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Reviel Netz
Frontmatter
More Information

A NEW HISTORY OF GREEK MATHEMATICS

The ancient Greeks played a fundamental role in the history of mathematics. Their innovative ideas were reused and developed in subsequent periods, all the way down to the scientific revolution and beyond. In this, the first complete history for a century, Reviel Netz offers a panoramic view of the rise and influence of Greek mathematics and its significance in world history. He explores the Near Eastern antecedents and the social and intellectual developments underlying the subject's beginnings in Greece in the fifth century BCE. He likewise leads the reader through the proofs and arguments of key figures, such as Archytas, Euclid, and Archimedes, while considering the totality of the Greek mathematical achievement, which includes – as well as pure mathematics – such applied fields as optics, music, mechanics, and above all, astronomy. This is the gripping story of not only a major historical development but also some of the finest mathematics ever created.

REVIEL NETZ is Patrick Suppes Professor of Greek Mathematics and Astronomy at Stanford University. He is the author of many books on Greek mathematics and culture, including *The Shaping of Deduction in Greek Mathematics* (Cambridge, 1999); *The Archimedes Codex* (coauthored with William Noel; Da Capo Press, 2007); *Ludic Proof: Greek Mathematics and the Alexandrian Aesthetic* (Cambridge, 2009); and *Scale, Space and Canon in Ancient Literary Culture* (Cambridge, 2020).

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To Maya, Darya, and Tamara

Contents

A plate section will be found between pages 272 and 273.

Plates

Preface

In 1586, Galileo Galilei was twenty-two years old and ready to do great things. He authored his first work, the *Bilancetta*, or *Small Balance*, a gem of a treatise. As so often with Galileo, it brings much together: literary erudition, mathematical sophistication, experimental precision.

Galileo begins with Vitruvius's story of Archimedes's solution to the problem of the crown. We all remember this story: The king's goldsmith was provided the materials to make a crown of pure gold – but could he have returned, instead, a crown made of an alloy containing silver? Could he have pocketed the difference? Stepping into the bath – so Vitruvius continued his tale – Archimedes suddenly realized that his body displaced a volume of liquid equal to that of his own body; excitedly, he ran home naked, shouting, "*Eureka*!" ("I have found it!"). It is possible to measure how much of a liquid is displaced by a given body and thus to measure the volume of any given body. And if so, we can also weigh the body and, by dividing the two quantities – weight divided by volume – we can find the body's (in modern terminology) "specific density" or, effectively, "specific gravity." It is well understood that gold is rather heavier than silver (has a higher specific gravity). So, the specific gravity of the crown would tell us whether it is or is not made of pure gold. It was not, we are told by Vitruvius. Archimedes – the mathematician detective – confronted the goldsmith, who confessed. A triumph for science!

This Galileo retells – and refutes. Of course Vitruvius must have been wrong. Why? Because Galileo read Archimedes himself (made newly available in Commandino's edition from 1565). The observation reported by Vitruvius – that the surface of a liquid rises as you step into it – is trivial; a really deep mathematical fact, however, which Archimedes did discover, is the law of buoyancy. A body immersed in a liquid *loses a weight equal to the weight of the liquid it displaces*. The fraction of the original weight lost as a body is immersed in a liquid can therefore be used directly to measure specific gravities – no need for messy (and inherently imprecise) measures of volume displaced. Instead, use the "small balance" of Galileo's treatise, in which a counterweight is moved so as to balance an arm immersed in water. Galileo shows the mathematical principles underlying the instrument and their grounds in actual Archimedean science, and he crowns this all with concrete, technical suggestions on how the small balance can be made truly precise.

