1 Introduction

1.1 Introduction

Uncertainties permeate many aspects of engineering decision-making in the design, construction, operation, and maintenance of structures such as buildings and bridges, and infrastructure systems such as transportation networks or distribution systems for electric power, gas, and water. Uncertainties are present in the characteristics of such systems including their material properties and dimensions, in the demands placed on these systems, in the mathematical models that are used to analyze their behaviors, in the measurements made to assess their health conditions, and even in the probabilistic models that are used to describe the relevant uncertain quantities. Under such conditions, the safety and serviceability of structures and infrastructure systems cannot be assured with certainty. Probabilistic and statistical methods are needed to define and assess measures of safety and serviceability.

In the engineering of constructed facilities under uncertainty, three goals are paramount: (1) to assess the safety of the system against failure; (2) to assess the serviceability of the system, i.e., the ability of the system to perform its intended function; and (3) to optimize decisions so that there is an optimal trade-off between safety and serviceability, on the one hand, and the cost of used resources on the other. The assurance of absolute safety and serviceability may be neither possible nor desired, and an optimal decision may allow for finite risks of failure and unserviceability. Whereas addressing the first two of the above goals is rooted in probabilistic analysis, the third requires methods of optimal design under uncertainty. All three goals are addressed in this book.

This book presents the probabilistic approach to assessing the reliability of structures and infrastructure systems. The word *reliability* is used to denote the complement of the failure probability or of the probability of unserviceability. A variety of methods are presented, providing alternatives depending on the nature of the problem and the computational tools available. One chapter addresses the topic of reliability-based optimal design. While the bulk of the book is tutorial in nature, a few chapters on recent developments are included that are more exploratory in nature and present opportunities for further advancement by researchers.

The field of structural reliability emerged from the discipline of structural engineering. However, the concepts and methods described in this book have much broader applicability. As we will see, mathematical formulation of the problem envisions characterization of structural components in terms of limit-state functions that define the transition between the states of each component and logical expressions of component states that define the state of a system. Any problem that can be formulated in terms of this framework can be solved by the methods described in this book. Although the vast majority of illustrative examples are taken from the field of civil engineering, the mathematical formulation of problems and solution methods are presented in as general a way as possible so as to allow consideration of possible applications in other fields.

1.2 Brief History of the Field

Uncertainties in the design of structures have been of concern for a long time. Historically, the issue was addressed through the *factor of safety* – the ratio of a measure of capacity over a measure of demand. Initially, the factor of safety was set on a purely judgmental basis with no explicit regard to variabilities in capacity and demand. The idea of using probability theory and statistics to arrive at an appropriate factor of safety was developed in the early- to mid-20th century in the works of Max Mayer (Mayer, 1926), Alfred M. Freudenthal (Freudenthal, 1947, 1956), and Alfred G. Pugsley (Pugsley, 1951). Mayer pioneered the idea of using probabilistic methods in mechanics, Freudenthal argued for employing probability distributions to model loads and capacities, and using elastic analysis for serviceability and inelastic analysis for safety assessment, and Pugsley provided statistical data on aircraft loads and argued for a safety factor defined in terms of stresses rather than capacities and loads. Vladimir V. Bolotin (Bolotin, 1967, 1971) in Russia was another pioneer, particularly in the development and use of random process theory in reliability analysis.

