CAMBRIDGE

Cambridge University Press
978-1-108-83441-4 — Paradoxes and Inconsistent Mathematics
Zach Weber
Index
More Information

Index

1 + 1 = 2, 193

absolute consistency, *see also* nontriviality, 14, 294
absolute sufficiency, 126, 151
absolutely separated, 277
absorption, *see also* contraction
absurdity, 14, 18, 79–80, 111, 141, 164, 166, 292
Ackermann constants, 142
addition, 193–195
adherent points, 245
adjunction, 116, 133
∀, 18
Anderson, Alan Ross, 101
annihilation, *see also* negation
antecedent strengthening, 135
antilogism, 141
antisymmetry, 153
Archimedes, 23, 83, 281, 299
Archimedes' axiom, 199
argument by cases, 134
Aristotle, 5, 51, 182, 238, 295
arithmetic
  classical, *see also* PA
  inconsistent, 96, 97, 189, 199
  relevant, 97
  Robinson, 193
Arruda, Ayda Ignez, 162
assertion, 114
asymmetry, 156
Aussersein, 95, 168, 298, 299
axiom of abstraction, 28, 151
axiom of choice, 185
axiom of comprehension, 29, 95, 152
  unresricted, 29
axiom of extensionality, 28, 151
axiom of infinity, 169

Badia, Guillermo, 84, 130, 132, 144, 188, 193, 292
Basic Law V, 29, 30, 35, 128, 152, 287
Beall, J. C., x, 6, 35, 59, 74, 78, 86–94, 111, 114, 119, 192, 289
Bell, John L., 230
Belnap, Nuel, 88, 101, 105, 236, 288
Berkeley, George, 213
Bernstein, Felix, 36
Berry's paradox, 33
Bishop, Errett, 257
bivalence, 4, 7, 18, 20
blip function, 24, 176
Bolzano, Bernard, 34, 253
Boole, George, 100
Boolos, George, 38
Borges, Jorge Luis, 80
⊥, 14
boundary, 50–51, 77, 108, 266–269
  eliminativism, 55
Brady, Ross, 29, 90, 114, 125, 126, 144, 170, 176
Brouwer's fixed point theorem, 25, 108, 281
Brouwer, L. E. J., 239
Burali-Forti paradox, 32, 39, 74, 151, 180, 183
Burgess, John P., 85, 87, 93
Butchardt, Sam, 48

cancellation, 195, 219, 222–224, 226, 231, 235
Cantini, Andrea, 122
Cantor, Georg, 31, 34, 39, 66, 80, 178, 180, 182, 183, 185, 245, 247
Cantor–Bernstein theorem, 187
Cantor's theorem, 31, 35, 68, 73, 167, 179, 185
cardinality, 31, 72, 73, 180, 187
Caret, Colin, 110
Cartesian product, 170
Casati, Roberto, 50, 269
category theory, 66

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-108-83441-4 — Paradoxes and Inconsistent Mathematics
Zach Weber
Index
More Information

Chase, James, 44
Church, Alonzo, 23
classical recapture, 96–99
closed (set/object), 52, 257
closed theory, 16, 69, 93, 101, 115, 117, 119, 129, 143, 145, 289, 292
closure operator, 256
  as logical consequence, 264
closure space, 258
cogito, 23, 83
collapsing lemma, 92
Colyvan, Mark, 43, 44, 72, 100
complement
  exclusive, 160
  relative, 160
  self-, 160
  unrestricted, 159
$\overline{X}$, 159
&, 18
conjunction (as intensional), 134
connectedness, 44, 53, 245, 270
$\vdash$, 132
consistency operators, 95
continuity, 6, 52, 53, 66, 230, 239, 251
  continuous mappings, 250–253, 273
contraction, 112, 116–119, 129, 132, 134, 137, 158, 161, 163, 181, 186, 190, 195, 197, 199, 200, 202, 204, 228, 232, 237, 258, 260, 264, 267, 291
  on identity, 121, 154, 202
  on theorems, 142
contradiction, 295–296
  in the world, x
contraposition, 5, 20, 73, 76, 126, 129, 151, 160, 185, 201, 263, 286–288, 294
Conway, John, 210
Cook, Roy, 103
Cotnoir, Aaron, 50, 257
counterexample, 21, 291
counterexample (principle), 21
Curry's paradox, 24, 78, 103, 110
  for operators, 110–114
  for validity, 116
Curry, Haskell, 111
cutoff, 48

