1 General Introduction

This brief introductory chapter outlines the broad coverage of the book and its intended contribution in the context of other available sources. It is recognized that there are many excellent books covering the "mechanics of materials," often with a strong bias towards metals, but relatively few that are focused strongly on testing procedures designed to reveal details about how they deform plastically. There are in fact many subtleties concerning metal plasticity and the information about it obtainable via various types of test. No attempt is made in this chapter to convey any of these, but the scene is set in terms of outlining the absolute basics of elastic and plastic deformation.

1.1 Rationale and Scope of the Book

The issue of how metals undergo plastic deformation (including creep – see the end of §1.3 below), and how this behavior can be measured and characterized, is a very old (and important!) one. Attempts to measure the “strength” of metals (and other materials) date back to antiquity. Understanding of the deformation mechanisms involved, and implications for systematic control over mechanical properties, are more recent, but they still extend back several decades. A number of texts were produced during the period in which most of the key advances were made, which was mainly between the 1950s and 1980s, and in many cases these have been followed up with a number of updated editions. Also, several new books have been published more recently. Many are strongly oriented towards engineering, with materials treated as (isotropic) continua and their properties described by analytical equations (constitutive laws). There are also several books containing a mixture of physical metallurgy and associated mechanical properties. Mechanical testing procedures are often covered in some way, although relatively few books have been dedicated to this area.

This book is partly aimed at comprehensive description of mechanical testing procedures, over a wide range of conditions (notably temperature and strain rate). Even the standard, “simple” procedures, such as uniaxial tensile testing, do in fact incorporate complexities that are not always well understood, but a large part of the motivation for the book relates to the fact that other testing geometries, particularly indentation, have become widespread recently. Also, the technique of finite element
method (FEM) modeling, which is a powerful tool for investigation of mechanical
deformation, is now mature and ubiquitous. Its use figures strongly in the book.

The theme of the book is not strongly oriented towards “nanoindentation” as such,
although it is included in the coverage. It is mainly related to the obtaining of bulk
mechanical properties (in the context of microstructure and deformation mechanisms,
and their relationship to such properties). What would commonly be regarded as
nanoindentation is not well suited to this objective, since the volume being mechanic-
ally interrogated is often too small for its response to be representative of the bulk.
However, the key recent advances concerning indentation do not really relate to it
being carried out on a fine scale, but rather to the process being instrumented so as to
obtain detailed information about the (bulk) mechanical characteristics of the sample.
This represents a fundamental advance, relative to its origins in hardness testing.

1.2 Structure and Readership of the Book

The structure of the book effectively comprises three sections. After this general
introduction, there is a chapter dedicated to the handling of stresses and strains (as
second rank tensors) and to the relationships between them during elastic deform-
ation. This is followed by a chapter focussed on how such relationships are modified
after the onset of plastic deformation (with the material treated as a homogeneous
continuum) and then one on the mechanisms of plastic deformation and how they are
affected by the microstructure of the metal. All three of these chapters can certainly
be regarded as providing background information, although it is clearly relevant to a
full understanding of plasticity. Similar material is available from a wide range of
sources, but it is considered to be potentially helpful to include it in the book, for
ease of reference. There is then a set of three chapters that cover the “standard”
procedures of tensile, compressive and hardness testing. These are undoubtedly
familiar to people involved with mechanical (plasticity) testing, but they do involve
some subtleties that are not universally understood. These chapters contain guide-
lines for obtaining and interpreting experimental data most effectively with these
procedures.

The final section of three chapters concerns testing techniques that have been
developed more recently, with the first of these, outlining the current state of the art
in indentation plastometry, given particularly close attention in view of its scope for
obtaining detailed plasticity characteristics in a convenient and flexible way. The
following one covers “nanoindentation,” which has had a high profile over recent
years, but actually suffers from some severe limitations in terms of obtaining infor-
mation about bulk plasticity characteristics. The last chapter covers a range of fairly
specialized test procedures, some in developmental form. There is thus a developing
emphasis throughout the book on more recent (and research-oriented) topics, which is
reflected in progressively increasing levels of citation of other published work. These
references are collected at the end of each chapter. There is also a nomenclature listing
at the beginning, with an attempt made to standardize the symbolism in use throughout the book.

