A Student's Guide to Special Relativity

This compact yet informative Guide presents an accessible route through Special Relativity, taking a modern axiomatic and geometrical approach. It begins by explaining key concepts and introducing Einstein's postulates. The consequences of the postulates – length contraction and time dilation – are unravelled qualitatively and then quantitatively. These strands are then tied together using the mathematical framework of the Lorentz transformation, before applying these ideas to kinematics and dynamics. This volume demonstrates the essential simplicity of the core ideas of Special Relativity, while acknowledging the challenges of developing new intuitions and dealing with the apparent paradoxes that arise. A valuable supplementary resource for intermediate undergraduates, as well as independent learners with some technical background, the Guide includes numerous exercises with hints and notes provided online. It lays the foundations for further study in General Relativity, which is introduced briefly in an appendix.

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I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

Isaac Newton, as quoted in Brewster, *Memoirs of the Life, Writings, and Discoveries of Sir Isaac Newton*, Vol. 2, p. 407.

The eternally incomprehensible thing about the universe is its comprehensibility.

Albert Einstein (1936*a*)

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Frontmatter
More Information

Contents

Preface

This book is an introduction to Special Relativity (SR), which intends to be both comprehensive, in the sense of covering most of what a physicist will be presumed to know about relativity, and accessible to those at a reasonably early stage of their physics education. These aims are not incompatible. Learning about relativity does not require a huge volume of material to be learned, and the maths is not elaborate (the most advanced maths we use is basic calculus).

The very significant challenge to approaching this area of physics is that it is probably the first area we encounter where we cannot rely on our intuition, and where we must rely on abstract arguments to navigate past our intuitions to somewhere we could not otherwise reach. I fully embrace this in the axiomatic approach of this book: we start with the two axioms or postulates of SR, and then follow their consequences.

But SR is not *just* an abstraction. It is instead a foundational theory of how our physical world is constructed, and the consequences we deduce must match the physical world we find ourselves in. Making the link from abstraction to nature is of course a little tricky – there are a lot of thought experiments, and a large number of trains moving at implausibly high speeds, in the pages ahead of you – but our destination is physics, not maths.

It is also possible to go too far the other way, and over-stress the continuity with familiar ideas, downplaying the dislocation that the second axiom necessarily implies. I don't think this is a useful approach, because we cannot really evade the disorientation which arises from the loss of simultaneity, nor evade the necessity of thinking of time as something other than a rope along which a sequence of snapshots of 'the present' are strung.

The nature of the middle path I wish to plot out in this book is therefore, I hope, clear. I want you as the reader to appreciate SR as an account of the geometrical structure of our universe, and also see that structure as something with concrete physical consequences.

I developed the material here over a number of years, in the course of teaching an early-undergraduate course in SR at the University of Glasgow. It has therefore been read, puzzled over, commented on, and corrected by multiple cohorts of students, and the distribution of exercises in each chapter reflects both their questions and difficulties, and the inevitable requirement that I formally examine their understanding. That is, this book is not an armchair exercise in finding 'interesting topics in SR', but an attempt to bring students to the understanding of SR that they will need in their future work, and which will expand their intellectual horizons. And of course those lectures were not just a discussion group, but a course with an exam at the end of it; the presence of that exam influenced the selection and shape of material in a way which may also be of use to you.

The text here is therefore suitable to support a course: I covered the content, without appendices, in ten fairly busy lectures (and did not declare all of it examinable). But in converting the text to a book I have also had in mind both the independent reader working their way into the subject, and the student of another course who wants light shone on the landscape from a different direction. I resisted adding new core material in this process, but have expanded explanations here and there, and added various supplementary comments, either historical, mostly in discursive footnotes, or technical, in 'dangerous-bend' sections.

Although I believe my students' experience validates this text as one good route to SR, I insist that it is not the only route, nor even the only good one. Amongst the unusual things about relativity is that there exists no single royal road to an understanding of it, nor even a single obvious way of mapping the territory. Relativity always makes more sense the second time you read about it (and makes still more sense the first time you explain it to someone else), and so it is always useful to expect to read more than one introduction; to that end, I point to a number of other books in Chapter 1. My goal for this book is not to disperse, but to join, this swarm of alternatives. Throughout, I have freely referred you to other textbooks, to review articles, and to original research articles. It is not necessary to follow each of these outward-pointing links to get an understanding of the subject, but I hope they indicate the richness of the subject's connections with the rest of physics, and with the history of physics and astronomy.

