MODELLING TURBULENCE IN ENGINEERING AND THE ENVIRONMENT

Rational Alternative Routes to Closure

Modelling transport and mixing by turbulence in complex flows are huge challenges for computational fluid dynamics (CFD). This highly readable book introduces readers to modelling levels that respect the physical complexity of turbulent flows. It examines the hierarchy of Reynolds-averaged Navier–Stokes (RANS) closures in various situations ranging from fundamental flows to three-dimensional industrial and environmental applications. The general second-moment closure is simplified to linear eddy-viscosity models, demonstrating how to assess the applicability of simpler schemes and the conditions under which they give satisfactory predictions.

The principal changes for the second edition reflect the impact of computing power: a new chapter devoted to unsteady RANS and another on how large-eddy simulation, LES, and RANS strategies can be effectively combined for particular applications.

This book will remain the standard for those in industry and academia seeking expert guidance on the modelling options available, and for graduate students in physics, applied mathematics and engineering entering the world of turbulent flow CFD.

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Rational Alternative Routes to Closure

Second Edition

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Preface

Over the decade since the first edition of *Modelling Turbulence in Engineering and the Environment* made its appearance, the wider topic of computational fluid dynamics – or CFD as it is now universally known – has become even more firmly established as the route to resolving important and possibly challenging questions of fluid motion in the turbulent flow regime. As the reader may judge from the Preface to that first edition (which follows), our view was that the progressive shift, then underway, from using the Reynolds-averaged Navier–Stokes (RANS) equations as the basis for accounting for turbulent transport (so-called RANS modelling) to large-eddy simulation (LES) was not assuredly the preferred practice for many applications.

The notion that, to improve the reliability of one's CFD computations, one needed to upgrade the modelling strategy from a RANS-based closure to LES largely arose from the presumption that RANS-based modelling was invariably associated with the use of a linear eddy-viscosity approximation. That presumption we emphatically rejected. Our emphasis in the first edition was rather at a closure level where turbulent momentum, heat and mass fluxes were found not from such quasi-laminar constitutive concepts but rather by approximation of their own transport equations, a path formally known as 'second-moment closure'. Indeed, the subheading to the book's title was *Second-Moment Routes to Closure*. This overall philosophy is one that we retain in the present edition, though, for reasons that will shortly become clear, the subtitle has been changed to recognize the broader range of modelling now included.

It is not argued that second-moment closure is *always* the best RANS approach to follow, however. In simple shear flows where turbulence transport is small, second-moment closures amount to what is tantamount to an eddy-viscosity model (EVM) of turbulence. There are then clear advantages to making simplifications to the physical model in order to achieve major savings in computational time, whether from solving fewer equations or from faster rates of convergence – or, more usually, from both. Readers familiar with the first edition will thus find that Chapters 1–7, where the key analysis and simplifications are made, are largely unchanged apart from modifications to the wording to simplify or otherwise clarify the meaning. Chapter 8, while presenting the same four alternative approaches for bridging the near-wall viscous layer, includes a challenging application relating to the urban environment to underline the applicability of the approaches beyond engineering.

The principal changes in the second edition are the two additional final chapters. The continuing rapid expansion in cheap computing power has stimulated two major areas of growth. The first is the solution of the transport equations in time-dependent mode, the *unsteady* RANS or URANS approach. This strategy was included in the first edition as a section of a chapter, but the number and complexity of the applications that have appeared in recent years now merit its figuring as a major chapter in its own right. Overall, the considerable success of the URANS approach, especially when adopted with a full or truncated form of second-moment closure, raises fundamental questions vis-à-vis LES in modelling the large-scale turbulence structures. Not all of those questions are yet resolved, but they are at least given a preliminary airing in Chapter 9.

Finally, Chapter 10 brings a collaboration between what are sometimes seen as opposing strategies. In many engineering or environmental problems, there are flow regions where LES is clearly the best approach (or even the only viable scheme) to employ. In many others, however, such as in flows bounded by solid walls, the solution of the relatively thin but important wall-adjacent areas can be entrusted to a RANS or URANS model. This practice mitigates the formidable grid density required in the wall region by the usual LES approach and equally, even with a URANS solution, enables substantially greater time steps. Then again, in some flow types, a RANS or URANS approach can perhaps cover the bulk of the flow while LES is employed only in critical regions involving complex physis not adequately accounted for by common RANS models. The final chapter thus considers how the two approaches may be brought effectively together within a single numerical solver, particularly considering the role and importance of the RANS model in different applications and the issues of interfacing between the two approaches.

As for the new subtitle to this second edition, by ‘Rational Alternative Routes to Closure’ we simply mean that the different approaches to modelling are based on a mixture of rigorous analysis, experimental inferences and, hopefully, sound physical insight while (echoing the first edition) giving particular emphasis to second-moment approaches to closure.

In bringing this edition to publication, the authors have benefitted from many individuals for information or advice, most of whom are already acknowledged in the first edition’s preface. In addition, we would here mention particularly

helpful interactions with Branislav Basara, Domenico Borello, Bruno Chaouat, Sharath Girimaji, Muhamed Hadziabdic, Michale Hrebtov, Rustam Mullyadzanov and Danesh Tafti. Finally, we are pleased to acknowledge a substantial contributor to the present edition, Professor Alistair Revell from Manchester University. As a specialist in the development and application of hybrid RANS-LES methods, he has made major contributions to the shape and scope of the final chapter and his name is, therefore, included on the title page.