It should be noted that the topic of fracture has a relatively low profile in the coverage. While indentation can sometimes be used to stimulate fracture, or at least crack propagation, it is not really an effect that can readily be investigated via indentation. Of course, some kind of fracture event does normally take place at the end of a tensile test, although local conditions during that event are often rather poorly defined. Testing aimed at obtaining fundamental fracture mechanics properties, such as the fracture energy or the fracture toughness, are carried out rather differently (and those tests are not described in the book). In fact, at least for most metals, it is largely the plastic deformation (yield stress and work hardening characteristics) that dictates their “strength” (as measured by the “ultimate tensile stress,” UTS), with their fracture toughness as such (important as that is for other purposes) being of limited significance during such testing. Moreover, by interpreting the outcome of a tensile test via its simulation in an FEM model, it is possible to obtain an estimate of the critical strain for fracture, which is a widely used “property.” Furthermore, coverage is included (in the final chapter) of certain types of cyclic loading test that may involve crack propagation and some of the basics of fracture mechanics are presented there. Such tests are closely related to real industrial service environments and, indeed, there is reference throughout the book to the relevance of various tests to industrial usage of metallic components.

A natural part of the readership is those already involved in mechanical testing (of metals), who need an update and a book for future reference. However, it is potentially of much broader appeal, since many people involved in the development of novel or improved materials and components have a need for understanding of the issues associated with mechanical deformation. Indentation plastometry has many attractions, and its usage is likely to increase over coming years. This book is aimed at providing a source of information for potential users of the technology. Furthermore, there is certainly potential for usage in university level teaching. The prerequisites are simply a very basic background in materials science and mechanics. Finally, there are many people involved in various manufacturing and processing sectors with an interest in improved understanding of the links between processing conditions and component performance in service. Metal plasticity and its characterization are central to this.

1.3 Basic Elastic and Plastic Property Ranges

There is, of course, a fundamental difference between elastic and plastic properties (of metals). Elastic deformation essentially arises because the distances between atoms are being changed (by an applied load). These distances increase in the direction of an applied tensile force, and decrease in directions transverse to this. Since the transverse reductions do not compensate fully for the axial extension, there is normally an
associated volume change (unless the Poisson ratio has a value of 0.5, which is not the case for any metal). The “building blocks” (unit cells of the structure for crystalline materials, i.e. for the vast majority of metals) have all been distorted. Of course, the deformation is fully reversible on removing the applied load. This is the basic meaning of “elastic.”

Plastic deformation, on the other hand, involves no volume change. It effectively occurs by moving some unit cells with respect to others (without distorting them). This most commonly occurs via the motion of dislocations (line defects within the structure). Other important differences, compared with the case of elastic deformation, are that the strains involved are commonly much greater and that, in general, the deformation is not readily reversible. (The word “plastic” effectively means “permanent.”)

Details concerning plastic deformation – and there are many – are covered elsewhere in the book. However, it may be useful here to summarize the range likely to be encountered for the main parameter values (of metals). These are the Young’s modulus ($E$), defined as the applied stress over the resultant (elastic) strain, the yield stress (applied stress level at which plastic deformation starts) and the ultimate tensile stress, which is the peak (nominal) stress, given by the maximum load attained during the test divided by the original sectional area of the sample. Typical values for various metals are shown in Table 1.1, although it should be emphasized that the yielding and strength values supplied there are simply indicative and are subject to large variations for the same material.