The next generation of researchers and developers in the field of structural reliability included Masanobu Shinozuka, a student of Freudenthal (Konishi and Shinozuka, 1956; Shinozuka, 1964, 1972, 1983), Jack Benjamin (Benjamin, 1964, 1968a, 1968b; Benjamin and Cornell, 1970), C. Allin Cornell, a student of Benjamin (Cornell, 1967, 1969), Alfredo H.-S. Ang, who was my doctoral advisor (Ang and Amin, 1968, Ang and Cornell, 1974), Niels Lind (Lind, 1971; Hasofer and Lind, 1974), and Emilio Rosenblueth (Rosenblueth and Esteva, 1972; Rosenblueth, 1975, 1976), among others. Shinozuka played a key role in developing simulation methods for reliability analysis, particularly under stochastic loads and random fields. Benjamin and Cornell's 1970 book, which was the first of its kind in civil engineering, facilitated learning and research in probabilistic methods and decision theory applied to civil engineering problems. Cornell's 1969 paper laid the foundation for development of probabilistic structural design code formats. This paper influenced Rosenblueth and Esteva (1972) to develop a probabilistic basis for the Mexican design code. Rosenblueth was also a pioneer in advocating optimal design methods and his 1975 paper on point estimation of probability moments was later employed by other researchers for approximate probabilistic analysis. The paper by Ang and Cornell (1974) marks the full development of what we now call the mean-centered, first-order, second-moment (MCFOSM) method. As described in Section 5.2.1, this method lacks invariance with respect to the formulation of the problem. Ang and Cornell were aware of this shortcoming and acknowledged it in a discussion section at the end of the paper. Interestingly, a paper by Hasofer and Lind (1974) published four months earlier had resolved this problem. That paper led to the development of what we now call the first-order, second-moment (FOSM) method (Section 5.2.2) and paved the way for further development of reliability methods in the next decade.

The two decades starting in the late 1970s marked a period of intensive development of the field. The theoretical foundations of the field were formulated and specialized software to carry out needed computations was developed. Specialized journals, including *Structural Safety*, *Probabilistic Engineering Mechanics*, and *Reliability Engineering and System Safety*, were established and other journals, including those of the American Society of Civil Engineers, published increasing numbers of research papers on probabilistic methods and structural reliability. Additionally, a host of books appeared that increased the level of interest in the field. These included Ang and Tang (1975, 1984), Ditlevsen (1981), Thoft-Christensen and Baker (1982), Augusti et al. (1984), Melchers (1987, 1999), Madsen et al. (1986), Thoft-Christensen and Murotsu (1986), Ditlevsen and Madsen (1996), Haldar and Mahadevan (2000a, 2000b), Nikolaidis et al. (2005), Lamaire (2005), Nowak and Collins (2012), and Melchers and Beck (2018). Many civil engineering departments started requiring courses on probability and statistics in undergraduate programs and offered senior-level or graduate courses on structural reliability. Major influencers of this development included Ove Ditlevsen and his students H. Madsen and P. Bjerager at the Technical University of Denmark, Rudiger Rackwitz and his coworkers at the Technical University of Munich, including M. Hohenbichler and K. Brietung, Gerhart Schueller, Y.-K. Wen, Mircea Grigoriu, Bruce Ellingwood, Ross Corotis, Dan Frangopol, John Sørensen, James Beck, Christian Bucher, Mark Stewart, Arvid Naess, Michael Faber, Marc Maes, the author's own group at the University of California, Berkeley, and others. Throughout this book, references are made to the works of these authors as the topics are developed.

The new generation of researchers in structural and system reliability is focused on broadening the field and integrating it with other fields such as structural health monitoring, finite elements, uncertainty propagation, optimization, early warning systems, data analytics, and artificial intelligence. Because the solution of complex problems usually requires the use of computationally intensive software, a topic of increasing interest is in developing surrogate (or meta-) models that can be used in place of implicit and computationally expensive models to assess uncertainty propagation and reliability analysis. Although much progress has been made in this direction, the development of efficient and accurate surrogate models for real-world complex systems is a challenge that needs much further work.

The topic of reliability-based optimal design is another area of current interest. The key here is developing a bridge between the fields of structural reliability and optimal design that takes advantage of the theoretical and operational strengths of both fields instead of developing specialized optimization algorithms that solve a particular application of structural reliability. In the area of structural health monitoring, with sensors becoming ubiquitous, naturally the interest is in continuous updating of reliability measures as information is gathered and processed, particularly for near-real-time decision-making. Bayesian techniques are particularly well-suited for this purpose and this is another area of current research and development. In particular, the Bayesian network approach can be a highly fruitful framework to pursue in this direction. Finally, the finite-element method has become the quintessential computational framework for solving all kinds of field problems defined by differential equations, including structural, mechanical, geotechnical, and environmental problems. Uncertainty quantification and reliability methods need to be integrated with finite elements in order to broaden the scope of their applications to real-world problems. Several chapters in this book present state-of-the-art reviews of methods in the above areas.

