Cambridge University Press & Assessment
978-1-108-83384-4 — A New History of Greek Mathematics
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CHAPTER 1

To the Threshold of Greek Mathematics

Plan of the Chapter

This chapter is introductory. I first survey, in a quick sweep, mathematics before Greece. This is followed by the historical context for the rise of mathematics in Greece itself (a discussion heavy with historiographical problems because so much is speculative). Finally, I conclude with a picture of the earliest known Greek mathematics.

I start with a section titled "Before Greece" – indeed, before any organized science at all. What are the universally shared bits of mathematics known even to simple societies? We find considerable but shallow knowledge. Familiarity with numbers and shapes is nearly universal – but does it amount to mathematics? "Empire and the Invention of Mathematics" brings in the rise of the state and with it, I argue, mathematical knowledge; "Beginning in Babylon" zooms in on the most important antecedent to the Greeks: the mathematics of Mesopotamia.

This, then, provides one kind of introduction. Another has to do with the Greeks themselves. The section titled "The Greeks: Standing Apart?" brings in the basic historical context: the unique characteristics of early Greek civilization. But where and how does mathematics emerge in Greece? "Greek Mathematical Myths" argues against some traditional narratives (most important: Pythagoras the mathematician was, I argue, indeed a myth). Another problematic context is that of Mesopotamian mathematics, and the following section, "Greeks and the Near East: A Historiographical Detour," tries to delineate a possible account of the debt owed by the Greeks to their predecessors.

With all of this in place, we may finally get to "The Threshold of Mathematics," which I identify as the mathematics attested to Hippocrates of Chios, and I conclude with "Assessing the Threshold": the historical meaning of this new Greek invention of mathematics.

Before Greece

Throughout this book, I will argue that Greek mathematicians had achieved something quite unprecedented. But of course, people everywhere know some mathematics, and the Greeks specifically must have owed something to past cultures. They did not start from scratch!

All of this sounds nearly obvious. In fact, we've merely started – and have already entered a minefield. The question of the cognitive universals underlying mathematics is invested with political meaning.

The issue can be stated quite simply, and it should be stated right as we begin. Students from disadvantaged backgrounds do much worse in mathematical tests. The response to this fact varies. Some take comfort. (They see the results of mathematical tests as proof of their belief in their group's superiority over others.) Most, aware of the enormous difference that social conditions make to cognitive growth, are less surprised that the underprivileged are also the mathematical underachievers. The explicit racist position is, frankly, preposterous, but it is stated by some and perhaps harbored by many. And so it is right that I should address it, head-on, right at the beginning. Consider the following two statements: (A) "The Greeks invented mathematics because they were white," and (B) "John is good at math because he is a white boy." If A strikes you as implausible, so should B. And if A does not strike you as implausible. . . . Well, this is, in part, why I've written this book.

So, what to do with mathematical tests? Some would say that they should not matter. Do math for the intellectual satisfaction it brings, not to get a good grade! But mathematical educators do not have the luxury of retreating into such fantasies. They have to go and teach in a world where mathematical tests do matter, and so the urgent task is this: How can we make mathematics more accessible to underprivileged students?

Now, this brings us back to the history of mathematics and to the question of universals. This question – how to make mathematics more accessible to the underprivileged – became especially acute in the global scene in the aftermath of decolonization in the 1960s and 1970s. New states in the Third World aimed to make education universal; however, this newly available education, more often than not, did not empower students but instead instilled in them a sense of helplessness and dependency. The mathematics was alien and forbidding, and so the best educators looked for ways to make it grow directly out of the students' own culture. Paulus Gerdes, for instance, as a young mathematics teacher in Mozambique, noticed that fishermen prepare their haul for sale by drying their fish near a fire built in the sand by the seashore. To make sure all the fish become dry at the same time, they follow a certain procedure. First, plant a stick in the ground, then attach a rope, and with a second stick attached at the other side of the rope, draw a circle in the sand. At this point, place all the caught fish along this drawn line, and finally, build the fire at the center. Gerdes's idea was revolutionary – and straightforward: Instead of starting with some abstract definitions, would it not make more sense to teach the children of those fishermen the concepts of "circle," "center," and "circumference" based on this procedure?[1]

Multiply this kind of example hundreds of times, and you have the discipline of ethnomathematics. Anthropologists, even apart from any application to the education of mathematics, came to be interested in the mathematical ideas available to preliterate societies; cognitive psychologists soon came to appreciate the significance of this research for the study of the universal human mind.

