

Introduction to Part I

The chapters of Part I will discuss why **complexity science** is important, how this science relates to other sciences and also a little bit about its philosophical status. The aim is to make clear what makes complexity science special and in which way it contributes to our understanding of the surrounding world. Part I concentrates on the conceptual level with two intentions. Firstly, the part will contribute to the discussion of the precise demarcation of complexity science. Secondly, a description of concepts, results and perspectives of this science is one way to illustrate its important relevance as a meta-science to subject fields which at present span from linguistics and economics to biology and physics. It will become clear that complexity science in this sense is comparable to mathematics, but although complexity science when fully at work may need to make use of mathematics, its conceptual basis can to a large extent be presented without mathematical formalism. We can, for example, discuss collections of agents and the kind of collective behaviour to expect at the aggregated systemic level without specifying the specific identity of the individual agents. They may represent certain aspects of people settling in a city or they may represent molecules moving on a surface. Both situations share aggregated behaviours, which can be captured by general concepts such as segregation, ordering or mixing.

Any science will use words from daily life and through refinement try to focus and sharpen the meaning in order to develop specific concepts that form the subject matter of the particular scientific activity. In Part I we discuss the way complexity science uses words such as **complex**, **complexity** and **emergence** to build up our understanding of the behaviour of systems consisting of many interacting components. It is important to be aware of the terminology and its distinct meaning, which sometimes can be different from the use encountered in other situations. For example, we may intuitively think of ‘complex’ and ‘complicated’ as being synonymous. Complexity science makes the distinction that a complicated phenomenon is quantitatively difficult to keep track of. It might be that we try to compute the properties of many different independent components, such as the particles in a gas. This will be computationally demanding but conceptually easy. Complexity arises when the collection of components interact and new collective phenomena emerge possessing properties entirely different from those of the individual components.

Part I will also discuss the modelling approach of complexity science. Contrary to what we may at first expect, the use of well-chosen simple models is needed to improve our understanding. The simplicity and transparency of the models are particularly important because we are trying to capture the behaviour of phenomena that are both complicated and complex. We will discuss how conceptually simple models allow us

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to identify behaviour shared across very different systems from sociology, physics and economics, for example.

We consider complexity science to be the investigation of emergent phenomena. This focus is behind what is included in Part I. We will explain the necessity of *interactions* between the constituent parts and try to classify a number of different types of emergent behaviour encountered in very different systems. We will discuss general aspects of modelling complexity and what features can make a model particularly useful. We will try to make our presentation concrete by relating ideas and concepts to applications and include references to further discussions.

1 The Science of Emergence

Synopsis: The subject matter of complexity science will be identified and placed in a historical perspective.

What is *complexity science*? There exists no universally agreed definition. Is complexity science a well-defined discipline with its own subject matter, or is it essentially just another term for science?

Let us consider what might be a useful and constructive working definition of the term 'complexity science'. Consider traditional disciplines such as mathematics, biology or physics. What such disciplines encompass appears more or less uncontroversial. Although with time the focus and methodology have changed, we have some fairly clear idea of which kind of problems biologists and physicists study. We are also broadly familiar with the methods employed by biologists or by physicists. Nor are we in doubt that mathematics, physics and biology are existing disciplines, each with a specific focus and subject matter and well-established institutions and educational traditions.