And so, for Galileo, a beginning. Not yet an original contribution, but immediately following that, Galileo set his eyes on his own path in science. Aristotle had argued that the motion of heavy bodies was simply downward, but Archimedes (so Galileo learned, from Commandino's edition) had shown the mathematical reason why objects, when immersed in a liquid heavier than themselves, are pushed upward. A thought then suggested itself to the young Galileo: Would it not be possible, in general, to account for the motions of bodies through the relative weights of the moving object and the medium through which it moved? Could one extend Archimedes's theory of floating bodies into a theory of free fall? This was Galileo's first major theoretical attempt, *De Motu* (*On Motion*), embarked upon when he was just twenty-five. Like the *Bilancetta* itself, it was never published in his lifetime, and the quest for a theory of free fall ultimately would lead Galileo in many other, different directions. The fruit of all this labor would wait for many years, until the *Dialogues Concerning Two New Sciences*, Galileo's mature contribution, published in 1638. A major book, now – it set out the foundations of modern classical mechanics. As a seventy-two-year-old, Galileo still reflected on where it all started from: "It was Archimedes' own books – which I had already read and studied with infinite astonishment – that rendered credible to me all the miracles described by various writers."[1]

Galileo set himself, from the beginning, as a critic of Aristotle, and this critique, indeed, would loom large in his career. We imagine him (probably a mere legend, though) hauling light and heavy balls up the Leaning Tower of Pisa (light and heavy, they fall at the same speed, thus refuting Aristotle!). More historically, we can spy him squinting at the sky with his homemade telescope, so crude to our eyes and yet so powerful in Galileo's hands. He detects moons circling Jupiter, mountains and seas on the surface of the moon. In other words, the heavens were not the ethereal, unchanging realm of pure circular motions all centered around the earth, as envisaged by Aristotle. They were a load of coarse matter pressing upon

[1] G. Galilei, *Dialogues Concerning Two New Sciences*, trans. H. Crew and A. de Salvio (New York: Macmillan, 1638/1914), p. 41.

coarse matter, just like the earth – Copernicus, after all, must have been right! The church, defending Aristotle's orthodoxy, found objection to all of this, and Galileo's ensuing trial set out a powerful image. Historians came to think of this era in such terms, breaking away, decisively, from the constricted dogmas of Aristotle. In 1957, Koyré called this the transition "from the closed world to the infinite universe"; Thomas Kuhn, a few years later, saw this transition as a *paradigm shift*, the prime example of his 1962 classic, *The Structure of Scientific Revolutions*.

More recently, historians have become wary of such sweeping and even teleological categories as the "scientific revolution." Steven Shapin, famously, began his study from 1996, titled simply *The Scientific Revolution*, with the following words: "There was no such thing as the Scientific Revolution, and this is a book about it." I am not here to beat a dead horse. But this should be emphasized. The narrative of a scientific revolution is mistaken, first of all, in that it hinges on encounters such as that of *Galileo with Aristotle*. But these, in fact, were not the decisive moments leading to the rise of modern science. Far more crucial were encounters such as that of *Galileo with Archimedes*. What gave rise to modern science was a new appreciation of the science of antiquity and the attempt, finally, to emulate and outdo it. To a large extent, modern science came not from a scientific revolution but a *scientific renaissance*.

Modern science is firmly rooted in the science of antiquity. To be clear: this ancient science was not *only* mathematical. William Harvey's revival of Galen's medicine would ultimately be as important, in its own way. And yet, the main line of development of modern science does begin with Copernicus, passing through Galileo, Fermat, Kepler, and Descartes, and leading on to Leibniz and Newton. This line of development, throughout, can be characterized as a revival of the ancient tradition of the exact sciences. And in this book, I set out to provide a new history of this ancient tradition.

A topic as important as this – the soil from which grew modern science – might be expected to attract significant scholarly attention. Astonishingly, this book is the first such contribution in precisely a century. Thomas Little Heath's *History of Greek Mathematics*, published in 1921, has served as a reliable guide to many generations of scholars and curious readers. Historiographies went in and out of fashion, but Heath still stands, providing a clear and readable survey of the contents of most of the works of pure mathematics attested from Greek antiquity. To a modern reader, used to more critical, analytical historical approaches, Heath's work reads most like an encyclopedia, arranged by chronological principles. One may turn to the entry on Apollonius of Perga, for instance, and find seventy highly informative pages summing up the contents of the Conics (forty-one pages), followed by smaller surveys of the contents of the minor, indirectly attested works. I keep Heath by my side, and I urge you to do so as well. This new history does not aim to replace Heath's, and I do not aim at his encyclopedic coverage. My goal, instead, is to provide a historical *account*. Something quite deep – indeed, transformational – took place in the ancient Greek world. A new kind of science emerged, ultimately providing the tools for modernity. Why and how did that happen? What we need is to understand the conditions and scope of this achievement.