Thanks to the work of the ethnomathematicians, several observations emerged. First, numbers are pretty much universal. To be clear: it has been observed that the Pirahã tribe in the Amazon has no words for numerals. (There is some scholarly debate over this: Do the Pirahã words *hói* and *hoí* mean "one" and "two," respectively, or do they mean – as the best experts now seem to believe – merely something like "small" and "larger"?) It is extremely interesting to cognitive psychologists if, indeed, even a single language could fail to develop numerical terms – and so, perhaps, number is not directly hardwired into the human brain.[2] However, from the point of view of the anthropologist or of the historian, the example of the Pirahã is striking primarily for its freakish rarity. Everywhere you go around the globe, languages possess varied systems of counting. A few might be more impoverished (in particular, the Amazon has a number of less numerical societies, of which the Pirahã are an extreme and relatively well-studied case). But more often, simple societies have highly sophisticated numerical systems, with addition, multiplication, and iteration encoded into language itself. (Only one among these is the base-ten numerical systems now used by nearly all humans; it is nearly universal, perhaps, because it is, if anything, mathematically simpler than many of its alternatives.)

[1] This example and more like it are detailed in P. Gerdes, "Conditions and Strategies for Emancipatory Mathematics Education in Undeveloped Countries," *For the Learning of Mathematics* 5 (1985): 15–20.

[2] For a fascinating account, see M. C. Frank, D. L. Everett, E. Fedorenko, and E. Gibson, "Number as a Cognitive Technology: Evidence from Pirahã Language and Cognition," *Cognition* 108 (2008): 819–824.

Second, geometrical terms are not as universally verbalized, but once again, one of the most persistent features of almost all cultures is some kind of attention to patterns – molded, painted, tattooed, drawn in the sand. Those patterns often display symmetries and occasionally involve more precisely drawn geometrical shapes. Does this amount, in and of itself, to geometry? Is any of this *mathematics*?

Authors in the tradition of ethnomathematics often elide this question, and one sometimes has the impression that they try to impute to indigenous cultures geometrical knowledge concerning figures, where in fact, all that those cultures have is the habit of producing those figures. Some ethnomathematicians probably are overenthusiastic in this sense, but mostly this is a misleading framing. Once again, let us take an example from Paulus Gerdes. He describes the following pattern in Mozambique weaving baskets:

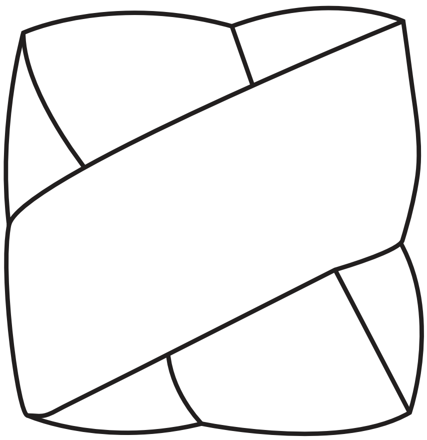

Figure 1

A nice geometrical pattern! But more than this, Gerdes observes, we may share this pattern in class and then proceed to discuss, with our students, how we may find here a relation between the various areas. In fact, with a little manipulation, we can derive, from this pattern, Pythagoras's theorem itself! (The main idea is that we see a big square – composed of four identical right triangles – and a smaller square enclosed in the middle. It is likely, I believe, that Pythagoras's theorem was indeed discovered around such drawings – by Babylonian teachers, working in a very different milieu. We shall return to see this in the discussion that follows.) Now, Gerdes does not mean that African basket-weavers are aware of Pythagoras's theorem; but it is nonetheless likely that the near-universal presence of patterns, of one form or another, is a significant precondition for the rise of geometrical reflection.

However, let us not get carried away. This is not yet reflective of explicit knowledge of geometrical properties, nor is the presence of a numerical vocabulary tantamount to the explicit knowledge of arithmetic. The discipline of ethnomathematics is useful for its scope – as well as for its limits. All humans, everywhere, talk about quantity and operate with shapes. But they almost never reflect on them explicitly, let alone develop a specialized craft of talking about numbers and figures. The discipline of mathematics and the profession of the mathematician are extremely rare.