∂, 266
da Costa, Newton, 90, 95
de Morgan laws, 18, 20
Dedekind, Richard, 8–9, 39, 52, 54, 169, 172, 178, 190, 228, 234, 239, 253
Dedekind cuts, 52, 239–243, 247
definite descriptions, 48, 78, 119
density, 44, 234
derivable, 131
Descartes, René, 83
diagonal lemma, 175
dialetheism, x–xi, 14
  fictionalism, 59
  and the inclosure schema, 82
  semantic, 59
$\vee$, 18
disjunction (as extensional), 134
disjunctive syllogism, 15, 19, 41, 47, 48, 75, 76, 78, 86, 87, 94, 98, 100, 102, 106, 111, 113, 119, 141, 144, 160, 193, 204, 210, 241, 270, 291
distribution, 138
division, 202–203
  algorithm, 202
doppelgänger problem, 128, 165
double negation, 18
Dummett, Michael, 38, 73
Dunn, J Michael, 88, 105, 114, 193, 214, 236, 288
Dunn–Mortensen problem, 213–215, 226, 235

empty, 164, 165, 237, 278
$\varnothing$, 164
$\varnothing_X$, 160
epistemicism, 45
$\in$, 29
equivalence relation, 152, 186
Escher, M. C., 81
Euclid, 37, 50, 100, 101, 201, 255
Euclid's lemma, 205
Euclid's theorem, *see also* Infinitude of Primes
Euler, Leonhard, 100
existence, 157, 172
$\exists$, 18
explanation, 26, 66, 69, 83, 296–300
explosion, 14, 97, 99, 108, 161, 166, 215, 224, 252
extensionality, 120, 123, 126–128
  and intensionality, 34–35, 135

f'', 171
factor (principle), 137, 139, 142, 153, 161, 166, 171, 175, 200, 258, 262, 266, 278
Feferman objection, 87, 88, 90, 96, 99, 105, 108, 211, 285
field, 227
Field, Hartry, 85, 87–90, 104, 119, 144, 289
Fitch, Fredric, 11, 16
fixed point property, 24, 178, 274
fixed point theorem, 23, 174
Forster, Thomas, 39
foundations of mathematics, x, 28, 35, 89, 94, 100, 183, 185
  crisis in, 30
Fréchet space, 259
Frege, Gottlob, 21, 29, 94, 97, 189
French, Rohan, 113
functions, 171
  and skolemization, 171, 178
  versus mappings, 171–173, 177, 250
  versus relations, 102, 104
Fundamental Theorem of Arithmetic, 205

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE
Cambridge University Press
978-1-108-83441-4 — Paradoxes and Inconsistent Mathematics
Zach Weber
Index
More Information

Gödel, Kurt, *see also* incompleteness theorems, 7, 285
gamma problem, 98
gaps, 12, 43, 46, 55, 176
Gentzen, Gerhard, 117
Girard, Jean Yves, 124
Girard, Patrick, 69, 84, 130
gluts, x, 3, 14, 46, 56, 83, 103, 271
God, 35, 59
  Nyarlathotep, 80
Goodship, Laura, 111
googolplex, 208
greatest common divisor, 203
greatest lower bound principle, 238
Grelling's paradox, 33
Grišin's paradox, 122
group, 221
gunk, 240