We can meaningfully talk about a specific scientific field without a very precise definition. In fact, it is important to realise that the scope and focus of a science will change as it develops and so will our understanding of the part of the world explored by the scientific activity. Accordingly, classifications of scientific fields should be flexible and accommodating and not restrictive. New methods will be launched and new phenomena included. Obviously, before the realisation that DNA carries the genetic code, genetics was very different from what it is now. Or before physics discovered quantum mechanics, it was an entirely different discipline with a completely different view of what constitutes matter and of the applicability of deterministic predictions. So we are looking for a flexible and informative description to define complexity science. One commonly used description of biology is that it studies animate matter in contrast to physics, which then is seen as the study of inanimate matter. This definition asks us to define animate and inanimate. Along the same lines, mathematics is the discipline concerned with a systematic and abstract study of patterns. So, we need to try to define what we mean by patterns. Of course these definitions can be deliberated endlessly, which we will not do; our purpose is simply to point out that it is possible in straightforward terms to make a reasonably useful demarcation of such scientific fields. It is helpful to think of biology as concerned with animate matter and physics as dealing with inanimate matter and mathematics as the study of patterns, even if we are unable to define rigorously the terms animate, inanimate and patterns.

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When we turn to complexity science, a similar consensus for a manageable brief description in terms of some salient aspects of the activity does not exist. Often complexity science is considered as synonymous with cross-disciplinary activities such as studies of brain dynamics involving, say, neuroscientists, biologists, mathematicians and physicists. Or complexity science is seen as identical to one of the many methodologies it makes use of. Some might see complexity science as essentially identical to data science; others might think of **game theory** as the essence of complexity science. Indeed when complexity scientists study a complicated system composed of very many individual components, they do make use of methodology from data science, and various versions of game theory are used to analyse complex systems ranging from internet dynamics to sociology.

Sometimes complexity scientists may even contribute to the development of strategies and approaches in data science, or refine the methods and approaches of game theory, but the complexity scientist's main aim is not to find efficient ways to analyse large data sets or to refine the theory of games. The complexity scientist focuses on extracting general patterns and essential behaviours from the data sets or understanding common consequences of the rules of games. This is done with the aim to improve our general understanding of how systemic structure emerges from the interaction and dynamics of the constituent parts.

Other times complexity science may be identified with **network** science. Again, to analyse many component systems, complexity science does indeed make use of, and often contributes strongly to, the development of the science of networks and mathematical graph theory. This is absolutely natural, since commonly various features of a complex system can be represented as sets of nodes connected by various types of links. The natural relationship between network science and complexity science is even more clear when one thinks of the view of mathematician and philosopher Alfred North Whitehead (1861–1947) ‘that scientists should concentrate on multi-perspective networks of relationships, rather than on the behaviour of the aggregated atomic unit’ [343, 493]. More about Whitehead and complexity science in a moment.

So, clearly complexity science participates very often in cross-disciplinary research and it makes use of, and contributes to, the theory of networks, data science and other disciplines. But its *raison d'être* goes beyond these activities. The viewpoint of the present book is that the **systematic investigation of the general patterns and structures of emergent phenomena** is what makes complexity science a distinct scientific activity. One may ask if this statement makes it clear what complexity science is about, since we will have to agree on what emergent phenomena are. And true enough, care is needed. We may use terms like ‘animate’ and ‘inanimate’ matter to pinpoint biology and physics, but obviously that does not answer all philosophical questions concerning the nature of biology and physics. Likewise when we start to think carefully about ‘emergent phenomena’ a wealth of philosophical questions suddenly present themselves [155].

While being aware of existing important philosophical concerns, we will for the moment make do with a pragmatic description of an emergent phenomenon. We

will stress the importance of interactions between the components that generate properties of the system as a whole which are not found among the properties of the individual building blocks. Our viewpoint is that complexity science sees the dynamics of interrelated processes as its subject matter, so it looks for the shared common behaviours between totally different parts of the world. This is why complexity science is eager to enter into collaborations with subfields such as finance, economics, neuroscience, ecology, etc.

