The Art of Mathematics – Take Two

Lovers of mathematics, young and old, professional and amateur, will enjoy this book. It is mathematics with fun: a collection of attractive problems that will delight and test readers. Many of the problems are drawn from the large number that have entertained and challenged students, guests and colleagues over the years during afternoon tea. The problems have their roots in many areas of mathematics. They vary greatly in difficulty: some are very easy, but most are far from trivial, and quite a few rather hard. Many provide substantial and surprising results that form the tip of an iceberg, providing an introduction to an important topic.

To enjoy and appreciate the problems, readers should browse the book, choose one that looks particularly enticing, and think about it on and off for a while before resorting to the hint or the solution.

Follow threads for an enjoyable and enriching journey through mathematics.

BÉLA BOLLOBÁS has been a Fellow at Trinity College, Cambridge, for over fifty years, for decades as a Director of Studies in Mathematics, teaching the very best undergraduates in England, and is the Chair of Excellence in Combinatorics at the University of Memphis. He has had over seventy Ph.D. students. He is a Fellow of the Royal Society and a Member of the Academia Europaea, and a Foreign Member of the Hungarian Academy of Sciences and of the Polish Academy of Sciences. Among the awards he has received are a Senior Whitehead Prize (2007), a Bocskai Prize (2016), a Széchenyi Prize (2017) and an Honorary Doctorate from Adam Mickiewicz University, Poznań. This is his thirteenth book.

Learning is the greatest pleasure in life

The Art of Mathematics – Take Two

Tea Time in Cambridge

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Preface

This collection of problems is a sequel to *The Art of Mathematics – Coffee Time in Memphis* (CTM). It is a playful tribute to four giants of mathematics and physics I was fortunate to know well: Paul Erdős, Paul Adrien Maurice Dirac, Israil Moiseevich Gelfand and John Edensor Littlewood. As in CTM, many of the problems in this volume are the kind they would have liked to think about. There are also echoes of the influence of my very early mathematical gurus, Baron Gábor Splényi and the geometer István Reiman.

I was horrified when I realized that as I have grown older, I have completely forgotten a number of the gems of classical elementary mathematics I had known so well in my early teens. It is for this reason that several of those gems have found their way into this collection. These ought to be well known by most people interested in mathematics.

This is not a volume for systematic study, but a book to enjoy. The problems have been selected for their beauty and the elegance of their solutions. Questions on the same topic are not collected into separate chapters, as I wanted to avoid the impression that this book can be used as a sound introduction to various topics. Rather, my hope is that the questions will whet the appetite of readers by giving them food for thought without much previous work.

Who are my intended readers? I have tried to make this volume appeal to people with vastly different backgrounds: students who enjoy doing mathematics, professional mathematicians looking for some relaxation, and also all who loved mathematics in their younger days and are still happy to think about it. Even more, I hope that just about everyone in academia will benefit from dipping into this volume.

Some of the problems are very easy, but others are likely to be pretty demanding even for excellent mathematicians. My hope is that a reader will be fascinated by a problem or two, and will be happy to turn them over in his head whether progress is forthcoming or not. I have always found that it is most pleasant to have a problem on my mind that I cannot do. Most problems have 'hints' that should give some help to the reader without taking away the pleasure of finding a complete solution.

The selection of problems shows a bias towards Cambridge mathematicians and, within Cambridge, towards members of Trinity College. As I have been a Fellow of Trinity College for over fifty years, I hope that this bias can be forgiven. My great respect for Trinity College was instilled in me close to sixty years ago by the Trinity mathematicians Harold Davenport and J.E. Littlewood, and the physicist Paul Dirac, who was actually a Fellow of St John's College.

My chief desire has been to make the book readable; in particular, in the proofs I have not aimed for brevity: I often remind the reader of the relevant definitions and facts, so that he does not have to rack his brain to continue the proof. For this reason mathematicians are likely to find that the proofs go too slowly, but less experienced readers might welcome proofs spelled out in full!

The structure of this book is identical with that of CTM: the first section has the Problems, the second the Hints, and the third the Solutions, i.e. the proofs of the appropriate assertions. Needless to say, the reader should try to solve a problem without reading its hint, and get that help only when he is in dire need of it.

Most of the solutions are followed by notes which tend to be longer than in CTM, as they contain remarks not only about the mathematics, but also about the mathematicians involved with the problems.

I would be disappointed to learn that some people may read one of the problems, ponder about it for a minute or two, and then go on to read its solution. That would be a complete misuse of this volume, like hammering in a nail with a screwdriver. If a problem is found to need more mathematical expertise than the reader has, I would recommend abandoning that problem until such time as he has acquired the relevant background: this volume has plenty of problems that need little mathematical sophistication.

There are many people who drew my attention to beautiful problems, and gave me the pleasure of discussing those problems with them. I have received especially much help from Paul Balister (Oxford) and Imre Leader (Cambridge): I owe them a great debt of gratitude. I am also grateful to Józsi Balogh (Urbana), Enrico Bombieri (IAS, Princeton), Tim Gowers (Cambridge), Andrew Granville (Montreal), Misi Hujter (Budapest), Rob Morris (IMPA, Rio de Janeiro), Julian Sahasrabudhe (Cambridge), Tadashi Tokieda (Stanford) and Mark Walters (London).

This book would not have been finished without the great help I have received

from my brilliant friend of many decades, David Tranah, the Editorial Director for Mathematics at Cambridge University Press – much more than I could reasonably have hoped for. From the other side of the pond, my excellent Editorial Assistant, Tricia Simmons, has also given me much help. I am deeply grateful to both of them.

I should like to thank my current research students, Vojtěch Dvořák, Peter van Hintum, Harry Metrebian, Adva Mond, Jan Petr, Julien Portier, Victor Souza and Marius Tiba, for reading parts of the manuscript, and saving me from numerous howlers, like proving the necessity of a condition twice and forgetting to prove its sufficiency. I am sure that many mistakes have remained, for which I apologize.

Finally, this volume would never have been completed without the help and understanding of my wife, Gabriella, who has also put artistic 'Art' into it.

Béla Bollobás
Cambridge, Ascension Day, 2021.