

GINZBURG–LANDAU THEORY OF CONDENSATES

Ginzburg–Landau theory is an important tool in condensed matter physics research, describing the ordered phases of condensed matter, including the dynamics, elasticity, and thermodynamics of the condensed configurations. In this systematic introduction to Ginzburg–Landau theory, both common and topological excitations are considered on the same footing (including their thermodynamics and dynamical phenomena). The role of the topological versus energetic considerations is made clear. Required mathematics, symmetry, including lattice translation, topology, and perturbative techniques are introduced as needed. The results are illustrated using arguably the most fascinating class of such systems, high T_c superconductors subject to magnetic field. This book is an important reference for both researchers and graduate students working in condensed matter physics or can act as a textbook for those taking advanced courses on these topics.

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GINZBURG–LANDAU THEORY OF CONDENSATES

Thermodynamics, Dynamics, and Formation
of Topological Matter

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Preface

The purpose of this book is twofold. The first goal is to present the Ginzburg–Landau approach to the description of various ordered phases of condensed matter, while the secondary goal is to consider in this framework the physics of “topological defects” in such phases or “condensates” like vortices. Despite the fact that the microscopic origin of the condensates is quantum mechanical, the Ginzburg–Landau–Wilson description on the mesoscopic scale is a much simpler classical field theory of the appropriate order parameter. Such a description is suitable (and in most cases the only available one) to describe dynamics, elasticity, and thermodynamics of the condensate, solitons, and their arrays.

It is not assumed that the reader is familiar with quantum mechanics, statistical physics, field theory, or condensed matter physics, so that the present course can be taught in parallel to these undergraduate courses. Necessary concepts and theoretical techniques are developed along the way.

Inclusion of the topological matter in the book (especially its timing) is not accidental. An unexpected discovery in 1987 of high-temperature superconductors opened the door for extensive confrontations of old and new ideas in this field with experiments on the Abrikosov vortex “matter” handily produced and manipulated in these unique materials by applying a magnetic field. Thirty years of fast, fruitful development produced an unprecedented body of new hypotheses, novel methods, and answers to old questions and to new ones that nobody had asked before. Since this activity somewhat subsided recently, the authors feel that it is the right time to summarize what had been achieved in a pedagogical form. Novel systems harboring the soliton matter are emerging. For example, during the last decade, skyrmions in quantum magnets became a focus of condensed matter and mesoscopic physics community.

The textbook resulted in part from experiences during a course taught by one of the authors over the years and incorporates some material included in the authors’ review article “Ginzburg–Landau Theory of Type II Superconductors in Magnetic Field,” published in *Review of Modern Physics*.

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General References

Several textbooks naturally complement the present book.

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[2] Physics of superconductors and superfluids are described on basic level, for example, in J. F. Annett, *Superconductivity, Superfluids, and Condensates*, Oxford University Press, 2004.

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