The sketch in Fig. 1.1 tries to indicate the difference between the focus of complexity science and that of the specific subject fields. To the left is a stereotypical representation of the structure as seen from the various subject fields. Each subject field is interested in a specific component at a certain level of the structure. For example, the main focus of cell biology consists in the internal workings of the cell. Similarly, psychology's subject matter is the dynamics of the human mind and typically the two fields are studied more or less independently of each other. The two columns to the right in Fig. 1.1

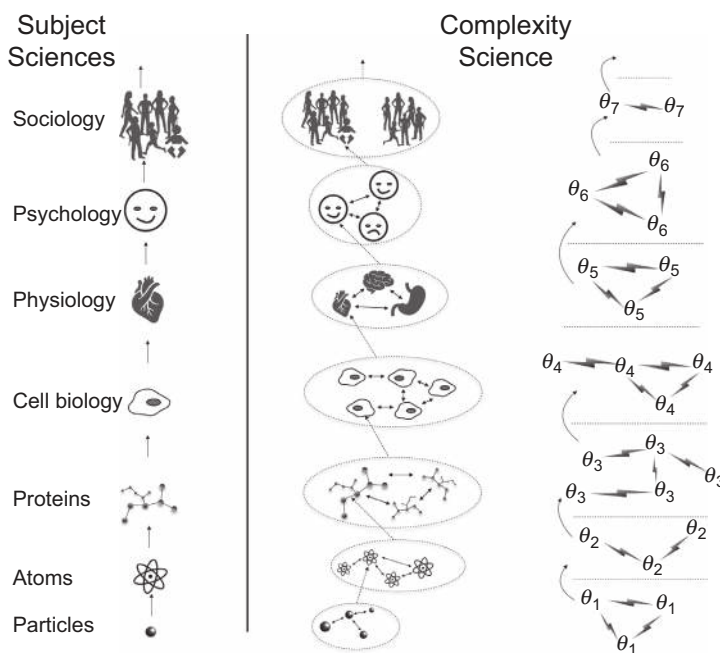


Figure 1.1 Sketch of how complexity science focuses on general features of the interaction between components and tries from these interactions to identify generalities shared across different systems and across the different levels of organisation. For example, psychology focuses on the individual. The interactions between individuals lead to sociology. Complexity science will investigate if similar processes and types of emergence can be identified at different levels. We can, for example, look for similarities at the level of sociology and the level of molecules. This may happen despite humans being very different from molecules since, viewed schematically, the interaction between humans, and between molecules, can share properties such as being attractive or repulsive, and this can in both cases, at the aggregate level, lead to phenomena such as segregation or mixing.

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indicate the perspective of complexity science. The middle column emphasises that complexity science is concerned with the behaviour of aggregates, and the rightmost column is meant to show that complexity science commonly extracts at a given level a few salient features of the interactions between the building blocks and represents these in a schematic way by a few parameters depicted as the θ s.

The rationale behind the approach of complexity science is to be able to shift the focus from the processes inside the components to the processes between the components. The inside at one level is ‘the between’ of the level below. For example, the inside of cell biology is the between of proteins. This shift in which one puts the focus on the interaction between components may allow us to identify general aggregate-level aspects of emergent dynamics. And what complexity science has found is that such generalities may be shared between different levels of the hierarchy in Fig. 1.1, For example, segregation or mixing can occur in a population of people and this behaviour can exhibit similarities with segregation or mixing of molecules. Sometimes the focus on interaction also allows us to understand, at least to some extent, the level above as emerging from the level below. We will discuss a mathematical example of this in Sec. 6.7, where we discuss how vortices that appear as structures at the aggregate level can arise from the spatial arrangement of the components.

Here follow a few examples which without mathematics illustrate how processes at the aggregate level, also often referred to as the systemic or collective level, may arise.

Let us first think about ‘thoughts’. Ponder the processes occurring in our brain while we think, for example when thinking about emergence. We do not know exactly what the processes we call thoughts are, but we do know that they involve different brain regions and zillions of neurones firing in some sort of coordinated manner. The individual participating neurone does not ‘think’, it simply undergoes a process of loading, firing and reloading. The thought process is a property of the collective interacting dynamics of all the participating neurones.