Hallett, Michael, 80
Halmos, Paul, 11
Harman, Gilbert, 87
Hausdorff, Felix, 32, 36, 185, 257
Hausdorff space, 237, 259
hierarchy, 10, 13, 36, 82, 115, 180, 287, 300
Hilbert, David, xi, 16, 33, 37, 101
Hinnion–Libert paradox, 122
$\supset$, 19
Humberstone, Lloyd, 43
Husserl, Edmund, 8

idempotence, 113, 117, 121, 140, 163
identity, 126–128
  and contraction, *see also* contraction
$=$, 152
$\equiv$, 154
image (of a relation), 171
$\Rightarrow$, 124
$\rightarrow$, 125
implosion, 23, 25, 163, 176, 184, 228, 279, 299
inclosure schema, 69–79, 111, 298
incompleteness theorems, 16–17, 100
inconsistent mathematics, xi, 82, 90, 96, 98–102, 108, 129, 171, 185, 214, 253
induction (mathematical), 189, 191, 201
  complete, 201
  transfinite recursion, 184
inexpressibility, 12
inf, 238
infinite descent, 32, 163, 201, 210, 211
infinitesimals, 212, 215, 247–249
Infinitude of Primes, 206
infinity, 9, 169, 180, 234, 239, 299
  actual, potential, absolute, 183
injection, 173, 187
  soft, 184
instrumentalism, 89
interior operator, 263
intermediate value theorem, 25, 46, 253
intersection, 158
  of closed sets, 262
$\cap$, 158
interval, 244
invalidity, 19, 21, 289, 290
inverse element, 221
Istre, Erik, 135, 178

Kierkegaard, Søren, 61
Kleene, Stephen, 23
Knaster's theorem, 173
König's paradox, 33
Kripke, Saul, 13, 85
Kroon, Fredrick, 59
Kuratowski, Kazimierz, 170, 257

λ-calculus, 23, 177
large cardinals, 36, 185
law of excluded middle, 12, 18, 73, 76, 79, 134, 141, 144, 167, 176, 207, 278
law of noncontradiction, 20
Lawvere's diagonal theorem, 66, 82
least number principle, 107, 201, 207, 211
least upper bound principle, 108, 237
Leibniz continuity condition, 44, 243
Leibniz's law, *see also* substitution
Leibniz, G .W., 20
Leibniz, G. W., xi, 52, 212, 230, 239, 247, 249, 272, 281
$\leqslant$ in $\mathbb{N}$, 197
$\leqslant$ in $\mathbb{R}$, 234
Lewis, C. I., 19
Lewis, David, 89, 119, 216, 294
liar paradox, 5, 30, 59, 68, 82, 83, 103, 104, 127, 176, 180, 294, 296, 298, 301
  and the inclosure schema, 71
limit on size, 38
linearity, 200
logic
  BCK, 124, 146
  classical, 12
  DKQ, 97, 125, 146
  of formal inconsistency, 95
  fuzzy, 46, 124
  intuitionistic, 12, 46, 85, 90, 239, 257
  is not a game, 96
  is the laws of truth, 21
  LP, 17, 80, 92, 102, 117, 156, 229, 289–291
  paracomplete, 12, 111
  paraconsistent, ix, 14, 111
  plural, 35
  R, 97, 229
  relevant, 97, 125
  subDLQ, 131
  substructural, 117, 117

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-108-83441-4 — Paradoxes and Inconsistent Mathematics
Zach Weber
Index
More Information

logic (Cont.)
  ultra-, 90, 97, 126
  universal, *see also* universality
logical pluralism, 84, 91, 295
Lucretius, 70

mapping, 172
  composition of, 173, 252, 273
  ultracontinuous, 281
Mares, Edwin, 59, 162
material conditional, 19, 41, 47–49, 78, 111, 129, 144, 173, 181, 193, 204, 243
McKubre-Jordens, Maarten, 204, 230
Meadows, Toby, 84, 92
mereological nihilism, 4, 45
mereology, 35, 188, 238
metalanguage, 10, 13, 85, 101, 115, 132
metatheory, 84, 90, 96
Meyer, Robert, 16, 59, 90, 97, 98, 108, 114, 132, 178, 191, 250
Mirimanoff's paradox, 32, 181
modus ponens, 5, 19, 78, 111, 112, 114, 117, 118, 121, 131, 133, 278
  pseudo, 113–114, 191
monad, 249
monoid, 226
Moore, G. E., 7
Mortensen, Chris, 97, 101, 178, 191, 212, 214, 243, 256, 257, 290
multiplication, 195–197
multisets, 132
Myhill, John, 115