This Greek achievement belongs, in my view, to the history of science. *Science* itself is not a word the Greeks used, although I think it is useful for our purposes. The modern word *mathematics* is indeed ultimately Greek, but what the Greeks called *ta mathēmata* was usually wider in meaning than the modern term implies (and wider than what Heath understood as the scope of his own history). Besides pure geometry and stereometry (as well as the much less central field of pure arithmetic), the ancient Greeks always included within mathematics fields such as astronomy and theoretical music, and they often added optics and mechanics as well. That there was a difference between "pure" and "mixed" mathematics was often acknowledged, but that the two belonged together was also clear. How could it not be? As I will point out throughout this book, the identity of Greek mathematics was, above all, that of a literary genre. And at this level – the way they were written about – the different fields were not all that far apart. Whether in geometry or astronomy, optics or mechanics, one would encounter nearly the same diagrams, nearly the same formulaic language. It would be ahistorical as well as misleading to produce a history of Greek mathematics and leave out the more applied fields. If you will, you may think of it as a history of the Greek exact sciences. But I prefer to keep the word *mathematics*: it is, in fact, closer to the way the Greeks, themselves, thought about it. Even in the more applied fields, it was a theoretical study, organized around the idea of proof, not around the idea of experiment. But I am getting ahead of myself. I shall argue for all of this throughout the book.

One final word. Heath required two volumes, even while excluding all the applied mathematical sciences. What I wish to produce is different: a single narrative account, of use for the general interested public, as well as for undergraduate classes and for those graduate students and scholars looking for some entry point into the historical foundations of science.

A wider public readership demands a slimmer apparatus. At the end of each chapter, I provide a list of additional readings – those that directly further the main topic – and offer only a handful of footnotes suggesting sources for more particular claims. My goal is simple: to make the story interesting enough so that my readers, indeed, look further.

Plan of the Book

The seven chapters of the book are mostly – but not entirely – arranged in chronological order. The first three chapters provide a chronological survey of (mostly) pure geometry as it developed from the beginnings to the era of Archimedes (so, roughly until the year 200 BCE). Chapter 1, "To the Threshold of Greek Mathematics," provides a comparative context, zooms in on the Babylonian antecedents to Greek mathematics, and then argues that the beginnings of Greek mathematics are to be found in the second half of the fifth century BCE. I then argue that much of the creative activity in Greek pure mathematics took place in two generational events. The first occurred early in the fourth century, and it is the subject of Chapter 2, "The Generation of Archytas." The second occurred late in the third century, and it is the subject of Chapter 3, "The Generation of Archimedes" (which also surveys the developments in the era in between the generations, including the important figure of Euclid). I argue that there is a significant difference between the two generations: in the generation of Archytas, mathematics was in dialogue with philosophy; in the generation of Archimedes, it was much more autonomous. Chapter 3 has the most mathematics: many authors of this era are extant, and their contributions are extremely important; as a consequence, it is also a longer chapter.

The two chapters that follow take up the more applied mathematical sciences: Chapter 4, "Mathematics in the World," looks at various "mechanical" and similar applications, and Chapter 5, "Mathematics of the Stars," looks at astronomy. Although these chapters break away from the chronological sequence, many of the developments in those more applied fields took place in the late Hellenistic era and then in the Roman imperial period, so those two chapters, taken together, extend the survey all the way down to around 200 CE. Astronomy is a huge field (which is why Heath put it aside); thus, Chapter 5, once again, is a longer chapter.