As our next example we will look at the phenomenon of colour. Perhaps at first we consider colours to exist in some objective sense. We may correctly define the colour red in physical terms as electromagnetic waves of frequency around 430 THz. This is the great physical insight of Isaac Newton (1642–1726), inspired by his observation that white light from the sun can split into different colours when passed through a prism. But this definition does not properly grasp the multitude of attributes the colour red possesses when we think of the colour experience. Wolfgang Goethe (1749–1832) developed a theory of colour, published in 1810, which studied colour as a combined physical, physiological and psychological phenomenon [476].

The mental experience of red is formed in our mind when light of the appropriate frequency passes through our eyes to generate a signal in the visual cortex that further propagates up through the hierarchies of cognitive processing. The qualities of the colour red as warm, or as the contrast colour to green, cannot be deduced from the value of its frequency as an electromagnetic wave. Even less can the emotional character of red, such as to do with romance or warmth, be reduced to a property of the wave. So where reside the properties of colours which stir us? Not in a single physical property of

the electromagnetic field, but rather in the collective effect of processes generated by the light absorbed through our eyes generating a hierarchy of other processes in our brain's neural system. All this leads in some not very well understood way to processes which we sense as thoughts and emotions in our mind. This example is simply meant to illustrate that some of the most familiar phenomena surrounding us are very much emergent in their nature. They exist in the form of some kind of amalgamated collaborative state across many participating components and not at all as some tangible *thing* that can be isolated and understood as an independent component of reality.

To highlight the aspects of emergence that are most important for complexity science, in the next section we consider two simple examples taken from physics and sociology, respectively, namely the **ideal gas** and **social segregation**.

1.1 The Importance of Interaction

Physics has a tradition for developing very schematic and simplified representations of our surrounding world. Physics describes the matter we encounter in our daily life as composed of molecules which interact more or less strongly. If interactions between molecules are relatively weak compared to the available thermal energy, which is the case at high temperatures, one may ignore that the molecules interact and can then describe the matter as an ideal gas consisting of independent non-interacting particles (see Fig. 1.2). In this situation the product of the pressure p and the volume V is proportional to the temperature T . This relation was established during the seventeenth and eighteenth century first as an empirical fact and then later understood in terms of the statistical behaviour of the molecules. At fixed temperature the product pV is proportional to the number of molecules N within the volume:

$$pV = Nk_B T. \quad (1.1)$$

The factor k_B denotes Boltzmann's constant, which we do not need to worry about right now. The mathematical description of the ideal gas makes it clear that each individual

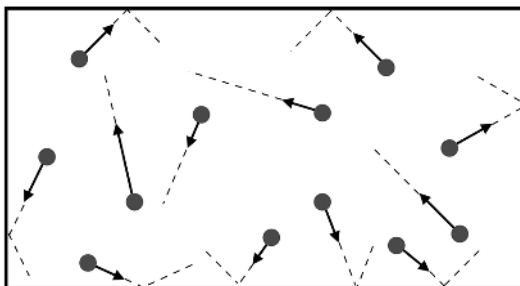


Figure 1.2 Sketch of an ideal gas. Each molecule is considered to move about independently of the others. When a molecule hits a wall, momentum is transferred between the molecule and the wall, leading to a force on the wall. The pressure on the wall is given by the total amount of force transferred from all the molecules hitting a unit area during a unit time interval.

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particle contributes the same amount to the pressure against the walls of the container. A single particle in the container and N particles behave identically. The only difference between having one particle in the container and N particles is that the walls are hit more frequently by the molecules, leading to the pressure growing proportionally with the number of molecules. In this sense the entire gas has the same properties as do the individual molecules. The pressure originates in the force transfer between the molecules and the walls when the molecules collide with the walls. The pressure is the sum of the force from the particles exerted on a unit area per unit time and therefore the measured pressure will fluctuate more if only a few molecules are inside the container. For a macroscopic number on the order of Avogadro's number 6×10^{23} , the relative fluctuations are negligible and therefore the measured pressure appears as independent of time.