Preface to the First Edition

Scientific papers on how to represent in mathematical form the types of fluid motion we call *turbulent flow* have been appearing for over a century while, for the last sixty years or so, a sufficient body of knowledge has been accumulated to tempt a succession of authors to collect, systematize and distil a proportion of that knowledge into textbooks. From the start, a bewildering variety of approaches has been advocated: thus, even in the 1970s, the algebraic mixing-length models presented in the book by Cebeci and Smith (1974) jostled on the bookshelves with Leslie's (1973) manful attempt to make comprehensible to a less specialized readership the direct-interaction approach developed by Kraichnan and colleagues. As the progressive advance in computing power made it possible to apply the emerging strategy of CFD to an ever-widening array of industrially important flows, however, EVMs based on the solution of two transport equations for scalar properties of turbulence (essentially, length and time scales of the energy-containing eddies) emerged as the modelling strategy of choice and, correspondingly, have been the principal focus in several textbooks on the modelling of turbulent flows (e.g. Launder and Spalding, 1972; Piquet, 1999; Wilcox, 2000).

Today, two-equation EVMs remain the workhorse of industrial CFD and are applied through commercially marketed software to flows of a quite bewildering complexity, though often with uncertain accuracy. However, there has been a major shift among the modelling research community to abandon approaches based on the RANS equations in favour of LES, where the numerical solution for any flow adopts a three-dimensional, time-dependent discretization of the Navier–Stokes equations using a model to account simply for the effects of turbulent motions too fine in scale to be resolved with the mesh adopted – that is, a *sub-grid-scale* (or *sgs*) model. While acknowledging that LES offers the prospects of tackling turbulence problems beyond the scope of RANS, a further major driver for this changeover has been the manifold inadequacies of the stress–strain hypothesis adopted by linear EVMs. While such a simple linkage between mean strain rate and turbulent stress seemed adequate for a large proportion of two-dimensional, nearly parallel flows, its weaknesses became abundantly clear as attention shifted to recirculating, impinging and three-dimensional shear flows. Although an LES approach will, most probably, also adopt an sgs model of eddy-viscosity type, the consequences are less serious for two reasons. First, the majority of the transport caused by the turbulent motion will be directly resolved by the large eddies, and second, the finer scale eddies that must still be resolved by the sgs model of turbulence will arguably be a good deal closer to isotropy. Thus, adopting an isotropic eddy viscosity as the sgs model may not significantly impair the accuracy of the solution.

However, to overcome many of the weaknesses of linear EVMs used within a RANS framework, it is quite unnecessary to upgrade one's modelling to LES level. Rather than adopting a linear algebraic relation to link stress and strain, one can obtain the turbulent stresses by solving closed forms of the exact Reynolds stress equations. It is this approach that represents the main focus of the present book, a modelling strategy known formally as *second-moment closure*, a label that also embraces the corresponding modelling of turbulent heat and species fluxes. This closure level, first advocated in the early 1950s (Rotta, 1951), has in principle a far greater capacity than EVMs for capturing the diverse influences of complex strain fields, body forces or substantial transport on the evolution of the turbulent stresses. This is because the direct effects of strain field, body forces and convective transport on the turbulent stresses appear directly in the second-moment equations in forms requiring no approximation! It is true that modelling is still needed, both in the second-moment equations and in the scale-determining equation, the latter of which must also be solved to complete closure. But, at the second-moment level, one can proceed further by way of analysis while several additional invariant parameters become available to help shape compliance with limiting states of turbulence.

Admittedly, even with a well-constructed code explicitly designed for second-moment closure (as many commercial solvers are not), such schemes require typically twice as much computational resource as corresponding EVMs. But this is a very small price to pay for predicting the flow correctly, while the computational costs will still usually be one or two orders of magnitude less than the cost of obtaining an LES of the same flow.

Why, the reader may legitimately ask, if second-moment closure represents such a major advance over eddy-viscosity approaches, has this situation not become evident and widely accepted by potential users? The present authors can offer no certain answer to that question. To those working at that closure level it *is* well known. Indeed, in the more comprehensive current textbooks, one will at least find signposts to modern forms of second-moment closures. But perhaps such broad-coverage treatments, while of inestimable value as reference sources, are unable to justify the space for providing a detailed examination of particular modelling forms or for showing a broad coverage of the successes and weaknesses of particular models. Perhaps, we concluded, one needed a textbook that focussed principally on second-moment closure, that provided the background in sufficient depth, bringing to light strategies from earlier decades that are still useful and also including the latest models available. Finally, one needed a textbook that discussed in detail a comprehensive range of applications so that potential users could judge the likely utility of the schemes in the flows that interest them. It has been our aim, in the pages that follow, to provide such a coverage.

The writers themselves began working together on second-moment closure in the late 1960s and over the ensuing forty-odd years have repeatedly interacted on research strategy in this field, both in specific collaborative research projects and through the ERCOFTAC[1] special interest group in turbulence modelling. Our views on closure modelling, if not identical, are sufficiently closely aligned that, when we learned that each of us was contemplating preparing a textbook on the subject, we quickly decided that we should pool our efforts and produce a joint volume. Throughout, this has been an equal partnership and, as in all our joint papers, our names are sequenced alphabetically.

