

## Stellar Structure and Evolution

*Stellar Structure and Evolution*, the second volume in the Ohio State Astrophysics Series, takes advantage of our new era of stellar astrophysics, in which modern techniques allow us to map the interiors of stars in unprecedented detail. This textbook for upper-level undergraduate and graduate students aims to develop a broad physical understanding of the fundamental principles that dictate stellar properties. The study of stellar evolution focuses on the “life cycle” of stars: how they are born, how they live, and how they die. As elements ejected by one generation of stars are incorporated into the next generation, stellar evolution is intertwined with the chemical evolution of our galaxy. Focusing on key physical processes without going into encyclopedic depth, the authors present stellar evolution in a contemporary context, including phenomena such as pulsations, mass loss, binary interactions, and rotation, which contribute to our understanding of stars.

MARC PINSONNEAULT received his Ph.D. in astronomy from Yale University in 1988. He is a full professor of astronomy at The Ohio State University, where he has been teaching since 1994. He has an extensive research record in theoretical models of stellar structure and evolution, with an emphasis on stellar rotation and magnetism, rotationally induced mixing, helio- and asteroseismology, solar models, and solar neutrinos. He was elected a Fellow of the AAAS in 2010 and was recognized as a Distinguished University Scholar at Ohio State in 2017.

BARBARA RYDEN received her Ph.D. in astrophysical sciences from Princeton University. After postdocs at the Harvard-Smithsonian Center for Astrophysics and the Canadian Institute for Theoretical Astrophysics, she joined the astronomy faculty at The Ohio State University, where she is a full professor. She has 30 years of experience in teaching, at levels ranging from introductory undergraduate courses to advanced graduate seminars. She won the Chambliss Astronomical Writing Award for her textbook *Introduction to Cosmology*, and she is co-author, with Richard Pogge, of *Interstellar and Intergalactic Medium*.

“Pinsonneault and Ryden’s book is a very welcome addition to the field of stellar evolution at a level appropriate to advanced undergraduate- or graduate-level study, since it manages to provide a clear, comprehensive overview of topics, without being intimidating in size or style. The textbook includes up-to-date results from contemporary missions such as Gaia and Kepler, with the final chapters discussing stellar rotation, pulsations, and binary evolution in depth. Most chapters include a few well-designed exercises, with a research-level reading list provided after the appendix. I would highly recommend it for Master’s-level courses on stellar structure and evolution.”

**Professor Paul Crowther, University of Sheffield**

“This text is a welcome addition to the pantheon of monographs and textbooks explaining the physical basics of stellar structure and evolution. Aimed primarily at an audience learning the material for the first time, this text explains the phases of the life of a star through a clear application of physical principles. Weaving together classical fluids, quantum mechanics, thermodynamics, and nuclear physics, it enables students and their instructors to gain the physical intuition needed for the study of stars in this time of their observational renaissance.”

**Professor Lars Bildsten, University of California, Santa Barbara**

“This is a welcome addition to the literature, providing a comprehensive overview of stellar structure and evolution, and including insights from the latest data, techniques, and results.”

**Professor William Chaplin, University of Birmingham**

# Stellar Structure and Evolution

**Marc Pinsonneault**

*The Ohio State University*

**Barbara Ryden**

*The Ohio State University*



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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

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*For Julie, my Sun and Moon.* MP

*For Pat Westphal, who started me down  
the inclined plane of physics.* BR



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## Preface

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This textbook is part of a series based on the curriculum for astronomy graduate students at The Ohio State University (OSU). In this curriculum, first-year graduate students take a five credit-hour course “Observed Properties of Astronomical Systems.” This is followed by six courses, each of two or three credit-hours: “Atomic and Radiative Processes in Astrophysics,” “Stellar Structure and Evolution,” “Dynamics,” “Cosmology,” “Numerical and Statistical Methods in Astrophysics,” and “The Interstellar Medium and the Intergalactic Medium.” The philosophy of the OSU graduate program, however, is best encapsulated in the two credit-hour course “Order of Magnitude Astrophysics,” which is offered every year to first- and second-year students. In this course, students work together to solve a wide range of astrophysical problems, using basic physical principles to find back-of-envelope solutions.

The Ohio State Astrophysics Series (OSAS), of which this is the second volume, is a projected series of books based on lecture notes for the six core courses and the first-year “Observed Properties” course. These textbooks will not be exhaustive monographs, but will instead adopt the back-of-envelope philosophy of the “Order of Magnitude” course to emphasize the most important physical principles in each subfield of astrophysics. The goal is to make our series a point of entry into the deeper and more detailed classic textbooks in our field. Although each volume in OSAS will stand on its own, care will be taken to unify notation and vocabulary as much as possible across volumes.

*Stellar Structure and Evolution* is based on the semester-long class of the same name. Stellar structure focuses on the underlying physics of stars. It naturally includes subjects that undergraduate astronomy and physics majors usually see in isolation: statistical mechanics, thermodynamics, electricity and magnetism, quantum mechanics, waves, fluid dynamics, and nuclear physics, among others. As such, the potential list of topics is vast, and we cannot address all of them. Our primary goal is to develop physical intuition for the fundamental principles

that dictate the main properties of stars. A secondary goal is for students to see how disparate tools can be harnessed to understand a rich physical system like a star. Stellar evolution is the study of the “life cycle” of stars: how they are born, how they live, and how they die. Stellar evolution is intertwined with chemical evolution, the origin of the elements on the periodic table. Classical texts in the field have focused mainly on the evolution of spherical isolated stars. This does not reflect the current state of the art in a dynamic field. We therefore present stellar evolution in a modern context, including phenomena such as mass loss, binary interactions, and rotation where relevant.

This textbook uses the cgs (centimeter, gram, second) system of units commonly used in graduate education in astronomy. It also uses the most common astronomical distance units: the solar radius ( $R_{\odot}$ ), the astronomical unit (au), and the parsec (pc). In addition, masses are given in units of the solar mass ( $M_{\odot}$ ) and luminosities in units of the solar luminosity ( $L_{\odot}$ ). On small scales, when we examine individual photons and other particles, the electron-volt (eV) will be a useful small unit of energy, with  $1 \text{ eV} = 1.602 \times 10^{-12} \text{ erg}$ . Other helpful conversion factors, and the values of physical and astronomical constants, are included in the Appendices. Online resources for this book hosted by Cambridge University Press include ancillary materials such as a solutions manual and links to Jupyter notebooks for recreating and modifying figures in the textbook.

The text of this book was greatly improved by the careful reading and insightful recommendations of Jennifer Johnson (OSU). Many of the figures and images in this book are derived from works in the published astronomical literature. We are grateful to the authors and journals who promptly granted permission to use their figures. We are especially grateful to those of our colleagues who dug out their original data for us to replot for this volume. Particular thanks are due to Emily Griffith (OSU) for Figure 1.9, Zeki Eker (Akdeniz University) for Figures 1.12 and 1.13, Franck Delahaye (Observatoire de Paris) for Figure 4.2, Kohji Takahashi (GSI) for Figure 5.2, Patrick Vallely (OSU) for Figure 9.3, Jamie Tayar (Institute for Astronomy) for Figure 10.4, Gibor Basri (University of California, Berkeley) for Figure 10.7, Radek Poleski (University of Warsaw) for Figure 11.1, and Mathieu Vriard (OSU) for Figure 11.9. All original figures were created by Richard Pogge (OSU), in his role as technical editor of the Ohio State Astrophysics Series.