

Index

composite function, 10

absolute moment, 143 accordion function, 414 adapted process, 300 admissible functions on [0,1], 384 all but countably many, 13 almost everywhere, 66 almost sure, 139 a.u. boundedness, 444, 445 a.u. càdlàg process, 406 a.u. càdlàg process on $[0,\infty)$, 479 a.u. continuity of r.f., 229 a.u. globally Hoelder process, 349 a.u. Hoelder coefficient, 349 a.u. Hoelder continuous process, 349

basis functions of an ε -partition of unity, 29 bijection, 10 binary approximation, 20 bounded set, 13 Brownian motion in R^m , 353 Brownian semigroup, 568

càdlàg completion, 382 càdlàg function on $[0,\infty)$, 479 càdlàg functions, 376 Cauchy in measure, 110 Cauchy in probability, 144 Cauchy-Schwarz inequality, 143 centered Gaussian r.f., 291 Central Limit Theorem, 221 characteristic function, 204 Chebychev's inequality, 87 Compact Daniell-Kolmogorov Extension, 242 compact metric space, 13 compactification of a binary approximation, 39 compactification of a Feller semigroup, 539 complete extension of an integration, 55 complete extension of an integration space, 55 complete integration space, 61

composite modulus of smoothness, 495 composite transition distribution, 495 conditional expectation, 184 conditional probability, 184 conditional probability space given an event, 184 conditionally integrable r.r.v.'s, 184 consistency condition, 232 consistent family, 232 consistent family of f.j.d.'s, 232 consistent family of f.j.d.'s from initial distribution and semigroup, 502, 533 continuity a.u. of r.f., 229 continuity point of an integrable function, 83, 84 continuity point of a measurable function, 101 continuity point of a P.D.F., 165 continuity point of an r.r.v., 140 continuity in probability of r.f., 229 continuous function on locally compact space, continuous function that vanishes at infinity, 15 convergence almost everywhere, 110 convergence almost surely, 144 convergence almost uniformly, 110, 144 convergence in distribution, 155 convergence in L_1 , 110 convergence in measure, 110 convergence in probability, 144 convergence of r.v.'s in distribution, 155 convergence uniformly, 110 convolution, 204 coordinate function, 11, 242 countable power of binary approximation, 25 countable power integration space, 134 countable set, 10 countably infinite set, 10 covariance, 143 C-regularity, 337



> 610 Index

Daniell-Kolmogorov Extension, 241, 251 Daniell-Kolmogorov-Skorokhod Extension, deletion, 233 direct product of two functions, 120 distribution function, 45 distribution induced by an r.v., 152 distribution metric for a locally compact space, distribution on a metric space, 151 division points of simple càdlàg function, 383 domain of a function, 9 Dominated Convergence Theorem, 117 D-regular family of f.j.d.'s on Q_{∞} , 410 *D*-regular process on \overline{Q}_{∞} , 410, 479

eigenvector, 190 empty set, 9 enumerated set, 10 enumeration, 10 ε -approximation of a totally bounded metric space, 13 ε -division points, 376 ε -entropy, 20 ε -partition of unity, 29 equivalent stochastic processes, 231 event, 139 expectation, 138 expected value, 138, 139 extension of family of f.j.d.'s, 231

dual function of a sequence, 232

family of functions that separates points, 73 Feller semigroup, 538 filtration, 300 filtration generated by a process, 300 finite integration space, 102 finite joint distribution, 232 finite power of binary approximation, 23 finite sequence, 10 finite set, 10 first Borel-Cantelli Lemma, 142 first exit time, 488 f.j.d., 232 f.j.d.'s generated by Feller semigroup, 538

f.j.d.'s, continuous in probability, 237 Fourier transform, 204 Fubini's Theorem, 127, 129

full set, 65, 141

function that separates two points, 73

Gaussian r.f., 290 greatest lower bound, 15

function, 9

Hoelder exponent, 349 Hoelder's inequality, 143

I-basis, 102 independent events, 182 independent r.v.'s, 182 indexed set, 10

indicator of an integrable set, 67 indicator of a measurable set, 94

infimum, 15 infinite sequence, 10 injection, 10 integrable function, 55

indicator, 65

integrable function, complex-valued, 202

integrable real random variable, 139

integrable set, 67 integral, 51 integration, 51

integration on locally compact metric space, 47

integration space, 51 integration subspace, 53

interpolated Gaussian process by conditional

expectations, 365

jointly normal, 193

Kolmogorov's ε -entropy, 20

 L_p -norm, 143 least upper bound, 15 Lebesgue integration space, 118, 133 Lebesgue measurable function, 118 left limit, 375