To a neutral and knowledgeable reader, the material presented may well be seen as giving too great an emphasis to the authors' own work. In part this 'bias' arises from wanting to show the performance of particular models for a wide range of test cases that (we have learned from experience) are sensitive to the modelling assumptions. We trust, however, that the cited references make the connection to (and the dependence on) the work of others plainly evident. Indeed, our hope would be that having had their enthusiasm for second-moment closure stimulated or reawakened by the present text, many readers will be encouraged to plunge into at least some of the other recent textbooks in turbulence modelling and, thereafter, to read the original journal papers that are cited.

In fact, one of the choices made in producing this book is directly aimed at encouraging the reader to progress into the original research literature. In presenting different models, while the main ideas and underlying principles have been included (along with examples of a model's performance), in many cases, we have not given a complete mathematical statement still less the boundary conditions or other essential numerical aspects of handling the equations appropriate to different classes of flow.

While, in some respects, the book is more comprehensive in its coverage of second-moment closure than most (perhaps all) alternative volumes on turbulence modelling, there are also omissions about which some brief explanation needs to be given. Although we make early reference to situations where the density fluctuations in the convective transport term need to be acknowledged and modelled, the

[1] European Research Community on Flow Turbulence and Combustion.

reader will find that this is not a subject to which we return. The reason is simple: we have ourselves done little work in the area, so our position statement could only be arrived at by borrowing conclusions from what others have written. It would, we felt, be better for the interested reader, instead, to digest directly the views of those with greater experience. In fact, two such individuals, Tom Gatski and Jean-Paul Bonnet (2009), have recently collaborated to produce a textbook specifically focussed on compressibility in high-speed flow, which we commend to the reader. Equally, while both of us have made proposals for obtaining the turbulent thermal timescale by solving an equation for the dissipation rate of temperature fluctuations, we nevertheless nowadays prefer to adopt simpler practices ourselves. Thus, here we leave Nagano's (2002) review to summarize the painstaking research and optimization in this area carried out by Nagano and his colleagues. A final important area where we offer no contribution is that of how to embed the concepts of turbulent intermittency within the closure. Long ago, Libby (1975) proposed a transport equation for intermittency that has been used and developed over the ensuing decades by numerous workers, especially those working in combustion and, more recently, those attempting to predict transition from laminar to turbulent flow. In the latter area, the review by Savill (2002b) gives an indication of the directions being followed to broaden the range of such flows that can be tackled.

Despite the care we have tried to apply in checking the typescript, we know there will inevitably be errors in what is written, whether just typographical slips or interpretational errors on our part. Readers are warmly invited to draw these to our attention (in writing, please) so that in any future reprinting they may be corrected.

In closing, we express our thanks to our host institutions for the infrastructure support they have provided. In the case of one of us (KH), this also includes La Sapienza University, Rome, where, as the holder of an EU-funded Marie Curie Chair, he spent much of the period during the book's preparation. Finally, we are especially conscious that the task of preparing this book would not have been realizable without the contributions of many past and present colleagues. In particular, we offer our thanks and appreciation to Tim Craft, Song Fu, Hector Iacovides, Suad Jakirlić, Saša Kenjereš, Remi Manceau, Kazuhiko Suga and the late Ibrahim Hadžić. We have also benefitted greatly over the years from inputs on various aspects of modelling from Peter Bradshaw, Paul Durbin, Tom Gatski, Bill Jones, Nobu Kasagi, Hiroshi Kawamura, Dominique Laurence, Michael Leschziner, John Lumley, Yasu Nagano, Steve Pope, Bill Reynolds, Wolfgang Rodi, Roland Schiestel, Ronald So, Dave Wilcox and Micha Wolfshtein. Finally, we extend a special thank you to the research students and postdoctoral researchers – too numerous to name individually – with whom we have shared the occasional frustrations but, ultimately, the pleasurable satisfactions of turbulence-modelling research.

Principal Nomenclature

Symbol	**Meaning**
A	Lumley's two-component stress ('flatness') parameter, $A \equiv 1 - \frac{9}{8}(A_2 - A_3)$
A_2	second invariant of stress anisotropy, $A_2 \equiv a_{ij}a_{ji}$
A_3	third invariant of stress anisotropy, $A_3 \equiv a_{ij}a_{jk}a_{ki}$
A_θ	scalar flux correlation function, $A_\theta \equiv (\overline{\theta u_i})^2/(\overline{\theta^2}\,\overline{u_k u_k})$, Eq. (3.32)
A^+	coefficient in van Driest's near-wall form of mixing-length hypothesis
a_{ij}	Reynolds stress anisotropy tensor, $a_{ij} \equiv \overline{u_i u_j}/k - 2\delta_{ij}/3$
B_i, $\mathbf{B}$	magnetic flux density
b_{ij}	$b_{ij} \equiv a_{ij}/2$
b^i_{lj}	third-order tensor in the model for $\Phi_{\theta j_2}$, Eq. (4.49)
b^{mi}_{lj}	fourth-order tensor in the model for Φ_{ij_2}, Eq. (4.39)
C	species concentration
C_p	pressure coefficient, $C_p \equiv 2(P_w - P_\infty)/\rho U_\infty^2$
C_κ	constant in Kolmogorov's $-5/3$ law for energy variation with wave number, Eq. (3.6)
C_{DES}, $C_{DDES}, \ldots$	coefficients in DES, DDES, IDDES
C_{ij}	cross (mixed) stress, Eqs. (9.9, 10.12)
$\mathcal{C}_{ij}$	convection of the Reynolds stress tensor, $\overline{u_i u_j}$
$\mathcal{C}_{\theta i}$	convection of the turbulent scalar flux, $\overline{\theta u_i}$
$\mathcal{C}_{\theta\theta}$	convection of scalar variance, $\overline{\theta^2}$
$\mathcal{C}_\phi$	convection of a turbulence variable, ϕ
c_μ	coefficient in eddy-viscosity formula
c_p	specific heat at constant pressure
$c_{\varepsilon 1}$, $c_{\varepsilon 2}, \ldots$	coefficients of source/sink terms in the modelled ε equation
c_1, $c_2, \ldots$	coefficients in the models of the pressure-strain term