1.3 The Nature of Uncertainties and Probability

The nature of uncertainties and probability has been the subject of debate for a long time. It is important to present the philosophical standpoint of this book on that subject.

As described in Chapter 10, there are several sources of uncertainty in engineering analysis and design. These include inherent variability or randomness, such as that arising in properties of materials and intensities of environmental loads; model uncertainty that results from the imperfection of mathematical models describing complex physical phenomena; statistical uncertainty in the estimation of model parameters, due to limited data; measurement error due to imperfection of measuring devices; and human error resulting from mistakes and omissions made by engineers in the processes of design, construction or operation of facilities. In practice, these uncertainties are often categorized as aleatory or epistemic in nature, the former characterized as irreducible, inherent variability and the latter as reducible uncertainty due to lack of knowledge. The two types are treated differently in classical statistics: aleatory uncertainties through probability distributions and epistemic uncertainties through confidence intervals.

The approach in this book is different from this in two ways. First, as described in Der Kiureghian and Ditlevsen (2009), we do not consider the distinction between aleatory and epistemic uncertainties to be crisp. What may appear as aleatory uncertainty may be epistemic if a higher-order model is used. For example, the wind speed on the surface of a building may be considered aleatory if no predictive wind model is used and data from measurements only are employed; however, it can be considered as at least partly epistemic if a physics-based predictive model is used that provides an estimate of the wind speed depending on various environmental and atmospheric factors. Hence, the distinction between aleatory and epistemic uncertainties makes sense only within the universe of models that we use to formulate a problem. Second, we use the rules of probability theory to model and analyze both types of uncertainty. Furthermore, with the eventual aim of using the results of probabilistic analysis in decision-making, the end result of our analysis incorporates all the relevant uncertainties into what we call a predictive estimate. Hence, the measure of reliability we compute incorporates the aleatory uncertainty arising from inherently random phenomena as well as the contribution of epistemic uncertainties arising from model error or limited data size.

The second issue concerns the interpretation of the probability of an event. Let $\Pr(E)$ denote the probability of event E. In classical statistics, $\Pr(E)$ is interpreted as a property of E. In fact, the frequentist notion of probability defines $\Pr(E) = \lim_{n\to\infty} n_E/n$, where n is the number of independent experiments where event E may or may not occur, and n_E is the count of the experiments where E occurs. In engineering applications, it is often not feasible to conduct a large number of experiments: there are many cases where even a single experiment is impossible. Consider, for example, determining the depth of the bedrock at the site of a future building. If a borehole could be dug, we could measure the depth exactly and there would be no need for probabilistic analysis. But suppose that resources are not available to dig a borehole and the information about the depth is needed right away. The engineer then has no choice but to make a probabilistic estimate of the depth with whatever information is at hand – e.g., by fitting a Gaussian surface to borehole data from nearby sites (so-called Kriging) or by transforming the measured travel time of shock waves that bounce back from the bedrock. In such an exercise, the probability of event E (say, the event that the depth is in the range 20–25m) is the property of the engineer who assigns it based on the information gathered. Of course, the accuracy of the estimate depends on the quality of the models used and analyses performed to arrive at the probability estimate. But this is typical of all engineering analysis, be it for predicting the stress at a point in a structure or determining the stability of an embankment. The probability estimate by the engineer must, therefore, be considered as the engineer's degree of belief based on the available information and the modeling and analysis performed. Naturally, transparency in the modeling and analysis is of paramount importance.

The above notions of uncertainty and probability are consistent with the Bayesian philosophy as originally conceived by Thomas Bayes with modern interpretations by de Finetti (1974) and Lindley (2014). Further treatment of this subject is presented in Chapters 10 and 15.