Lipschitz constant, 27 Lipschitz continuity, 13

Lipschitz continuous function, 27 locally compact metric space, 13 Lyapunov's inequality, 143

mapping, 9

marginal distributions, 231 marginal metric, 237 Markov process, 494 Markov property, 494 Markov semigroup, 500 martingale, 306 mean, 143

measurable extension, 286

measurable function, complex-valued, 202

measurable r.f., 276 measurable set, 94 measure of integrable set, 67 measure-theoretic complement, 67, 94

mesh, 45



Index 611

metric complement, 12 metric space, 12 metrically discrete subset, 13 Minkowski's inequality, 143 modulus of a.u. boundedness, 445 modulus of a.u. càdlàg, 406 modulus of a.u. continuity of r.f., 228 modulus of càdlàg, 376 modulus of continuity, 13 modulus of continuity a.u. of r.f., 228 modulus of continuity of f.j.d.'s, 237 modulus of continuity in probability, 228 modulus of C-regularity, 337 modulus of D-regularity, 410 modulus of integrability, 90 modulus of local compactness, 20 modulus of non-explosion of Feller semigroup, modulus of pointwise tightness, 249 modulus of smoothness of Feller semigroup, 538 modulus of smoothness of Markov semigroup, modulus of smoothness of transition distribution, 495 modulus of strong continuity of Feller semigroup, 538 modulus of strong continuity of Markov semigroup, 500 modulus of strong right continuity, 445 modulus of tightness, 159 moment, 143 Monotone Convergence Theorem, 64 mutually exclusive, 65 nonempty set, 9 nonnegative definite function, 291 nonnegative definite matrix, 190 normal distribution, 193, 197 normal p.d.f, 193 normal P.D.F., 193 normally distributed, 193 null set, 67

observation of a process at a simple stopping time, 302 one-point compactification, 34 one-point compactification from binary approximation, 35 one-step transition distribution, 498 operation, 9 outcome, 139

parameter set, 227 partition of *R*, 45 partition of unity, 277

partition of unity of locally compact metric space, 30 path space, 242 P.D.F., 165 P.D.F. of an r.r.v., 165 point at infinity, 34 point mass, 52 pointwise continuity, 376 positive definite function, 291 positive definite matrix, 190 positivity condition for integration, 51 positivity condition of integration on locally compact metric space, 47 power integration space, 129 principle of finite search, 3 principle of infinite search, 3 probability density function, 164 probability distribution function, 165 probability of an event, 139 probability function, 139 probability integration space, 61 probability metric, 145, 265 probability space, 138 probability space induced by an r.v., 152 probability subspace, 150 probability subspace generated by family of r.v.'s, 150 process with Markov semigroup, 502 product integration, 125 product integration space, 125, 129 product metric space, 14 product of a sequence of complete integration spaces, 134 profile bound, 74 profile system, 73

quantile mapping, 169

random càdlàg function, 405, 406, 480 random field, 227 random variable, 138 range of a function, 10 real random variable, 139 refinement of a partition, 45 regular point of integrable function, 83, 141 regular point of measurable function, 101 regular point of an r.r.v., 139 representation of integrable function, 55 restriction of a consistent family, 236 restriction of a family of functions, 35 restriction of a function, 9 restriction of an r.f., 227 r.f., a.u. continuous, 228 r.f., continuity a.u., 228

r.f., continuous in probability, 228



612

Riemann–Stieljes integral, 46 Riemann–Stieljes sum, 45 right complete, 375 right continuity, 375 right continuous filtration, 300, 301 right-Hoelder constant, 469 right-Hoelder exponent, 469 right-Hoelder process, 469 right-limit extension of a filtration, 301 right-limit extension of process, 418 r.v. observable at stopping time, 302

sample, 139 sample function, 227 sample space, 139, 227 semigroup, 500 set, 8 set of distributions on complete metric space, 151 set-theoretic complement, 9 set-theoretic equality of functions, 9 σ -finite integration space, 102 simple càdlàg functions, 383 simple first exit time, 304 simple function, 120 simple modulus of integrability, 90 simple stopping time, 302 size of a finite set, 10 Skorokhod metric, 384 Skorokhod representation, 170 Skorokhod space on $[0, \infty)$, 479 special convex function, 316 standard deviation, 143 standard normal p.d.f., 193 state space, 227 state-uniformly a.u. càdlàg, 548

Index

stochastic approximation, 276, 279
stochastic process, 227
stopping time, 302
strictly convex function, 314
strong Markov process, 494
strong right continuity in probability, 444, 445
submartingale, 306, 307
subsequence, 10
sum of a sequence in an integration space, 62
support, 15
surpremum, 15
surjection, 10

tight family of distributions, 159 tight family of r.v.'s, 159 time parameter, 227 time-uniformly a.u. càdlàg process on $[0,\infty)$, 480 time-uniformly a.u. continuous, 229 time-uniformly *D*-regular process on \overline{Q}_{∞} , 479 transition distribution, 495 transition distributions generated by Feller semigroup, 539

unequal elements of metric space, 13 uniform distribution on [0,1], 169 uniform integrability, 90 uniform metric, 334 uniformly continuity, 13

variance, 143

weak convergence of distributions, 155 weak convergence of r.v.'s, 155 wide-sense submartingale, 306 wide-sense supermartingale, 307