $U^\mu \equiv {\rm d}x^\mu/{\rm d}\tau$
Π_{ij}	anisotropic stress
Ψ	gravitational potential in Newtonian gauge, $\delta g_{00} \equiv -2a^2\Psi$
Φ	curvature perturbation in Newtonian gauge, $\delta g_{ij} \equiv -2a^2\Phi\, \delta_{ij}$
ζ	curvature perturbation in uniform density gauge
$\mathcal{R}$	curvature perturbation in comoving gauge
h_{ij}	tensor metric perturbation, $\delta g_{ij} \equiv a^2 h_{ij}$
$A_{\rm s}$	amplitude of scalar fluctuations
$A_{\rm t}$	amplitude of tensor fluctuations
$n_{\rm s}$	scalar spectral index
$n_{\rm t}$	tensor spectral index
r	tensor-to-scalar ratio
ε	Hubble slow-roll parameter, $\varepsilon \equiv -\dot{H}/H^2$
κ	Hubble slow-roll parameter, $\kappa \equiv \dot{\varepsilon}/(\varepsilon H)$
ϕ	inflaton field
$V(\phi)$	inflaton potential
δ	dimensionless acceleration, $\delta \equiv -\ddot{\phi}/(H\dot{\phi})$
ε_V	potential slow-roll parameter, $\varepsilon_V \equiv (M_{\rm Pl}^2/2)(V_{,\phi}/V)^2$
η_V	potential slow-roll parameter, $\eta_V \equiv M_{\rm Pl}^2(V_{,\phi\phi}/V)$
$\mathcal{P}_f(k)$	power spectrum, $\langle f(\mathbf{k})f(\mathbf{k}')\rangle \equiv (2\pi)^3\delta_{\rm D}(\mathbf{k}+\mathbf{k}')\,\mathcal{P}_f(k)$
$\Delta_f^2(k)$	dimensionless power spectrum, $\Delta_f^2(k) \equiv (k^3/2\pi^2)\mathcal{P}_f(k)$
C_l	CMB power spectrum, $\langle a_{lm}a^*_{l'm'}\rangle \equiv C_l\,\delta_{ll'}\delta_{mm'}$
Δ_T^2	rescaled CMB power spectrum, $\Delta_T^2 \equiv [l(l+1)/2\pi]C_l\,T_0^2$
g_*	effective number of relativistic degrees of freedom
$N_{\rm eff}$	effective number of neutrino species
μ	chemical potential

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This book would not exist without the many emails I have received from students around the world expressing their appreciation of my lecture notes. I am very grateful for their encouragement and I hope that this book will be a useful resource for them and others.

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