Introduction to Complex Variables and Applications

The study of complex variables is both beautiful from a purely mathematical point of view, and very useful for solving a wide array of problems arising in applications. This introduction to complex variables, suitable as a text for a one-semester course, has been written for undergraduate students in applied mathematics, science, and engineering. Based on the authors' extensive teaching experience, it covers topics of keen interest to these students, including ordinary differential equations, as well as Fourier and Laplace transform methods for solving partial differential equations arising in physical applications. Many worked examples, applications, and exercises are included. With this foundation, students can progress beyond the standard course and explore a range of additional topics, including the generalized Cauchy theorem, Painlevé equations, computational methods, and conformal mapping with circular arcs. Advanced topics are labeled with an asterisk and can either be included in the syllabus or form the basis for challenging student projects.

MARK J. ABLOWITZ is Professor of Applied Mathematics at the University of Colorado, Boulder. He is the author of five books, including *Nonlinear Dispersive Waves* (Cambridge, 2011) and *Complex Variables: Introduction and Applications* (Cambridge, 2003), now in its second edition.

ATHANASSIOS S. FOKAS is Professor of Nonlinear Mathematical Science in the Department of Applied Mathematics and Theoretical Physics at the University of Cambridge. He is also Adjunct Professor in the Viterby School of Engineering at the University of Southern California. He is the author of four books, including *Complex Variables: Introduction and Applications* (Cambridge, 2003) and *A Unified Approach to Boundary Value Problems* (2008).

Cambridge Texts in Applied Mathematics

All titles listed below can be obtained from good booksellers or from Cambridge University Press. For a complete series listing, visit www.cambridge.org/mathematics.

Geometric and Topological Inference
JEAN-DANIEL BOISSONNAT, FRÉDÉRIC CHAZAL & MARIETTE YVINEC

Introduction to Magnetohydrodynamics (2nd Edition)
P. A. DAVIDSON

An Introduction to Stochastic Dynamics
JINQIAO DUAN

Singularities: Formation, Structure and Propagation
J. EGGERS & M. A. FONTELOS

Stochastic Modelling of Reaction-Diffusion Processes
RADEK ERBAN & S. JONATHAN CHAPMAN

Microhydrodynamics, Brownian Motion and Complex Fluids
MICHAEL D. GRAHAM

Discrete Systems and Integrability
J. HIETARINTA, N. JOSHI & F. W. NIJHOFF

Think Before You Compute: A Prelude to Computational Fluid Dynamics
E. J. HINCH

An Introduction to Polynomial and Semi-Algebraic Optimization
JEAN BERNARD LASSERRE

The Fluid Dynamics of Cell Motility
ERIC LAUGA

Self-Exciting Fluid Dynamos
KEITH MOFFATT & EMMANUEL DORMY

Numerical Linear Algebra
HOLGER WENDLAND

Introduction to Complex Variables and Applications

MARK J. ABLOWITZ
University of Colorado Boulder

ATHANASSIOS S. FOKAS
University of Cambridge and University of Southern California

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108832618
DOI: 10.1017/9781108961806

First published 2021

Printed in the United Kingdom by TJ Books Limited, Padstow Cornwall

A catalogue record for this publication is available from the British Library.

ISBN 978-1-108-83261-8 Hardback
ISBN 978-1-108-95972-8 Paperback

Contents

Preface

The study of complex variables is both beautiful from a purely mathematical point of view, and provides a powerful tool for solving a wide array of problems arising in applications. It is perhaps surprising that to explain real phenomena, mathematicians, scientists, and engineers often resort to the "complex plane." In fact using complex variables one can solve many problems that are either very difficult or virtually impossible to solve by other means. The text provides a broad treatment of both the fundamentals and the applications of this subject.

This text can be used in an introductory undergraduate course. Alternatively, it can be used in a beginning graduate-level course and as a reference. Indeed, this book provides an introduction to the study of complex variables. It also contains a number of applications which include evaluation of integrals, methods of solution to certain ordinary and partial differential equations, and ideal fluid flow. It also provides a broad discussion of conformal mappings and many of their applications. In fact, applications are discussed throughout the book. Our point of view is that students are motivated and enjoy learning the material when they can relate it to applications.

To aid the instructor we have denoted with an asterisk certain sections which are more advanced. These sections can be read independently or can be skipped. However, in teaching the course we have found that the more advanced sections can be effectively used as a source of valuable material for student projects. Every effort has been made to make this book self-contained. Thus, advanced students using this text will have the basic material at their disposal without dependence on other references.

We realize that many of the topics presented in this book are not usually covered in complex variables texts. This includes the generalized Cauchy integral formula, ODEs in the complex plane, the solution of linear PDEs by integral transforms, conformal mappings of polygons with circular sides, etc. Actually some of these topics, when studied at all, are only included in advanced graduate-level courses.

However, we believe that these topics arise so frequently in applications that early exposure is useful. It is fortunate that it is indeed possible to present this material in such a way that it can be understood with only the foundation presented in the introductory chapters of this book.

We are indebted to our families who have endured all too many hours of our absence. We are thankful to B. Fast and C. Smith for an outstanding job of word processing the manuscript and to B. Fast who has so capably used mathematical software to verify many formulae and produce figures.

Several colleagues helped us with the preparation of this book. B. Herbst made many suggestions and was instrumental in the development of the computational section. C. Schober, L. Luo, and L. Glasser worked with us on many of the exercises.

We are deeply appreciative that (the late) David Benney encouraged us to write this book. We would like to take this opportunity to thank those agencies who have over the years consistently supported our research efforts. Actually this research led us to several of the applications presented in this book. We thank the Air Force Office of Scientific Research, the National Science Foundation, the Engineering and Physical Research Council of the UK, and in particular Arje Nachman, Program Director (Air Force Office of Scientific Research), for his continual support.