c_S	Smagorinsky coefficient, Eq. (10.14)
D	diameter, channel width
D_{ij}	complementary stress production tensor $D_{ij} \equiv -(\overline{u_i u_k}\partial U_k/\partial x_j + \overline{u_j u_k}\partial U_k/\partial x_i)$
$\mathcal{D}_{ij}$	total diffusion of the Reynolds stress tensor
$\mathcal{D}_{ij}^p$	turbulent diffusion of the Reynolds stress tensor $\overline{u_i u_j}$ by pressure fluctuations, Eq. (2.20)
$\mathcal{D}_{ij}^t$	turbulent diffusion of the Reynolds stress tensor $\overline{u_i u_j}$ by velocity fluctuations, Eq. (2.18)
$\mathcal{D}_{ij}^\nu$	molecular diffusion of the Reynolds stress tensor $\overline{u_i u_j}$, Eq. (2.18)
$\mathcal{D}_{\theta i}$	total diffusion of scalar flux $\overline{\theta u_i}$, Eq. (2.25)
$\mathcal{D}_{\theta i}^p$	turbulent diffusion of scalar flux $\overline{\theta u_i}$ by pressure fluctuations, Eqs. (2.22, 2.25)
$\mathcal{D}_{\theta i}^t$	turbulent diffusion of scalar flux $\overline{\theta u_i}$ by velocity fluctuations, Eqs. (2.22, 2.25)
$\mathcal{D}_{\theta i}^\alpha$	thermal molecular diffusion of scalar flux $\overline{\theta u_i}$, Eqs. (2.22, 2.25)
$\mathcal{D}_{\theta i}^\nu$	viscous diffusion of scalar flux $\overline{\theta u_i}$, Eqs. (2.22, 2.25)
$\mathcal{D}_{\theta\theta}$	total diffusion of scalar variance $\overline{\theta^2}$, Eq. (3.20)
$\mathcal{D}_\phi$	total diffusion of a turbulence variable ϕ
$\mathcal{D}_\phi^p$	turbulent diffusion of variable ϕ by pressure fluctuations
$\mathcal{D}_\phi^t$	turbulent diffusion of variable ϕ by velocity fluctuations
$\mathcal{D}_\phi^\nu$	molecular diffusion of variable ϕ
$\tilde{d}$ $(\equiv L_{DES})$	effective length scale in DES, $\tilde{d} = \min(d_w, C_{DES}\Delta)$, Eq. (10.42)
d_w	distance to the nearest wall in Eq. (10.42)
E	two-component-limit parameter for dissipation tensor, $E \equiv 1 - \frac{9}{8}(E_2 - E_3)$
E	integration constant in log-law, $E \approx 8.4$ for a smooth wall
E_2	second invariant of e_{ij}, $E_2 \equiv e_{ij}e_{ji}$
E_3	third invariant of e_{ij}, $E_3 \equiv e_{ij}e_{jk}e_{ki}$
$E(\kappa)$	contribution by the Fourier-mode wavenumber κ to the turbulent kinetic energy
e_i	fluctuating electric potential
e_{ij}	stress dissipation-rate anisotropy tensor, $e_{ij} \equiv \varepsilon_{ij}/\varepsilon - \frac{2}{3}\delta_{ij}$
$\mathcal{F}_{ij}$	turbulent stress production due to all body forces, Eq. (2.23)
$\mathcal{F}_{\theta i}$	turbulent scalar flux production due to all body forces, Eq. (2.23)

Symbol	Meaning
$\mathcal{F}_\phi$	production of a turbulence variable ϕ by all body forces
f	scalar variable in Durbin's elliptic relaxation EVM
f	natural shed frequency
f_D	van Driest wall damping function, Eq. (10.15)
f_i	fluctuating body force
f_k	ratio of unresolved to total turbulent kinetic energy in PANS, $f_k = k_u/k$
f_L ($\equiv \alpha$)	RANS/LES switching function, Eqs. (10.31, 10.34) $f_L = \max(1, L_{RANS}/L_{LES})$
f_w	wall damping function in GL and HJ low-Re RSM
f_Δ	blending function in VLES, Eq. (10.33)
G	spatial filter function, Eqs. (10.8, 10.9)
$\mathcal{G}_{ij}$	turbulent stress production due to gravitational force, Eqs. (2.19, 4.74)
g	gravitational acceleration constant
g_i, $\mathbf{g}$	gravitational vector
H	height of the step in flow over a backward-facing step
Ha	Hartmann number
H, H_{12}	boundary-layer shape factor, δ^*/θ (note $\delta_1 \equiv \delta^*$, $\delta_2 \equiv \theta$, $H_{12} \equiv H$)
h	half width of a plane channel
h	enthalpy, $h \equiv \int c_p dT$
h	heat transfer coefficient, $h \equiv q''_w/(\Theta_w - \Theta_{ref})$
II	alternative notation for the second invariant of stress anisotropy, $II \equiv b_{ij}b_{ji}/2 = A_2/8$
III	alternative notation for the third invariant of stress anisotropy, $III \equiv b_{ij}b_{jk}b_{ki}/3 = A_3/24$
J	Jayatilleke function (relative resistance of sublayer to heat and momentum transfer from a smooth wall), Eq. (8.5)
K	acceleration parameter, $K \equiv (\nu/U_\infty^2)(dU_\infty/dx)$
K	mean flow kinetic energy, $K \equiv \frac{1}{2}U_i^2$
k	turbulent kinetic energy, $k \equiv \frac{1}{2}\overline{u_i u_i}$
k^*	sub-grid-scale turbulence energy normalized by total (sgs plus resolved) k, Eq. (10.40)
k_{ssv}	'scale-supplying variable' in PANS (resolved k), Eq. (10.63)
L, $\mathcal{L}$	characteristic flow dimension
L	integral turbulent length scale (usually defined as $k^{3/2}/\varepsilon$; for definitions of bounded length scale in elliptic relaxation models see Eqs. (6.74, 7.45))