1.4 Objectives

The main objective of this book is to present a state-of-the-art treatment of the methods of structural and system reliability to serve as a textbook for upper-division or graduate courses in engineering, particularly in civil and environmental engineering programs. With the aim of making the book maximally accessible, the narrative avoids excessive mathematical formalism and uses as simple a notation as possible. To facilitate learning, numerous examples are presented, some with detailed step-by-step derivations and intermediate numerical results. It is assumed that the student possesses good command of the basic principles of probability theory. Nevertheless, Chapter 2 presents the minimum on this topic that is necessary in order to follow the remainder of the book. In addition, the student must have strong knowledge of multivariate distributions, a topic that is covered in Chapter 3.

Other subjects, such as Bayesian statistics and random processes, are introduced in later chapters, where they are employed.

This book is also intended for use by researchers and practicing engineers. The illustrative examples are intended to facilitate self-learning. Furthermore, while the material in Chapters 2–11 is well established, Chapters 12–15 present snapshots of current research in still-evolving topics of finite-element reliability methods, nonlinear stochastic dynamic analysis, reliability-based optimal design, and Bayesian networks for risk and reliability assessment. These chapters offer opportunities for exploration to those interested in pursuing research and further development in these fields.

There are many other topics that could be included in this book. Examples include probabilistic design codes and code calibration, modeling of specific types of loads (e.g., live, wind, and earthquake loads), reliability assessment of existing structures, and decision theory. However, a decision was made to focus on the mathematical foundations of the methods of structural and system reliability rather than aspects that would focus too narrowly on specific application areas or topics whose proper treatment would require significant deviation from the main focus of the book. No doubt the experience and expertise of the author also had a role in the choice of topics.

1.5 Software

The structural and system reliability methods described in this book require the use of computer software for all but the most trivial problems. During the past three decades, a number of commercial and free software packages have been developed. Some of these were described in two special issues of the journal *Structural Safety* (Volume 28, Issues 1 and 2, 2006) and a comparative analysis of a larger collection is provided by Chehade and Younes (2020). Notable commercial software packages for structural and system reliability analysis include:

- PROBAN (https://manualzz.com/doc/7264297/proban---dnv-gl reached on December 2, 2020), developed by DNV GL, Oslo, Norway, as a part of the SESAM system.
- STRUREL (www.strurel.de/index.html reached December 2, 2020), developed by the late Professor Rudiger Rackwitz's group at the Technical University of Munich, Germany.
- COSSAN (https://cossan.co.uk/ reached December 2, 2020), developed by the late Professor Gerhart Schueller's group at the University of Innsbruck, Austria; now maintained at the University of Liverpool, UK.
- NESSUS (www.swri.org/nessus reached December 2, 2020), developed at the Southwest Research Institute, San Antonio, TX.

Notable free software packages for structural and system reliability analysis include:

- CalREL (https://bitbucket.org/sanjayg0/calrel/src/master/ reached December 2, 2020), developed by the author's group at the University of California, Berkeley.

- FERUM (www.sigma-clermont.fr/en/ferum reached December 2, 2020), a MATLAB toolbox originally developed by the author's group at the University of California, Berkeley, and further developed and maintained by Jean-Marc Bourinet at the Université Clermont Auvergne, France.
- OpenSees (https://opensees.berkeley.edu/ reached December 2, 2020), a general-purpose structural analysis program developed at the Pacific Earthquake Engineering Research (PEER) Center of the University of California, Berkeley, that contains some capabilities for structural reliability analysis.
- DAKOTA (https://dakota.sandia.gov/ reached December 2, 2020), developed at the Sandia National Laboratories, Albuquerque, NM.
- Rt (http://terje.civil.ubc.ca/the-computer-program-rt/ reached December 2, 2020), developed by Professor Terje Haukaas' group at the University of British Columbia, Vancouver, Canada.