It's easy for me to talk about the simplicity and minimalism of SR: you may find the claim a little surprising, if you look ahead and see a lot of solid text, including a dense undergrowth of primes and subscripts. A lot of this is *re*-explanation, however. In Chapter 5, for example, I discuss some of the 'paradoxes' of SR, deliberately pointing to results of relativistic arguments which are so unexpected as to seem surely wrong, and then going on to discuss how those results are not only consistent, but merely a different way of looking at ideas we have already encountered, so that we acquire a more textured understanding of ideas that seemed insubstantial the first time around. There is a lot of internal cross-reference knitting the book together – pointing to another version of an idea introduced previously, or presenting a preview of something done more carefully later – but I think of it as having a single short thread running from the first chapter to the last, and when you have undone and re-done these chapters in your head, along with any other broader reading, I hope you will see that thread, too.

Each chapter starts with a few high-level aims. These are collected on page xvi, and I think of them as the 'intellectual table of contents' of the book.

The exercises at the end of each chapter are, I think, important in solidifying your understanding of SR. A number of them are annotated with d^+, d^- or u^+ to indicate ones which are more or less difficult, or more particularly useful, than those around them.

I have included an appendix on the link to General Relativity and gravitation. Although it is of course auxiliary to a book on Special Relativity, I believe it is useful to show that the gap between the two areas is narrower than it may at first appear, and that we can build a bridge which lands a small but significant way into the new territory. The approach I have taken to SR is designed to give this bridge as firm a foundation as possible, on the SR side.

Similarly, the appendix on relativity's contact with experiment is auxiliary but, I hope, both useful and interesting. It it not concerned with the question 'is relativity right?' – the answer to which it takes as obviously 'yes' – but instead steps back and discusses the nature of the relationship between relativity, both special and general, and experimental corroboration, and indeed has something to say about the relationship between corroboration, science in general, and scientists as a community.

Throughout the book, I have included a number of historical asides. Special Relativity does not have an unusually intricate history, but these asides are present partly because they add an extra dimension to our knowledge of the topic, but also because, by hinting at an alternative intellectual path not followed, they can colour in our understanding of the ideas as they have developed in fact. This also seems a good point at which to mention my deliberate habit of styling scientists' names, when they become adjectives, in lowercase: thus 'Newton's laws', but 'newtonian physics'. This is partly because, by the time a topic acquires an adjective like this, it has absorbed the work of a multitude of people beyond any original creator (see also 'Stigler's law'), and evading the ownership question with a lowercase letter seems both fairer and less cumbersome than a hyphenated list of all an idea's retrospectively discovered co-discoverers. Even Newton would have to go back to school if presented with a book on contemporary classical mechanics.

Throughout these notes, there are occasional sidenotes, and one or two complete sections, which are marked with a symbol like this. These 'dangerous bend' paragraphs provide supplementary detail or precision, or discuss extra subtleties, which are tangential but interesting, or which comprise extra discussion of ideas which students have in the past found confusing or easy to misunderstand. You will typically want to ignore these on a first reading, and I will sometimes presume you are re-reading, here, by referring in passing to ideas which are introduced only later in the book.

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Aims

My goal is that you should:

1.1. understand the importance of events within Special Relativity, and the distinction between events and their coordinates in a particular frame;
1.2. appreciate why we have to define very carefully the process of measuring distances and times, and how we go about this;
2.1. understand the two axioms of SR;
2.2. appreciate the significance and inevitability of the immediate consequences of those axioms;
2.3. understand the ideas of a coordinate transformation, and of the covariance of an equation under a coordinate transformation;
3.1. understand why the concept of simultaneity is problematic in the context of SR, and how we resolve these problems;
4.1. appreciate the role of geometry in understanding spacetime, specifically the importance of the invariant interval and the Minkowski diagram;
4.2. internalise the utility of units where $c = 1$, as the natural units for discussing events in spacetime;
5.1. understand the derivation of the Lorentz transformation, and recognise its significance;
6.1. understand the concept of a 4-vector as a geometrical object, and the distinction between a vector and its components;
7.1. understand relativistic energy and momentum, the concept of energy-momentum as the magnitude of the momentum 4-vector, and conservation of the momentum 4-vector; and
7.2. understand the distinction between invariant, conserved and constant quantities.