negation, 105, 130, 142, 192, 288–289, 295
  annihilation, 112, 125, 130, 134, 141, 192, 193, 291
$\neg$, 18
nested intervals theorem, 245
Newton, Isaac, 212
Nietzsche, Friedrich, 17, 71, 110, 300
no retraction theorem, 253, 277
non-self-identical, 157, 205, 278
non-self-identity lemma, 157
nonstandard analysis, 247
nontriviality, 14, 124, 125, 144–145, 176, 294
numbers
  cardinal, *see also* cardinality
  even and odd (not both), 208
  hyperreal, 247
  irrational, 189, 207, 209
  natural, 189, 231
  natural, axioms for, 191
  ordinal, *see also* ordinals
  prime, 204
  real, 213, 230
  real, axioms for, 230
  surreal, 247

objects, ordinary, 51, 61, 94, 95, 298, 301
Omori, Hitoshi, 84
On (set of all ordinals), 32, 182
$\mathbf{1}_x$, 216
Ono, Hiroakira, 124
open (set/object, 52, 264, 269
$\langle x, y \rangle$, 170
ordinals, 32, 37, 74, 80, 89, 182, 189, 280

PA, 97–99, 190, 193
paraconsistent mathematics, *see also* inconsistent mathematics
paradox
  definition, 3, 287
  options, 9, 13, 21, 41, 45, 54, 84, 94, 104, 286, 297
  solution, 22, 26, 297, 300
parallax, 299
Parmenides, 25
partial order, 153
  strict, 156
partition, 185
Peano postulates, *see also* PA
Peano, Giuseppe, 29, 94, 97, 100, 190
permutation, 114
Pessoa, Fernando, 9, 119, 283, 302
Petersen, Uwe, 7, 29, 176
phenomenology, 8, 50
Plato, 5, 17, 34
Poincaré, Henri, 36
point, 7, 25, 52, 218, 238–239, 255, 270, 277–281, 297
possibilism, 290
powerset, 162
$\mathscr{P}$, 162
Priest, Graham, x, 13, 18, 45, 69–74, 86, 92, 94, 96, 103, 111, 114, 144, 285, 288, 289, 294, 298
principle of sufficient reason, xi, 51, 52, 83, 281, 294, 297, 302
principle of uniform solution, 65, 69, 70
proper classes, 38
Pythagoras, 127, 209

quantification, 139
  restricted, 143, 263
  universal, 11, 18, 82, 143, 178
quantifier duality, 21, 139, 156
quasivalid, 88, 94, 96, 290
Quesada, Miró, x
Quine, W.V.O., 3, 86, 157

$\mathbb{R}$, 230
r, 157
rationality, 80, 94
reductio, 5, 12, 71, 73, 76, 77, 140–141, 189, 197, 199, 209, 211, 234, 277
relation, 170
  relational semantics, 17, 102, 292

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE
Cambridge University Press
978-1-108-83441-4 — Paradoxes and Inconsistent Mathematics
Zach Weber
Index
More Information

residuation, 114, 136
Restall, Greg, 70, 91, 122, 156, 165, 189, 191, 290
retraction, 253, 273
  soft, 276
revenge, 4, 11, 13, 42, 45–47, 55, 56, 59–61, 72, 89, 103, 106, 143, 286, 291, 294, 297, 298
revisionism in mathematics, 98–100
Richard's paradox, 33
ring, 225
Ripley, David, 14, 113, 134, 189
Robinson, Abraham, 247
Rosenblatt, Lucas, 113
Rossberg, Marcus, 55, 103
Routley, Richard, x, 6, 16, 29, 85, 90, 93, 98, 114, 126, 151, 185, 285, 300
Routley Set, 95, 167–168, 179, 186, 236, 272, 276, 285, 290, 293, 297, 300, 302
Russell, Bertrand, 10, 48, 65, 70, 73, 94, 101, 157, 180
Russell paradox, 31, 35, 60, 73, 80, 296
  and the inclosure schema, 75–78
Russell set, 31, 68, 73, 76–77, 80, 95, 107, 111, 157, 159, 160, 162, 167, 169, 285, 294, 298