In this sense one might be tempted to consider the pressure as an emergent property of the gas. But this is misleading. The pressure is simply the direct sum of properties of the individual parts and nothing more. The reason the pressure becomes a negligibly fluctuating and therefore well-defined quantity for a gas containing Avogadro's number of molecules is not that some new state foreign to the individual particles has formed, but because of a simple and universal mathematical fact following from the **Central Limit Theorem** [139]. Namely that the fluctuations in a sum of *independent* terms vanish compared with its average as the number of terms increases. This ensures that the pressure, being the net effect of the bombardment of the walls by the molecules in the container, is very stable for a volume containing a macroscopic amount of gas.

The lack of interaction between the components of the ideal gas prohibits emergent systemic behaviour qualitatively different from the behaviour of the individual particles. Perhaps this is a little bit surprising for anyone who has used a bicycle pump. The piston does feel like it is compressing some sort of elastic medium and one might imagine that this has to do with squeezing the air molecules together. But the effect would also be there if the pump only contained a single molecule – though the pressure exerted by a single molecule would be minute and strongly fluctuating. The pressure increases because the molecules fly around inside the container and will hit the walls more frequently when the distance to the walls decreases as the volume becomes smaller. So even with one molecule inside the container, the pressure will increase when we decrease the available volume because the molecule will have a shorter distance to travel between collisions with the wall and hence hit the wall more often, so the time-averaged force exerted on the wall increases.

The situation becomes very different if we take into account that real atoms, or molecules, do interact. The description in terms of an ideal gas, i.e. in terms of non-interacting particles, eventually breaks down when the density in the gas becomes sufficiently high. This density depends on the specific material. Let us think of water. Not too dense water vapour behaves like an ideal gas. But water in liquid or ice form obviously behaves in entirely different ways. Liquid water or solid ice have properties that are completely different from the properties possessed by individual water molecules. The wetness of water or the hardness of ice are the result of the interactions between

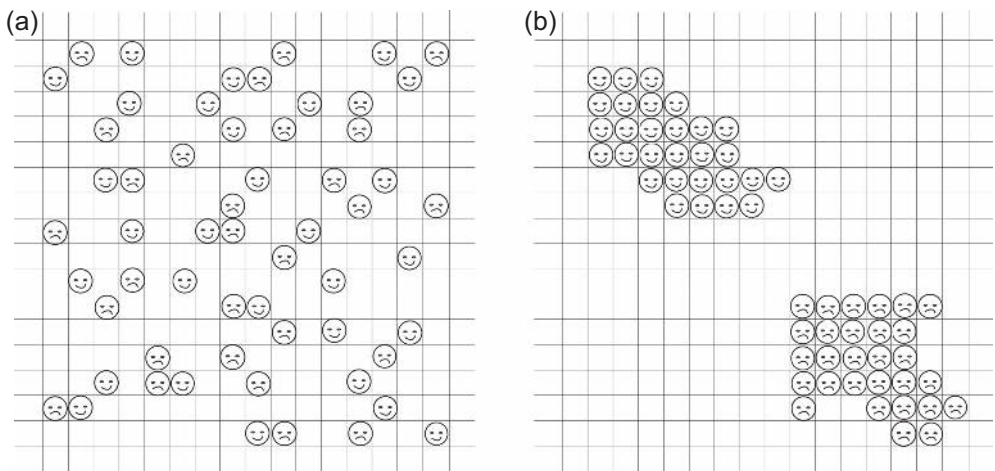


Figure 1.3 Lattice population of two types of agents: smilies and grumpies. In panel (a) the agents do not interact, i.e. they are indifferent to how their neighbour sites are occupied and as a result we see a homogeneous mix of types across the lattice. In panel (b) the agents are conscious about their neighbours. They prefer to be surrounded by agents of the same type as themselves, i.e. an attraction between like types is present. This interaction leads to a structured arrangement, namely a segregated distribution of the two types on the lattice.

the water molecules and are examples of emergent phenomena with no equivalence amongst the properties of the individual components. It is the forces acting between the water molecules that allow water to feel wet, which for example makes water able to climb up the wall of the container, and similarly at lower temperature the solid and elastic behaviour of ice is a result of the ability of the water molecules to interact with each other.