l	turbulence length scale, $k^{3/2}/\varepsilon$
ℓ	alternative turbulence length scale (used in Wilcox–Rubesin model), $\ell = c_\mu l$
L_{DDES}	effective length scale in DDES, Eq. (10.45)
L_{IDDES}	effective length scale in IDDES, Eq. (10.48)
L_K	von Karman length scale, Eq. (10.36)
L_{ij}	Leonard stress, Eqs. (9.9, 10.12)
$\mathcal{M}_{ij}$	stress production due to fluctuating (electro)-magnetic (Lorenz) force, Eq. (4.95)
N	bulk-flow Stuart number, $N \equiv \sigma B_0^2 L/\rho U_b$
Nu	Nusselt number, $\mathrm{Nu} \equiv hD/\lambda$, D denotes relevant length dimension, for example, pipe diameter
n_i, $\mathbf{n}$	wall-normal unit vector
$\hat{P}$, P, p	instantaneous, mean and fluctuating pressure
P^+	non-dimensional pressure gradient, $P^+ = \nu(\partial P/\partial x)/\rho U_\tau^3$
P	wall-adjacent grid node
$\mathcal{P}_{ij}$	stress production due to mean velocity gradient, Eq. (2.18)
$\mathcal{P}_{\theta i}$	production of turbulent scalar flux $\overline{\theta u_i}$, Eq. (2.22)
$\mathcal{P}_k$	production of turbulent kinetic energy k, $\mathcal{P}_k = \mathcal{P}_{ii}/2$, Eq. (1.5)
$\mathcal{P}_{\theta\theta}$	production of the mean-square scalar variance $\overline{\theta^2}$, Eq. (3.20)
$\mathcal{P}_\phi$	production of a turbulence variable ϕ by gradients of mean and fluctuating properties
Pr	molecular Prandtl/Schmidt number
Q	criterion for eduction of coherent vortical structures, $Q \equiv -(S_{ij}S_{ij} - W_{ij}W_{ij})/2$
$\dot{q}$	internal heat source
q_w''	wall heat flux
q_w	kinematic wall heat flux, $q_w = q_w''/\rho c_p$
R	pipe radius
R, r	thermal-to-mechanical timescale ratio, $\mathsf{R} = \overline{\theta^2}\varepsilon/k\varepsilon_{\theta\theta}$; $\mathsf{r} = 1/\mathsf{R}$
R_{ij}	Reynolds stress, Eqs. (9.8, 10.12)
Ra	Rayleigh number, $\mathrm{Ra} \equiv \beta g(\Theta_w - \Theta_{ref})L^3/\alpha\nu$, where L is a characteristic flow dimension, Θ_w and Θ_{ref} denote the wall and reference temperatures, respectively
Re_L	Reynolds number based on a characteristic flow dimension, L and velocity, U_0, $\mathrm{Re}_L \equiv U_0 L/\nu$
Re_m	channel flow Reynolds number based on the mean (bulk) velocity, $\mathrm{Re}_m \equiv U_m 2h/\nu$