Ethnomathematics is, of course, part of ethnography, and ethnographers tend to focus on what people *do* – how people interact, form kinship structures, cook, talk, sing. Anthropologists are trained to observe action, and so ethnomathematicians, quite properly, observe actions that are rich in mathematical meaning: counting, calculating, patterning. Those actions are real and form the background for the history we are about to explore in this book. Still, we should try to draw a line between an action that can be explained, *by us*, through our own mathematical understanding and the actors' own mathematical knowledge. Fishermen in Mozambique draw lines in the sand to dry their fish, and it is right and proper that we describe those lines in terms of circle, center, and circumference. It is also important to draw the conclusion that those fishermen have what it takes to create geometry. And finally, it is reasonable to say that the fishermen act in a geometrically intelligent way, without possessing any knowledge of a theoretical field such as geometry.

Many of you would probably agree that drawing a circle in the sand does not display, yet, knowledge of the theoretical field of geometry. I would say that the same is true about drawing a route, from point A to point B, along a straight line. This is a geometrically intelligent practice – but not a display of geometrical theory. I would also say the same about the building of a straight canal of irrigation leading to your fields. If you construct such a canal, then it is still the case that you may, or may not, have some theoretical understanding of "lines." I would also say the same about a straight road, faced by straight walls that form rectangular houses. And I would continue to say the same even if the houses become very imposing and perhaps assume the more complicated forms of various temples and pyramids. A pyramid, in and of itself, implies no more science than a line drawn for drying fish on the sand.

All of this is relevant to the question of the rise of mathematics as a theoretical discipline. We can find extremely sophisticated architecture and town planning around the globe – from the imperial cities of China to those of the Aztecs – and it is often assumed, especially by nonspecialists, that such imposing structures must involve theoretical mathematical knowledge. They certainly could, but the buildings, themselves, are not dispositive. And in fact, when we do find mathematics emerging, the context seems to be somewhat different.

Empire and the Invention of Mathematics

We can locate several historical moments where mathematics was independently invented. Taking them together, we may form certain conclusions about the natural context of such an invention – which brings us back to politics.

The Inca empire, ruling over a vast region of the Andes in South America, left behind many monuments – but no writing. From the very beginning, Western observers noted a puzzling and rather humble artifact. Known as the "quipu," this is a system of knotted threads (often made of cotton) that can usually be spread out as pendants – one main thread, with many others hanging on the main one; occasionally, this can become a many-layered object. Each of the threads has a pattern of knots attached to it, and throughout the twentieth century, as more of these artifacts were surfaced and analyzed, the system came to be understood as essentially numerical (and base ten). Roughly speaking, the knots on a cord form clusters. To simplify things a little, it works like this: if you have a cluster of three knots, a space, and then a cluster of two, this can stand for "32." Such individual numbers on the hanging cords are summed up as the number recorded on the main cord. This, then, seems like an accounting device. The research leading up to this basic decipherment, based purely on a mathematical analysis of the extant quipus (of which there are now several hundred), can be found in the work of Ascher, *Code of the Quipu* (1981). I mention this because Ascher is also one of the most brilliant scholars in ethnomathematics and the author of the basic monograph in the field, *Ethnomathematics: A Multicultural View of Mathematical Ideas* (1991). For her, quipus are an example of ethnomathematics: an indigenous culture's preliterate display of mathematical sophistication. We should, in fact, note an ambiguity: Is that display, strictly speaking, *pre*literate? Or was the quipu, instead, simply the Inca form of literacy? As more evidence came to light in the last generation, based on more careful excavations, we came to understand better the original function of the quipu. As was often suspected in the past, it seems to represent a tax system based on geographical allocation through subdivisions. We find that several quipus replicated each other (a guarantee of accounting consistency), and some quipus may be identified as summing up the results in other quipus (apparently, this represents lower and higher layers of the geographical subdivision). Most spectacularly – a veritable Rosetta Stone – a very late set of quipus from after the Spanish conquest was seen to match a Spanish written list of tributes from across many villages. It now seems likely that the colors of the threads were also meaningful, perhaps encoding geographical regions – thus, quipus were an even more informative system than we had ever assumed. The upshot of this research is that the Inca empire produced a specialized class of quipu masters whose job was to maintain information on the tribute required from across the empire. Now, as a matter of fact, we cannot really say how much "mathematics" those quipu masters knew, precisely because the Inca produced no writing. Whatever education was involved in the perpetuation of the quipu-master technique was purely oral and is now lost. But some education of this kind certainly existed, and so we can say this: in the Andes, prior to Pizarro, there must have been some mathematics actively produced, with people explicitly discussing rules of calculation and accounting.