Next we consider emergent, and perhaps surprising, behaviour in models of sociology. Game theory is concerned with the resulting behaviour when **agents** interact according to a given set of rules. In its simplest form, two participants follow a table of pairs of actions in the form of player one does that and player two does that. A table allocates a payoff for each possible pair of actions and the theory then studies the accumulated payoff for a player choosing a certain string of actions. The rigorous mathematical analysis of such games was pioneered by von Neumann [478]. The approach has been generalised in various ways to include multiple players and the evolutionary aspects included by Maynard Smith [194].

The approach was generalised by combining ideas from game theory with dynamic action (see Fig. 1.3) in the 1960s by the American sociologist and game theoretician Schelling¹ [386, 387]. Schelling wondered why American cities were often found to segregate entirely into White and Black regions. Schelling had noticed that when asked,

¹ The model Schelling made use of in his study is now known as the Schelling model, although it is a specific instance of a more general model introduced earlier by James M. Sakoda [376]. The story of why the segregation model became known solely as the Schelling model is presented in detail in [188].

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people were not very concerned about the exact composition of the colour of people in their neighbourhood. The percentage of co-inhabitants of the opposite colour an individual feels comfortable with, Schelling calls the individual's tolerance level. If the ethnic composition of the population in a neighbourhood is directly determined by the tolerance levels of the individual inhabitants, one would expect cities to consist of a mix of White and Black with a specific ratio that might fluctuate somewhat from place to place, but as long as the individual tolerance levels are larger than zero there is no reason to expect total segregation into White and Black neighbourhoods.

However, nearly complete segregation was very often observed in American cities. The discrepancy between the expectation based on the preferences of the individual and the actual behaviour at the systemic population level led Schelling to consider collective dynamical effects induced by the fact that each individual contributes to the environment of the others. We will return to Schelling's model in detail later in Sec. 11.2, all we need to be aware of here is that Schelling's simple mathematical model demonstrated that the dramatic difference between behaviour expected from an extrapolation of the inclinations of individuals and the actual aggregate behaviour may be related to what one might call the synchronised collective dynamics of the individuals. This tendency towards herd behaviour can be seen as caused by a kind of trend setting. When an individual, say White, moves out from a region, the density of White goes down and hence it becomes more likely that the tolerance level of the remaining Whites is breached, motivating more Whites to leave. The systemic dynamics is a result of a collective magnification of the effect of the individual's limited tolerance.

That the collective of many interacting components can possess properties that are different and richer than those of the individual components is surely not a big surprise. But how different can the systemic level be? And will the properties of the aggregate be a specific and unique reflection of the components of this particular system? If this is the case, we will not be able to determine general principles and identify classes of behaviour. Each case would be totally unique and a science of emergence would not be possible. But this is not how aggregates behave.

We considered the examples of how the wetness of water and the hardness of ice are emergent features with no equivalence at the level of the individual water molecules. And we discussed how Schelling's model of social segregation demonstrates that segregation may appear at the collective level although the inclinations of the individuals do not suggest a preference for separation.

Moreover, fortunately it turns out that very different systems can share the same kind of emergent properties that are understood as manifestations of general principles or 'laws'. The observation that all matter is normally found to exist in three forms – gas, liquid and solid – is just one such example of regularities of very general scope. Segregation is another example of similar collective systemic behaviour in very different situations, such as amongst human populations and molecules in materials. Complexity science studies these regularities and how they are related to the different classes of interactions amongst components. In later chapters we will elaborate on conceptual and mathematical descriptions.