Symbol	Meaning
Re_H	Reynolds number of flow behind a backward-facing step of height H
Re_M	magnetic Reynolds number, $\mathrm{Re}_M \equiv \mu_0\sigma UL$, $[(\mu_0\sigma)^{-1}$ is known as the *magnetic diffusivity*]
Re_t	turbulent Reynolds number, $\mathrm{Re}_t \equiv k^2/(\nu\varepsilon)$
Re_{δ_s}	Reynolds number based on Stokes thickness and maximum free-stream velocity
Re_θ	Reynolds number based on momentum thickness, $\mathrm{Re}_\theta \equiv U_\infty\theta/\nu$
Re_τ	Reynolds number based on friction velocity and channel half width, $\mathrm{Re}_\tau = U_\tau h/\nu$
Re_λ	Taylor microscale Reynolds number, $\mathrm{Re}_\lambda \equiv \sqrt{\overline{u_1^2}}\lambda/\nu$
R_f	flux Richardson number, $\mathrm{R}_f \equiv -\mathcal{G}_k/\mathcal{P}_k$
Ri	gradient Richardson number, $\mathrm{Ri} \equiv R_f\sigma_\theta$
$\mathcal{R}_{ij}$	stress production due to system rotation, Eqs. (2.19, 4.68)
R_{ij}	Reynolds stress, Eqs. (9.8, 10.12)
$R_{ij}(\mathbf{x},\mathbf{x}')$	two-point correlation tensor, $R_{ij}(\mathbf{x},\mathbf{x}') \equiv \overline{u_i(\mathbf{x})u_j(\mathbf{x}')}$
Ro	bulk rotation number (various definitions according to specific application comprising rotating velocity divided by some other reference velocity)
r	radial coordinate
r_i, $\mathbf{r}$	position vector
r	mechanical-to-scalar timescale ratio, $\mathsf{r} \equiv k\varepsilon_{\theta\theta}/(\overline{\theta^2}\varepsilon) \equiv 1/\mathsf{R}$
S	salt concentration (‘salinity’)
S_w	swirl intensity, a dimensionless ratio of the axial fluxes of angular to axial momentum, $S_w \equiv 2\pi \int_0^R UWr^2dr/\pi R^3U_b^2$ or $S_w = \int_0^R UWr^2dr/R\int_0^R U^2rdr$
S	invariant of the non-dimensional mean strain tensor, $\mathrm{S} \equiv \sqrt{\tilde{S}_{mn}\tilde{S}_{nm}}$
S	invariant of the strain rate tensor, $\mathsf{S} \equiv \sqrt{S_{ij}S_{ji}}$
S^*	alternative invariant of mean strain tensor used by Yakhot’s group, $\mathrm{S}^* = \sqrt{2}\mathrm{S}$, Eq. (5.4)
S	dimensionless mean strain (in simple shear), $\mathsf{S} \equiv 2(k/\varepsilon)(S_{12}S_{12})^{1/2} = (k/\varepsilon)dU/dy$
Sr	Strouhal number, $\mathrm{Sr} \equiv fL/U$, dimensionless vortex shedding frequency

St	Stanton number, $\mathrm{St} \equiv h/\rho U_\infty c_p$
S_{ij}	mean rate of strain tensor, $S_{ij} \equiv \frac{1}{2}(\partial U_i/\partial x_j + \partial U_j/\partial x_i)$
$\tilde{S}_{ij}$	non-dimensional mean rate of strain, $\tilde{S}_{ij} \equiv S_{ij}k/\varepsilon$
$\mathcal{S}_{\varepsilon 1}$,$\mathcal{S}_{\varepsilon 2}$,$\mathcal{S}_{\varepsilon 3}$	general symbols for the source and sink terms in the ε equation, respectively
s_{ij}	fluctuating rate of strain, $s_{ij} \equiv \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$
T	temperature [°K]
$\mathcal{T}$	characteristic turbulence timescale, (usually $\mathcal{T}$ is taken as k/ε, but not in Eq. (5.23)); for definitions of bounded time-scale in elliptic relaxation models see Eqs. (6.74, 7.35 and 7.44)
$\mathcal{T}(\kappa)$	spectral energy transfer rate
$\mathbb{T}_{ij}^{(n)}$	tensor integrity bases
t	time
U	streamwise mean velocity component
$\overline{U}$	filtered velocity in LES
U, V, W	Cartesian components of mean velocity
$\hat{U}_i$, U_i, u_i	instantaneous, mean and fluctuating velocity vector
$\tilde{U}$	local time-averaged velocity, Eq. (9.1)
$\widetilde{U}$	phase/ensemble averaged velocity, Eqs. (2.8, 2.9, 9.7)
U_m, U_b, $\bar{U}$	bulk velocity
U_q	buoyancy velocity, $U_q \equiv (\beta g q_w \alpha^2/\nu)^{1/4}$
U_w	wall velocity
U_∞	free-stream velocity
U_τ	friction velocity, $\sqrt{\tau_w/\rho}$
U^+	mean velocity non-dimensionalized with friction velocity, $U^+ \equiv U/U_\tau$
U^*	mean velocity for use in wall functions, $U^* \equiv Uk^{1/2}/U_\tau^2 \equiv \rho U k^{1/2}/\tau_w$
ΔU	streamwise velocity change across free shear flow
$-\overline{u_i u_j}$	kinematic Reynolds-stress tensor
u, v, w	Cartesian representation of turbulent velocities
V	mean velocity component in direction y
Va	Valensi number, $\mathrm{Va} \equiv R^2\omega/\nu$
W	invariant of the non-dimensional rotation rate, $\mathrm{W} \equiv \sqrt{\tilde{W}_{ij}\tilde{W}_{ij}}$
W	spanwise and circumferential velocity component
Wo	Womersley number, $\mathrm{Wo} \equiv R\sqrt{\omega/\nu} = \sqrt{\mathrm{Va}}$
W_{wall}	circumferential velocity of rotating wall

Symbol	Meaning
W_{ij}	mean rate-of-rotation tensor, $W_{ij} = 1/2(\partial U_i/\partial x_j - \partial U_j/\partial x_i)$
$\tilde{W}_{ij}$	non-dimensional mean rate-of-rotation tensor, $\tilde{W}_{ij} \equiv W_{ij}k/\varepsilon$
w_{ij}	fluctuating rate-of-rotation tensor, $w_{ij} = 1/2(\partial u_i/\partial x_j - \partial u_j/\partial x_i)$
x_i, $\mathbf{x}$	Cartesian coordinates in index and vector notation
x, y, z	Cartesian coordinates
y	wall distance,
y^+	non-dimensionalized wall distance, $y^+ = U_\tau y/\nu$
y^*	alternative normalized wall distance, $y^* \equiv k^{1/2}y/\nu$
$y_{1/2}$	half width of plane jet or wake