Table 1.1 Overview of the basic elastic and (approximate) plastic properties of a range of metals.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus, $E$ (GPa)</th>
<th>Yield stress, $\sigma_Y$ (MPa)</th>
<th>Yield strain, $\epsilon_Y$ (millistrain)</th>
<th>Ultimate tensile stress, $\sigma$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild steel</td>
<td>205</td>
<td>250</td>
<td>1.2</td>
<td>370</td>
</tr>
<tr>
<td>304 stainless steel</td>
<td>200</td>
<td>250</td>
<td>1.2</td>
<td>600</td>
</tr>
<tr>
<td>4130 steel</td>
<td>205</td>
<td>400</td>
<td>1.9</td>
<td>700</td>
</tr>
<tr>
<td>Hadfield’s Mn steel</td>
<td>210</td>
<td>350</td>
<td>1.7</td>
<td>1000</td>
</tr>
<tr>
<td>4340 steel</td>
<td>205</td>
<td>700</td>
<td>3.4</td>
<td>1100</td>
</tr>
<tr>
<td>A228 piano wire</td>
<td>200</td>
<td>1000</td>
<td>5</td>
<td>2000</td>
</tr>
<tr>
<td>Cast iron</td>
<td>170</td>
<td>350</td>
<td>2</td>
<td>500</td>
</tr>
<tr>
<td>1050 Al</td>
<td>71</td>
<td>100</td>
<td>1.4</td>
<td>150</td>
</tr>
<tr>
<td>6061 Al</td>
<td>69</td>
<td>250</td>
<td>3.6</td>
<td>300</td>
</tr>
<tr>
<td>7075 Al</td>
<td>71</td>
<td>500</td>
<td>7</td>
<td>600</td>
</tr>
<tr>
<td>OFHC Cu</td>
<td>130</td>
<td>350</td>
<td>2.7</td>
<td>370</td>
</tr>
<tr>
<td>Cu-2%Be</td>
<td>130</td>
<td>500</td>
<td>3.8</td>
<td>700</td>
</tr>
<tr>
<td>Cu-40%Zn brass</td>
<td>97</td>
<td>250</td>
<td>2.5</td>
<td>400</td>
</tr>
<tr>
<td>Solder (Pb-40%Sn)</td>
<td>30</td>
<td>30</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>Inconel 718</td>
<td>200</td>
<td>600</td>
<td>3</td>
<td>1000</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>115</td>
<td>950</td>
<td>8</td>
<td>1050</td>
</tr>
<tr>
<td>Be</td>
<td>300</td>
<td>250</td>
<td>0.8</td>
<td>370</td>
</tr>
<tr>
<td>Al-20%SiC$_p$ (MMC)</td>
<td>90</td>
<td>250</td>
<td>2.8</td>
<td>300</td>
</tr>
</tbody>
</table>
It may be noted that the Young’s modulus (stiffness) depends only on the types of atoms present, which determines the interatomic forces. For example, all steels have a similar value of $E$, since they are all composed predominantly of Fe atoms. In contrast to this, the onset of plasticity (i.e. the yield stress) depends in a complex manner on the “microstructure” of the metal, a term that encompasses many features that can change with purity level, (relatively small) alloying additions, processing conditions, heat treatments etc. This arises mainly because these features are likely to have an effect on the ease of dislocation motion. Details of these effects are provided in Chapter 4. The yield stresses of different steels thus cover a wide range. This is also true of the ultimate tensile stress. (In fact, for much of the data in Table 1.1, a strong caveat should be appended to the yield stress and tensile strength values, to the effect that they could change significantly if the component concerned were to be heated or subjected to some other kind of treatment.) It also follows that the strain at the onset of yielding varies between different metals, although, as can be seen, the value is in all cases quite small ($<1\%$, i.e. 10 millistrain). In contrast, plastic strains are commonly large (typically several tens of %).

At this juncture, only a few broad points of this type should be noted concerning the data in Table 1.1. Reliable and meaningful information concerning the plasticity of metals is far from simple in terms of both experimental procedures and interpretation of the data. Of course, the values in the table do refer to material response at ambient temperature. It will tend to be different at both lower and (particularly) higher temperatures than this. It should also be noted that the behavior is assumed to be time-independent, so the rate at which the load is applied is taken to have no effect. In practice, this is often fairly reliable over the range of loading rates that is likely to be used, but both very high rates and very low rates can lead to a significantly different response. In particular, what is usually termed creep – i.e. progressive plastic deformation at a constant applied load – may become noticeable at low loading rates. However, such behavior should be characterized using laws that incorporate time, rather than making some sort of adjustment to the apparent plasticity parameters. All of these issues are covered in detail elsewhere in the book.