In addition to the above, useful tools for reliability analysis, uncertainty quantification, Bayesian updating, and other probabilistic tools are available at the following sites:

- https://systemreliability.wordpress.com/software/ (reached December 2, 2020) contains tools for surrogate modeling and reliability analysis by sampling techniques developed by Professor Junho Song's Structural & System Reliability Group at the Seoul National University, South Korea.
- www.bgu.tum.de/era/software/ (reached December 2, 2020) contains tools for Bayesian inference, surrogate modeling and reliability and risk analysis developed by Professor Daniel Straub's Engineering Risk Analysis Group at the Technical University of Munich, Germany.
- https://sudret.ibk.ethz.ch/software.html (reached December 2, 2020) contains tools for metamodeling and uncertainty quantification developed by Professor Bruno Sudret's group at ETH, Zurich, Switzerland.

A variety of software is available for Bayesian network analysis (the topic of Chapter 15) for inference and learning purposes. A comprehensive but somewhat outdated list prepared by Murphy (2014) is available at www.cs.ubc.ca/~murphyk/Software/bnsoft.html (reached December 2, 2020). Among these is the MATLAB toolbox described in Murphy (2001):

- BNT (https://github.com/bayesnet/bnt reached December 2, 2020), freely downloadable MATLAB toolbox for Bayesian network analysis.

Furthermore, Murphy (2007) presents a review of several Bayesian network software. Among these, the following two commercial codes are noteworthy:

- GeNie (www.bayesfusion.com/ reached December 2, 2020), developed by Professor Mark J. Druzdzel's group at the University of Pittsburg and available through BayesFusion, LLC. A free version for educational purposes is provided.

- Hugin (www.hugin.com/ reached December 2, 2020), developed at the University of Aalborg and available through HuginExpert A/S. A free version for educational purposes is provided.

Details about the specifications and capabilities of the above software can be obtained from the listed websites.

1.6 Organization of Chapters

Following this introductory chapter, Chapter 2 presents a review of the basic concepts and rules of probability theory. Mastery of this material is essential for a thorough understanding of the material in the subsequent chapters. This chapter can be skipped if the student has a fairly advanced understanding of probability theory. However, in my own teaching, I have found it necessary to review this material, even for students who have taken a prior course. I normally spend six lecture hours reviewing this material in a semester-long course of 45 lecture hours.

Chapter 3 describes multivariate distribution models and the transformation of variables to the standard normal space. The inverse transforms as well as the Jacobians of both transforms are presented. Several reliability methods make use of these transformations. I have found it useful to cover this material in two lecture hours. A focused presentation of this material rather than covering it as a part of a particular reliability method gives it the weight that it deserves.

Chapter 4 introduces the basic formulation of the structural reliability problem. Exact solutions are provided for the special cases of capacity and demand values having jointly normal or jointly lognormal distributions. Formulations are presented in terms of the safety margin and safety factor. The important issue of sensitivity to the tail of the assumed distributions is investigated. Finally, the structural reliability formulation is generalized and expressed in terms of limit-state functions of basic random variables, a formulation that is used throughout subsequent chapters. Several example formulations of the limit-state function provide the link between this theory and classical domains of civil engineering. In my teaching, I normally spend one lecture covering this topic.

Chapter 5 presents methods of structural reliability analysis under incomplete probability information. These include second-moment methods, i.e., the MCFOSM method, the FOSM method, and the generalized second-moment method, as well as methods that employ information beyond the second moments such as higher-order moments or marginal distributions. An algorithm for finding the “design point,” which is a point of approximation in several reliability methods, is also introduced in this chapter. In my opinion, the methods described in this chapter now have only historical value. In the concluding section of the chapter, I argue that the notion of having perfect information about a limited number of moments and no information on higher moments or other probabilistic characteristics is counter to reality. In practice, one estimates the moments from available data with decreasing accuracy for higher moments. Furthermore, the moment-based formulations do not provide a logical framework for accounting for statistical uncertainties. In my teaching of the course, I make only brief introduction of these methods (no more than one lecture) but require that students read the chapter on their own.