From 200 CE onward, the legacy of Greek mathematics was formed and carried forward by many civilizations. This is the subject of the final two chapters. Chapter 6, "The Canonization of Greek Mathematics," discusses the absorption of mathematics into Neoplatonist philosophy, and specifically into the practice of philosophical commentary, in Late Antiquity. The final chapter, Chapter 7, "Into Modern Science: The Legacy of Greek Mathematics," considers both the survival of Greek mathematics, through the transmission of the works in manuscript in Byzantium, and its impact in later scientific civilizations, such as the Islamic world and, finally, early modern Europe – with the renaissance of Greek mathematics giving rise, ultimately, to the rise of modern science.

Acknowledgments

Back in 1986, the Tel Aviv University's Bulletin was still an actual printed book. Flipping through it, I chanced upon an intriguing class: "Euclid's Elements," taught by Professor Sabetai Unguru. I went in, very soon realized I had much more to learn; I still do. Thank you, Sabetai.

To list the people to whom this book owes a debt would require a fully-fledged memoir. Let me just say that I had plenty of good luck in my encounters since. Avoiding a long apparatus, this book provides instead suggestions for further reading. Looking again through them all, I notice how many of the names I cite are those of my teachers or of my peers. This book emphasizes the role of the scientific network: the people you talk to shape the scholarship you produce. I was formed as a scholar in the 1990s, and the network organized around the study of early science was, back then, bursting with creative energy, with Geoffrey Lloyd as its philosopher-king. I just list those of my friends that I cited in the Suggestions for Further Reading (apologies to the many of you I have learned from in other ways): Len Berggren, Alain Bernard, Alan Bowen, Christián Carman, Karine Chemla, Jean Christianidis, Leo Corry, Serafina Cuomo, Daniella Dueck, Michael Fried, Jens Høyrup, Carl Huffman, Alexander Jones, Henry Mendell, William Noel, Josiah Ober, Eleanor Robson, Courtney Roby, Francesca Rochberg, Ken Saito, David Sedley, Michalis Sialaros, Nathan Sidoli, Natalie Tchernetska, Nigel Wilson, Ido Yavetz, and Leonid Zhmud. A particular thank-you to Christian Carman and to Alexander Jones, who read and vastly improved an early version of Chapter 5, and to Geoffrey Lloyd and Yuval Wigderson, who commented insightfully on the entire manuscript. A particular sorrow is that one's new work can no longer be shared with Andrew Barker, Myles Burnyeat, and Ian Mueller; I constantly remember David Fowler, the loveliest of friends.

I never met Wilbur Knorr. This book owes everything to his research. In 1999 I stepped into his giant shoes in the department of Classics at Stanford. This book would have been impossible without the generosity of my department and of my deans. I wrote it during a sabbatical year at the Center for Advanced Studies in Behavioral Sciences (CASBS), where I benefited from the idyllic site and from the delightful company of my co-scholars of the last academic year before the pandemic, 2018–2019. Sixty years before, in the same place – a few studies away from my own – Thomas Kuhn wrote his *The Structure of Scientific Revolutions*. As I was writing this book, I became more and more convinced that Kuhn was completely wrong about ancient science. But then again, I came to wonder, how would my own book fare? Scholars come and go; places such as CASBS remain; scholarship progresses from one wrong book to the next.

For this, we have publishers to thank. The profession of classics is lucky to have Michael Sharp as the editor of Classics at Cambridge University Press, bringing into being, with his assured touch, so much of the literature on which we rely. Bethany Johnson, as content manager, deserves special honor for steering so many books through the ravages and backlogs of the pandemic. Special thanks to Kirsten Balayti, my copy-editor, and Amy Carlow, my research assistant who helped with preparation of the index and also discovered typos I am too ashamed to reveal. And now, thanks to all of you, it is finally ready, a printed book. I can now hope for chance encounters: future readers, chancing upon the book, flipping through its pages. Maybe, some of my readers might even decide to embark upon the study of Greek mathematics. . . . Thank you, my readers.