

Contents

<i>Foreword by Francis Bach</i>	<i>page ix</i>
<i>Symbols</i>	<i>xi</i>
<i>Preface</i>	<i>xiii</i>
1 Introduction	1
PART ONE HISTORICAL AND THEORETICAL BACKGROUND	
2 Positive-definite Kernels and Moment Problems	7
2.1 Hilbert Space with a Reproducing Kernel	9
2.2 The CD kernel and the Moment Problem on the Real Line	16
2.3 Relation with Other Kernels	19
2.4 Notes and Sources	24
3 Univariate Christoffel–Darboux Analysis	27
3.1 Zeros of Orthogonal Polynomials	27
3.2 Asymptotics on the Complex Unit Circle	29
3.3 Asymptotics of the Christoffel Function on the Real Line	31
3.4 Asymptotics Outside the Support	33
3.5 Further Results and Remarks	37
3.6 Notes and Sources	38
4 Multivariate Christoffel–Darboux Analysis	42
4.1 Preliminaries	43
4.2 Explicit Asymptotics for Special Geometries	47
4.3 Quantitative Asymptotics and the Bernstein–Markov Property	50
4.4 Qualitative Asymptotics	52
4.5 Notes and Sources	55

5	Singular Supports	61
5.1	Preliminary Material	61
5.2	The CD Kernel on a Real Variety	63
5.3	Asymptotics	69
5.4	Notes and Sources	71
PART TWO STATISTICS AND APPLICATIONS TO DATA ANALYSIS		75
6	Empirical Christoffel–Darboux Analysis	77
6.1	The Empirical Christoffel Function	77
6.2	A Stability Result and a Law of Large Numbers	81
6.3	Finite Sample Convergence and Bounds	82
6.4	Notes and Sources	85
7	Applications and Occurrences in Data Analysis	87
7.1	Parameteric Regression	88
7.2	Optimal Design of Experiments	90
7.3	Inference from a Sample	97
7.4	Notes and Sources	110
PART THREE COMPLEMENTARY TOPICS		113
8	Further Applications	115
8.1	Approximation of Nonsmooth Functions	115
8.2	Spectral Analysis of Dynamical Systems	119
8.3	Notes and Sources	122
9	Transforms of Christoffel–Darboux Kernels	125
9.1	Perturbations of Christoffel–Darboux Kernels	125
9.2	Eliminating Outliers in a 2D Christoffel–Darboux Kernel	128
9.3	Notes and Sources	134
10	Spectral Characterization and Extensions of the Christoffel Function	136
10.1	A Spectral Characterization of the Christoffel Function	137
10.2	Extension to $L^k(\Omega, \mu)$ Spaces	140
10.3	Extension via Convex Cones of Positivity on Ω	144
10.4	Multi-point Interpolation	147
10.5	Notes and Sources	153
<i>References</i>		155
<i>Index</i>		167