And another remarkable observation: numeracy was so central, in this particular civilization, that it completely supplanted literacy. To explain: the tool that the state needed was some kind of numerical record. This was efficiently achieved by the quipu, and this did not give rise to literacy as a spin-off.

In other places, of course, states did rely much more on writing. Once again, it is useful to start from as far away from Greece as possible: let us get a sense of the entire range of possibilities. We may begin with China, where finally, we see a very clear tradition of theoretical mathematics. Here it is useful to focus on a relatively late work, *The Nine Chapters on the Mathematical Art*, a work that may have reached something like its current form under the Han dynasty (perhaps in the second century CE?). The Chinese court always required a large retinue of scholars, the bulk of whom were masters of religious rituals, but many specialized in fields such as astrology or other forms of scientific knowledge. It seems that at the latest by the Middle Ages, but perhaps even in the very earliest times, some were trained, and examined, based on their knowledge of the *Nine Chapters* – which is appropriately, then, seen to concentrate around accounting-like needs.[3] The measurement of fields and of heaps of rice and grain, taxation, and distribution by proportion – all brought under a set of general, well-understood algorithms, which then become a subject of study in their own right. The needs of the state, generalized – and turned into a mathematical art. Once again, our evidence in this case is late, and it is hard to tell how mathematics first emerged in China. But more recent archeological excavations do provide us with more context and push the evidence further back. One dramatic find is that of "The Book on Numbers and Computation," a set of inscribed bamboo strips that a civil servant took to his tomb, sometime early in the second century BCE. Much earlier, then, in Chinese imperial history – but still well after the formation of the first Chinese states – yet we see here the same kind of material as that found in the *Nine Chapters*. Problems that relate to concrete bureaucratic needs – solved with considerable general sophistication.

Beginning in Babylon

This brings us to the best-documented and most significant emergence of mathematics – and also, much closer to Greece itself. To the extent that the emergence of Greek mathematics was in debt to previous civilizations, it was to Babylonian mathematics.

This begins very early, along the shores of the Tigris and the Euphrates, and especially near their southern marshes.[4] This is one of the origins of urban civilization, and from the beginning, we find a system of accounting – not unlike that of the Quipu, perhaps – based, this time, on clay. (In the steep Andes, transportation is at a premium, and one looks for light tools; in the flat, river-based civilization of Mesopotamia, heavy but durable inscriptions are favored.) Archeologists have noted small, variously shaped pieces of clay found in many sites from the late Neolithic. Schmandt-Besserat was the first to offer a general account of those tools, and although she is not without her critics, very few doubt her basic interpretation (Schmandt-Besserat's critics mostly point out that the

[3] For the relation between mathematics and administration in the early Chinese state, see K. Chemla and B. Ma, "How Do the Earliest Known Mathematical Writings Highlight the State's Management of Grains in Early Imperial China?" *Archive for History of Exact Sciences* 69, no. 1 (2015): 1–53. Chemla and Ma, remarkably, are able to extract detailed information on the working of the administrative state, based on theoretical mathematical writings!