Greek Symbols	Meaning
α	thermal diffusivity, $\alpha = \lambda/(\rho c_p)$
β	thermal expansion coefficient, $\beta = -(1/\rho)(\partial\rho/\partial\Theta)\big\|_{C,P}$
γ	molecular diffusivity of a scalar
γ	concentration (salinity) expansion coefficient, $\gamma = (1/\rho)(\partial\rho/\partial C)\big\|_{\Theta,P}$
Δ	characteristic mesh size in direct and large-eddy simulations
δ	boundary layer thickness
δ_1, δ^*	displacement thickness, $\delta_1 = \int_0^\infty (1 - U/U_\infty)dy$ for a uniform density
δ_2, θ	momentum thickness $\delta_2 = \displaystyle\int_0^\infty \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right)dy$ for uniform density
δ_s	Stokes thickness, $\delta_s = \sqrt{2\nu/\omega}$
δ_ν	viscous length scale, $\delta_\nu = \nu/U_\tau$
δ_{ij}	Kronecker unit symbol
ε	dissipation rate of the turbulence kinetic energy k, $\varepsilon = \nu\overline{(\partial u_i/\partial x_j)^2}$
ε^h	homogeneous dissipation rate of k, $\varepsilon^h = \varepsilon - 1/2\mathcal{D}_k^\nu$
ε_P	turbulence energy transfer rates from production region in multi-scale model
ε_T	turbulence energy transfer rates across the transfer region in multi-scale model

ε_w	wall value of the kinetic energy dissipation rate
$\tilde{\varepsilon}$	'quasi-homogeneous' dissipation rate of k, $\tilde{\varepsilon} = \varepsilon - \mathcal{D}_k^\nu \equiv \varepsilon - 2\nu(\partial k^{1/2}/\partial x_n)^2$
ε^+	dimensionless dissipation rate (in wall units), $\varepsilon^+ \equiv \varepsilon\nu/U_\tau^4$
ε_{ij}	stress dissipation rate tensor, $\varepsilon_{ij} \equiv 2\nu\overline{(\partial u_i/\partial x_k)(\partial u_j/\partial x_k)}$
ε_{ij}^h	homogeneous stress dissipation rate tensor, $\varepsilon_{ij}^h = \varepsilon_{ij} - {}^1\!/_2\mathcal{D}_{ij}^\nu$
ε_{ijk}	viscous dissipation of triple velocity moments, Eqs. (4.102, 4.103)
$\varepsilon_{\theta\theta}$	dissipation rate of the scalar variance, $\varepsilon_{\theta\theta} = 2\alpha\overline{(\partial\theta/\partial x_j)^2}$
ϵ_{ijk}	third rank alternating unit symbol (= +1 for *i, j, k* all different and in cyclic order; −1 for *i, j, k* all different in anti-cyclic order; 0 in other cases)
ζ	normalized effective wall-normal velocity $\overline{v^2}/k$ in ζ- f EVM
ζ	enstrophy (mean square of the vorticity fluctuations), $\zeta = \overline{\omega_i\omega_i}$
η	Kolmogorov length scale, $\eta \equiv (\nu^3/\varepsilon)^{1/4}$,
Θ	mean scalar property in general (primarily used for mean temperature)
θ	momentum thickness
θ	scalar property fluctuations
$\overline{\theta^2}$	mean-square scalar fluctuations (scalar variance)
ϑ	Kolmogorov timescale, $\vartheta \equiv (\nu/\varepsilon)^{1/2}$
κ	wave number, $\kappa \equiv 2\pi/\lambda$
κ	von Karman constant in log-law, $\kappa \approx 0.41$
κ^*	von Karman constant in the velocity log-law normalized with $k^{1/2}$, $\kappa^* = c_\mu^{1/4}\kappa$
$\tilde{\kappa}$	von Karman constant in the log-law for temperature, $\tilde{\kappa} \approx 0.38$
$\tilde{\kappa}^*$	von Karman constant in the temperature log-law normalized with $k^{1/2}$, $\tilde{\kappa}^* = c_\mu^{1/4}\tilde{\kappa}$
λ	Taylor microscale, $\lambda^2 \equiv \overline{u_1^2}/\overline{(\partial u_1/\partial x_1)^2}$, Eq. (3.11)
λ	thermal conductivity
λ	wave length
λ	ratio of shear stress at wall to that at edge of viscous layer, Eq. (8.27)
λ_α	eigenvalue of mean strain rate
μ	molecular viscosity of a fluid