Chapter 6 develops the first-order reliability method (FORM), which is one of the linchpins of this book. This is a full-distribution reliability method that employs a first-order approximation of the limit-state function in the standard normal space, thus requiring transformation of random variables into the standard normal space, as described in Chapter 3. It is important to recognize the approximate nature of FORM, and this is discussed in a separate section dealing with measures of its accuracy. The chapter then presents FORM measures of variable importance and parameter sensitivities. Other topics in this chapter include addressing the problem of multiple design points, the inverse reliability problem, and determining the distribution of a function of random variables by FORM. In my teaching, I devote eight lecture hours to the material in this chapter.

Chapter 7 develops the second-order reliability method (SORM). This is a refinement of FORM in which a second-order approximation of the limit-state function is used. Aside from the classical SORM that requires computation of second derivatives of the limit-state function, two additional SORM approximations are developed that require first-order derivatives only. Several examples compare results obtained from the three SORM approximations together with those of FORM analysis. I devote two lecture hours to this chapter.

Chapter 8, another linchpin of this book, introduces system reliability analysis. It starts with the definition of a general system in terms of the system function and its characterization in terms of cut and link sets. For systems with statistically independent component states, methods are described for computing the system reliability in terms of the component failure probabilities. For systems with dependent components, methods for computing system reliability in terms of minimum cut and link sets are presented and bounding formulas are derived. For cases with incomplete probability information, bounds on system reliability are obtained by the use of linear programming. For the case of complete probability information, the latter formulation leads to an efficient matrix-based reliability method for certain classes of problems. Next, attention is focused on structural systems, where approximations and bounding formulas based on FORM and an event-tree approach are presented. The chapter ends with the development of component importance and sensitivity measures. I devote around nine lecture hours to the material in this chapter.

Chapter 9 presents simulation methods for assessing the reliability of structural systems. The chapter starts with the presentation of methods for generating pseudorandom numbers for specified distributions. Then the basic Monte Carlo simulation method is described, showing its shortcoming for estimating small probabilities that are typical of structural reliability problems. Next, several methods are presented for enhancing the efficiency of the Monte Carlo method. These include use of antithetic variates, importance sampling, directional sampling, orthogonal-plane sampling, and subset simulation. An adaptive importance sampling method for numerical integration in high dimensions is also presented, which is useful for Bayesian posterior analysis. The chapter ends with the presentation of parameter sensitivity analysis by simulation. My recommended time for this topic is four lecture hours.

Chapter 10 deals with Bayesian parameter estimation and reliability updating. The chapter starts with a discussion of the sources and types of uncertainties then develops the basic notions of Bayesian parameter estimation and model assessment. Next, analysis of structural reliability under parameter and model uncertainties is described and various measures of reliability that account for these uncertainties are introduced. Methods for Bayesian updating of structural reliability and the distribution of basic variables in light of observational data concludes the chapter. I recommend devoting four or five hours to this chapter.

Chapter 11 starts by introducing three types of time- and space-variant reliability problems: the outcrossing problem, the encroaching problem, and the outcrossing-encroaching problem. This provides motivation for the remainder of the chapter. After a brief review of random process theory, approximate or bounding solution approaches are presented for the three classes of time- and space-variant reliability problems. Next, the Poisson process and related distributions of waiting and inter-arrival times are derived. These are used to develop stochastic load models. The chapter ends with a presentation of the load combination problem based on the load-coincidence approach. Given the time constraints of the semester, I devote only two lectures to this chapter, focusing more on the Poisson process and stochastic load models and load combination.

Chapters 12–15 present the more advanced topics of finite-element reliability methods, stochastic structural dynamics, reliability-based optimal design, and Bayesian networks for system reliability and risk analyses. These are topics of current research and are still evolving. Time permitting, I devote about one lecture to introduce the more salient aspects of each topic, emphasizing example applications. I encourage students to read these chapters on their own, particularly if they are interested in conducting research in probabilistic methods.