A final point can be made that concerns composite materials. Of course, most composites are based on polymers, which are outside the scope of the book, but metal matrix composites (MMC) are in industrial use and are certainly worthy of study. In most cases, they contain ceramic reinforcement, in the form of either particles or fibers. These raise the stiffness and also usually lead to a higher yield stress and UTS, although often at a cost in reduced ductility and toughness. Table 1.1 does contain data for an MMC, although such materials have not really reached the levels of development and stability typical of many alloys, so this material, and these values, are not very well defined. It is perhaps also worth recognizing that metals containing relatively high levels of porosity can be treated as a special type of composite. Metal components, particularly castings, do sometimes contain porosity, although in many cases the levels are too low ($<1\%$) to have much effect on elastic or plastic properties. On the other hand, metals containing high levels of porosity ($>30\%$), produced deliberately in some way, have found
some applications: they would commonly be described as metallic foams. Their properties, both elastic and plastic, are naturally very different from the corresponding pore-free metal. Again, however, such materials are not well defined or mature, so it’s difficult to justify the inclusion of a specific example in Table 1.1. Nevertheless, it is worth noting that the presence of pores in a metal, at or above a level of a few %, can affect the mechanical response.
2 Stresses, Strains and Elasticity

Comprehensive treatment of metal plasticity requires an understanding of the fundamental nature of stresses and strains. A stress can be understood at a basic level as a force per unit area on which it acts, while a strain is an extension divided by an original length. However, the limitations of these definitions rapidly become clear when considering anything other than very simple loading situations. Analysis of various practical situations can in fact be rigorously implemented without becoming embroiled in mathematical complexity, most commonly via usage of commercial (finite element) numerical modeling packages. However, there are various issues involved in such treatments, which need to be appreciated by practitioners if outcomes are to be understood in detail. This chapter covers the necessary fundamentals, relating to stresses and strains, and to their relationship during elastic (reversible) deformation. How this relationship becomes modified when the material undergoes plastic (permanent) deformation is covered in the following chapter.

2.1 Stress and Strain as Second Rank Tensors

Although stress and strain can sometimes be handled as if they were simple scalars (i.e. numbers with no directions associated with them), more rigorous treatment is often required. Stress and strain are in fact second rank tensors. The utility of tensors is mainly concerned with treating differences in the response or characteristics of a material in different directions within it. Their usage is thus particularly required when treating anisotropic materials. However, even for isotropic materials, tensor analysis is necessary, or at least very helpful, when treating many types of mechanical loading.

A tensor is an \( n \)-dimensional array of values, where \( n \) is the “rank” of the tensor. The simplest type is thus a tensor of zeroth rank, which is a scalar – i.e. a single numerical value. Properties like temperature and density are scalars. They are not associated with any particular direction in the material concerned, and the variable does not require any associated index (suffix\(^1\)). A first rank tensor is a vector. This is a 1-D array of values. There are normally three values in the array, each corresponding to one of three (orthogonal) directions. Each value has a single suffix, specifying the

\(^1\) The term “suffix” is in common use for these indices, although they are employed as subscripts.
direction concerned. These suffices are commonly numerical (1, 2 and 3), although sometimes other nomenclature, such as \((x, y \text{ and } z)\) or \((r, \theta \text{ and } z)\), may be used. Force and velocity are examples of vectors. The components of a vector can thus be written down in a form such as

\[
F = F_1 = [F_1 \ F_2 \ F_3]
\]

with each of the suffices (1, 2 and 3) referring to a specific direction, such as \((x, y \text{ and } z)\).

There are other variables, including stress, for which each component requires the specification of two directions, rather than one, so that two suffices are needed. In the case of stress, these two suffices specify, firstly, the direction in which a force is being applied and, secondly, the normal of the plane on which the force is acting. Stress is thus a second rank tensor and the components form a 2-D array of values.

\[
\sigma_{ij} = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix}
\]

(2.2)

When the suffices \(i\) and \(j\) are the same, the force acts parallel to the plane normal, and so the component concerned is a normal stress. When they are different, it is a shear stress (and sometimes the symbol \(\tau\) is used instead of \(\sigma\) for such components).

Some of the stresses that could act on a body are depicted in Fig. 2.1. Provided that the body is in static equilibrium, which is commonly assumed, then the normal forces acting on opposite faces so as to generate a normal stress (e.g. \(\sigma_{33}\) in Fig. 2.1) must be equal in magnitude and anti-parallel in direction. (If this were not the case, then the body would translate.) For shear stresses, a further condition applies. Not only must the two forces generating the \(\sigma_{23}\) stress (see Fig. 2.1) be equal and opposite, but the magnitude of the \(\sigma_{32}\) stress must be equal to that of the \(\sigma_{23}\) stress. (If this were not the
case, then the body would rotate.) Shear stresses thus act in pairs. This applies to all shear stresses, so that $\sigma_{ij} = \sigma_{ji}$, and the tensor represented in Eqn. (2.2) must be symmetrical.