Greek Symbols	Meaning
μ_t	turbulent (eddy) viscosity of a fluid
μ_0	magnetic permeability
ν	kinematic molecular viscosity of a fluid, $\nu \equiv \mu/\rho$
ν_t	kinematic turbulent viscosity of a fluid, $\nu_t \equiv \mu_t/\rho$
ν_t^+	non-dimensional turbulent viscosity, $\nu_t^+ \equiv \nu_t/\nu$
ν_{sgs}	sub-grid-scale eddy viscosity
$\tilde{\nu}$	kinematic turbulent viscosity in the SA model
Π_{ij}	velocity–pressure-gradient correlation, $\Pi_{ij} \equiv (1/\rho)[\overline{u_i(\partial p/\partial x_j)} - \overline{u_j(\partial p/\partial x_i)}]$
$\hat{\rho},\ \rho,\ \rho'$	instantaneous, mean and fluctuating fluid density
σ	electrical conductivity of fluid
σ_φ	turbulent Prandtl–Schmidt number for diffusion of φ
τ	total shear stress (viscous plus turbulent)
τ^+	non-dimensional shear stress, $\tau^+ \equiv \tau/\tau_w$
τ_{ij}	stress tensor
τ_{ij}^t	turbulent stress tensor, $\tau_{ij}^t \equiv -\rho\overline{u_i u_j}$
τ_{ij}^ν	viscous stress tensor, $\tau_{ij}^\nu = 2\mu(S_{ij} - \frac{1}{3}S_{kk}\delta_{ij})$
τ_w	wall shear stress
$\hat{\Phi},\ \Phi,\ \varphi$	general variable: instantaneous, mean/filtered and fluctuation
$\tilde{\Phi}$	local time-averaged general variable, Eq. (9.1)
Φ_{ij}	pressure-strain correlation in the $\overline{u_i u_j}$ equation, $\Phi_{ij} \equiv (1/\rho)\ \overline{p(\partial u_i/\partial x_j + \partial u_j/\partial x_i)}$
$\Phi_{\theta j}$	pressure-scalar gradient correlation in $\overline{\theta u_j}$ equation, $\Phi_{\theta j} \equiv (1/\rho)\overline{p\partial\theta}/\partial x_j$
$\overline{\varphi u_i}$	scalar flux vector
ϕ	general symbol for a turbulence variable
Ψ	generalized turbulent scale variable, $\Psi \equiv k^m\varepsilon^n$,
ψ	parameter in SAWF scheme accounting for departures from equilibrium, Eq. (8.31)
Ω	angular velocity
Ω	magnitude of the mean vorticity
Ω_k	system rotation vector, angular velocity vector
Ω_i, $\mathbf{\Omega}$	mean vorticity vector, $\Omega_i \equiv \epsilon_{ijk}\partial U_k/\partial x_j$, ($\mathbf{\Omega} \equiv \nabla \times \mathbf{V}$)
ω_i, $\boldsymbol{\omega}$	fluctuating vorticity vector, $\omega_i \equiv \epsilon_{ijk}\partial u_k/\partial x_j$, ($\boldsymbol{\omega} \equiv \nabla \times \mathbf{v}$)
ω	turbulence 'frequency' or specific dissipation rate, ε/k

Superscripts, subscripts	
+	quantity normalized by wall units ν and U_τ
*	quantity normalized by wall units ν and $k^{1/2}$ (but note U^* definition above)
ˆ	instantaneous value of variable
′	turbulent fluctuating value of variable
~	root-mean-square value of turbulence variable (e.g. $\tilde{u}_1 \equiv \sqrt{\overline{u_1^2}}$)
c	centre-line value (of a symmetric free shear flow)
c	coherent
h	homogeneous
int	RANS/LES interface
mod	modelled
n	normal-to-the-wall direction
res	resolved
s	stochastic
sgs	sub-grid-scale
u	unresolved
v	evaluated at edge of viscous sublayer
w	wall value
∞	free-stream conditions

Abbreviations and acronyms (subjects)	
AWF	analytic wall functions
APG	adverse pressure gradient
ASM	algebraic second-moment (closure)
A(R)SM	algebraic (Reynolds) stress model
AFM	algebraic flux model
BWT	blended wall treatment
CFD	computational fluid dynamics
CFL	Courant, Friedrichs and Lewy number, Eq. (10.18)
CV	control volume
DDES	delayed DES

DES	detached eddy simulation
DIHRL	dynamically interfaced HRL
DNS	direct numerical simulation
DSM	differential second-moment closure
EA(R)SM	explicit algebraic (Reynolds) stress model
EB	elliptic blending
ELES	embedded LES
ER	elliptic relaxation
ER	expansion ratio
EVM	eddy-viscosity model
Exp.	experiment
FPG	favourable pressure gradient
GGD(H)	generalized gradient diffusion (hypothesis)
GWF	generalized wall functions
HRL	hybrid RANS-LES
HTM	hybrid turbulence model
IP	isotropization of production
LES	large eddy simulation
NWF	numerical wall functions
NLEVM	non-linear EVM
PANS	partially averaged Navier–Stokes
PITM	partially integrated transport model
QI	quasi-isotropic
RANS	Reynolds-averaged Navier–Stokes
RDT	rapid distortion theory
RNG	renormalization group theory
RSM	Reynolds stress model
SA	Spalart–Allmaras
SAWF	simplified analytical wall functions
SMC	second-moment closure
SST	shear-stress transport (model)
TRANS	time-resolved (triple-decomposition based) RANS
URANS	unsteady RANS
VLES	very large eddy simulation
WF	wall function(s)
WIN	wall integration (model)
WMLES	wall-modelled LES
ZPG	zero pressure gradient

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More Information

Author abbreviations	
CLS	Craft, Launder, Suga
CKL	Craft, Kidger, Launder
DFJ	Dianat, Fairweather, Jones
FLT	Fu, Launder, Tselepidakis
GL	Gibson, Launder
GS	Gatski, Speziale
HJ	Hanjalić, Jakirlić
HL	Hanjalić, Launder
JM	Jones, Musonge
JMG	Jongen, Mompean, Gatski
LRR	Launder, Reece, Rodi
LS	Launder, Sharma
LT	Launder, Tselepidakis
SA	Spalart, Allmaras
SL	Shih, Lumley
SSG	Speziale, Sarkar, Gatski
WJ	Wallin, Johansson