[4] The history of Mesopotamia is complicated: not a single state but a plethora of city-states and kingdoms, whose kaleidoscope kept shifting over millennia. I skip all the details (this is a history of Greek mathematics!), but read, for instance, N. Postgate, *Early Mesopotamia: Society and Economy at the Dawn of History* (New York: Routledge, 1994).

small pieces of clay could have been used for a variety of purposes beyond those she emphasizes; this is a reasonable critique). Most likely, different shapes stood for different commodities – so, for instance, could it have been a particular shape, say, for one head of cattle? Economic obligations – in the form of contracts or even taxation – could have been certified by an archive of such small tokens. This is all still ethnomathematics, a direct reliance on basic calculation and simple tools. And then, Schmandt-Besserat noted, something dramatic happened: it was realized that one could make impressions on clay, whose shape resembled the actual tokens. Late in the fourth millennium BCE, people in Mesopotamia began to use such tracings as economic records. A new idea, then: visual traces to mark numerical quantities. Pretty soon, instead of being tied to particular commodities, symbols emerged to represent *number as such*, and at this point, it took a mere step (or, if you will, a leap of genius) to begin to record other linguistic elements as well – at first, names of the objects counted and, very soon, language itself with its full vocabulary. By the end of the fourth millennium BCE, one of the major Mesopotamian languages – Sumerian – became fully written, the first ever. Literacy emerged, piggy-backing on numeracy.

Skipping many centuries of Mesopotamian history, we may look at the same shores of the Tigris and the Euphrates almost a millennium later. They are now dominated by different people, speaking a different language (Akkadian, a Semitic language that is somewhat similar to Hebrew or Arabic), still using the same script, the same inscriptions on clay. The technical knowledge of the Sumerians was not lost, in this and in other matters. The rivers themselves required constant attention – digging the canals and irrigating the fields. A lot of engineering, planning, and control was necessary, and throughout, Mesopotamia saw the rise of strong central authorities, powerful temple centers, and kings and their retinue. In the late third millennium, we see clear evidence for a specialized bureaucracy. Scribes were trained in writing, keeping accounts, and advising the rulers. What is most important: they did not just use the basic techniques of writing and calculation; they took pride in becoming genuine masters in all of those. Thus, besides simply writing down bureaucratic records in Akkadian, they also transcribed (a much harder task) the old literary legacy in Sumerian. And they did not just calculate, say, how many workers were required to dig a canal or how much tax should be levied on a field – they also invented particular fictional problems of a more abstract character, where one calculated volumes, plane areas, and work rates. In the Chinese *Nine Chapters* (or in the somewhat earlier "The Book on Numbers and

Computation"), we see the end result of, perhaps, a similar trajectory: bureaucratic training becoming its own raison d'être, giving rise to the problem-set version of a mathematics, which, although quite elementary, is already sophisticated. In Mesopotamia, our evidence is much more plentiful (early Chinese writing used a variety of delicate surfaces, such as the bamboo strip; from Mesopotamia, we have the clay tablet, history's most robust writing material). And so we get a closer sense of the entire transition: tokens, then writing, a bureaucracy, and this, finally – sublimated into mathematics. We have massive evidence, from the end of the third millennium to the beginning of the second millennium BCE. The evidence stops quite abruptly a little after 1800 BCE, for reasons we cannot quite fathom (for indeed, we no longer have substantial evidence!). It appears that the same old cities came under different sets of rulers and that the scribal traditions were disrupted. Little is known, then, for over a millennium – but clearly, there was some continuity. Beginning in the eighth century BCE, we find, once again, Mesopotamian palaces – preserving masses of clay tablets and a lot of the ancient culture. There is little mathematics to be found, though, in this later material (but plenty of astronomy; we shall return to this in Chapter 5). The object we study, then, is fantastically distant in time: the mathematics produced early in the second millennium BCE, or roughly four thousand years ago.

Just what is this mathematics? Let me paraphrase a very simple tablet (BM 13901 #1):

> I have it that the surface of the square, and its side, taken together, are three quarters.
>
> [Implicitly, our task becomes to find the numerical values of the side and area of this square. We're no longer just calculating taxes on fields; we're doing clever problems that build off such calculations! I attach Figure 2; notice that here, as in most cases, we do not have a figure on the clay tablet itself.]
>
> Here is what you should do. Make one as a projection to the side.
>
> [We now have in Figure 2 an elongated rectangle, divided into two parts, of which the right one is the original square, and the left one is a rectangle, one of whose sides is the original side of the square, its other side – one. The area of this left rectangle, then, is equal to 1 × the original side of the square, so its area is taken to be equal to the original side of the square. At this point, we can say that the entire elongated rectangle is equal to the original square plus the original side of the square. This is all equal to three-quarters, then.]