### 2.2 Transformation of Axes

It follows that there are just six independent components in a general stress state – three normal stresses and three shear stresses. Their magnitude will, of course, depend on the directions of the axes chosen to provide the frame of reference. However, the state of stress itself will clearly be unaffected if we choose an alternative frame of reference. Any tensor can be transformed so as to be referred to a new set of axes, provided the orientation of these with respect to the original set is specified. Furthermore, any stress state can be expressed solely in terms of normal stresses (i.e. all shear stresses are zero), provided that a certain set of axes is chosen. Partly because it’s often helpful to express a stress state in terms of this unique set of normal stresses, the procedures for transforming tensors are important. They are illustrated first for a vector (force) and then for a stress.

#### 2.2.1 Transforming First Rank Tensors (Vectors)

Consider a vector (a force, for example), $F = [0, F_2, F_3]$, with components that are referred to the axis set (1, 2, 3). A specific reorientation of this set of axes is now introduced, namely a rotation by an angle $\phi$ about the 1-axis, to create a new axis set (1’, 2’, 3’) – see Fig. 2.2. In this case, the new 1’-axis coincides with the old 1-axis, but the 2’- and 3’-axes have been rotated with respect to the 2- and 3-axes.

**Fig. 2.2** Rotation, in the 2–3 plane, of the axes forming the reference frame for a vector $F$. 
The values of $F_{20}'$ and $F_{30}'$ are found by resolving the components $F_2$ and $F_3$ onto the $2'$ and $3'$ axes and adding these resolved components together:

$$F_{20}' = F_2 \cos (2' - 2) + F_3 \cos (2' - 3)$$
$$F_{30}' = F_2 \cos (3' - 2) + F_3 \cos (3' - 3)$$

(2.3)

where the symbolism $(x - y)$ represents the angle between the $x$ and $y$ axes. In terms of the angle $\phi$, these two equations can be written as

$$F_{20}' = F_2 \cos (-\phi) + F_3 \cos (-90 - \phi) = F_2 \cos \phi - F_3 \sin \phi$$
$$F_{30}' = F_2 \cos (90 - \phi) + F_3 \cos (-\phi) = F_2 \sin \phi + F_3 \cos \phi$$

(2.4)

Clearly, these cosines (of angles between new and old axes) are central to such transformations. They are commonly termed direction cosines and represented by $a_{ij}$, which is conventionally the cosine of the angle between the new $i$ direction ($= i'$) and the old $j$ direction. Of course, the rationale can be extended to cases in which all three axes have been reoriented, leading to the following set of equations:

$$F_{10}' = a_{11}F_1 + a_{12}F_2 + a_{13}F_3$$
$$F_{20}' = a_{21}F_1 + a_{22}F_2 + a_{23}F_3$$
$$F_{30}' = a_{31}F_1 + a_{32}F_2 + a_{33}F_3$$

(2.5)

It can be seen that the direction cosines form a matrix and this set of equations can be written more compactly in matrix form:

$$\begin{bmatrix}
F_{10}' \\
F_{20}' \\
F_{30}'
\end{bmatrix} =
\begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2 \\
F_3
\end{bmatrix}$$

(2.6)

in which the transformation matrix is given by

$$[T] =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}$$

(2.7)

Sets of equations such as Eq. (2.6) can be written even more concisely by using the Einstein summation convention. This states that, when a suffix occurs twice in the same term, then this indicates that summation should be carried out with respect to that term. For example, in the equation

$$F_{i} = a_{ij}F_j$$

(2.8)

$j$ is a dummy suffix, which is to be summed (from 1 to 3). The $i$ suffix, on the other hand, is a free suffix, which can be given any chosen value. For example, Eqn. (2.8) could be used to create the equation

$$F_{10}' = a_{11}F_1 + a_{12}F_2 + a_{13}F_3$$

(2.9)

and also the two other equations, corresponding to $i$ being equal to 2 or 3.