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TIME-DOMAIN SCATTERING

The wave equation, a classical partial differential equation, has been studied and applied since the eighteenth century. Solving it in the presence of an obstacle, the scatterer, can be achieved using a variety of techniques and has a multitude of applications. This book explains clearly the fundamental ideas of time-domain scattering, including in-depth discussions of separation of variables and integral equations. The author covers both theoretical and computational aspects, and describes applications coming from acoustics (sound waves), elastodynamics (waves in solids), electromagnetics (Maxwell's equations) and hydrodynamics (water waves). The detailed bibliography of papers and books from the last 100 years cement the position of this work as an essential reference on the topic for applied mathematicians, physicists and engineers.

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To Thomas, Emma, Samantha and, last but not least, Ann

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Preface

I have been studying scattering problems since I was a graduate student in the late 1970s. Specifically, I have studied time-harmonic problems where the solutions depend on time t through a factor $\exp(-\mathrm{i}\omega t)$ in which ω is the frequency; they are *frequency-domain problems*. More recently, I decided to study *time-domain problems*; the prototype problem is: solve the wave equation in the unbounded region exterior to a bounded obstacle (the scatterer) subject to appropriate boundary and initial conditions. One result is this book.

My frequency-domain work resulted in a book, *Multiple Scattering: Interaction of Time-Harmonic Waves with N Obstacles*, published in 2006. The present book is written in the same spirit. The problems considered come mainly from acoustics (governed by the three-dimensional scalar wave equation) with some discussion of problems from electromagnetics, elastodynamics and hydrodynamics. The emphasis is on exact methods, primarily separation of variables and boundary integral equations. As far as I know, there is no comparable book. For more information on the topics covered, see the Table of Contents and Section 1.7.

I am grateful to Gerhard Kristensson for his detailed comments on an early draft of the whole book. Thomas Anderson made many useful comments on a later draft. I also thank many others for comments and help with various parts of the book.

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