s, 190
Sartre, Jean Paul, 161, 215
Schönflies, Arthur, 36
Scott, Dana, 37, 178
self-reference, 10, 23
semantic closure, 10, 12, *see also* universality
separation, 53, 270
separation axioms, 259
set concept
  iterative, 36–38
  naive, 28, 33–35
set theory, 28–40
  classical, *see also* ZFC
  naive, 28
  non-well-founded, 155
  versus property theory, 38, 125, 128
shadows, 50
Shapiro, Stewart, 15, 38, 39, 89, 96, 239
Shirahata, Masaru, 121
shrieking, 88, 192
simple consistency, 14
Skolem, Thoralf, 40
Slaney, John, 39, 78, 102, 113, 189, 197, 210
Smith, Nicholas J.J., 21, 69
smooth infinitesimal analysis, 101, 247
Socrates, 17, 26, 29
Sorensen, Roy, 42
sorites paradox, 40–43
  and the inclosure schema, 72–73, 78–79
  continuous, 43, 108, 243
  discrete, 41, 107, 207
  inductive, 41
  line drawing, 42
  topological, 57, 108, 272
Spinoza, Baruch, 52
$\subseteq$, 153
$\sqsubseteq$, 154
substitution, 120–122, 124, 126, 129, 131, 258
subvaluationism, 46
sup, 237
supervaluationism, 46
suppression, 136
surjection, 173
Sylvan, Richard, *see also* Routley, Richard
symmetry (principle), 52, 239, 240, 269, 281

T-schema, 5, 10, 22, 112, 176, 288, 289, 294
Tappenden, James, 59
Tarski, Alfred, 5, 85, 103
Tarski's theorem, 10, 104
Tennant, Neil, 85
Terui, Kazushige, 26, 87, 121
theorem, 19, 132
theory, 15
They Might Be Giants, 298
tolerance (principle), 41–44, 53, 54, 57, 72, 78, 81, 107, 208, 243
$\top$, 165
topology, 262
totally separated space, 277
transconsistent, 93, 96, 109, 135, 298
transfinite, 32, 182
transfinite recursion, *see also* induction (mathematical)
transitivity, 5, 14, 20, 85, 113, 117, 132
trichotomy, 200, 234
triviality, 14, *see also* implosion, 122
truth
  deflationism, 115
  inconsistentcy theory of, 10
  in a model, 89, 288, 292
  naive theory of, 5, 17, 30
  values, 17, 102–103
truth tables, 104
Turing, Alan, 23

ultraparaconsistent, 296–297, 300
union, 158
  of closed sets, 261
$\cup$, 158
uniqueness, 48, 49, 78, 119, 155, 171, 201, 206, 220, 241, 250
unit element
  absolute, 221
  relative, 221
universal set, 11, 24, 31, 35, 39, 71, 77, 79, 121, 128, 164–167, 179, 268, 275, 276, 297–300
universality, 16, 23, 93–94, 115, 178

$\mathcal{V}$, 164
vagueness, 43

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-108-83441-4 — Paradoxes and Inconsistent Mathematics
Zach Weber
Index
More Information

Val, 115
validity, 19, 115, 132, 287, 289
  predicate, 115
van Dalen, Dirk, 239
Varzi, Achille, 51, 269
vectors, 216–221
von Neumann, John, 32, 36, 69, 102, 182

Wang, Hao, 73
weakening, 20, 125, 129
Weir, Alan, 14, 38
well-founded, 181
well-ordered, 182
Weyl, Hermann, 35, 37, 239
Whitehead, Alfred North, 55
Williamson, Timothy, 49, 240
Wittgenstein, Ludwig, 12, 69, 83, 157, 288, 301
Woods, Jack, 85
world
  actual, 15
  inconsistent, x, 15
  is not a story, 295
Wright, Crispin, 38, 39, 41

$\mathcal{Z}$, *see also* Routley Set
Zardini, Elia, 117
Zeno, 238
Zermelo, Ernst, 36, 168
zero, 190
$\mathbf{0}_x$, 216
zeros, laws of, 232
ZFC, 36, 38, 39, 87, 89, 90, 92, 98, 145, 168, 193

© in this web service Cambridge University Press www.cambridge.org