

## Social Aggregations and Distributional Ethics

This coursebook looks at four distinct areas of social choice theory and welfare economics: nonstrategic choice, Harsanyi's aggregation theorems, distributional ethics, and strategic choice. It analyzes Arrow's social welfare ordering and addresses issues relating to social choice functions. It studies practical applications of social welfare functions and looks at welfare-theoretic approaches to the measurement of both single and multidimensional inequality. Harsanyi's social aggregation theorems are studied to explain how individual utility functions can be aggregated into a social utility function under the assumption that individual and social preferences satisfy expected utility hypotheses.

The methodology employed in the book is analytical and geometric. Each chapter is supplemented with bibliographical notes, and technical terms and operations employed to discuss the results are explained in a comprehensive and accessible way.

The book has developed out of two different courses on social choice theory regularly taught at the level Master of Science in Quantitative Economics. It will be useful material for students and researchers interested in this frontline area of study, both in India and globally.

Key features include:

- Review questions
- Numerical examples and non-technical explanations
- Graphical representations of concepts and results

**Satya R. Chakravarty** is a non-resident honorary distinguished fellow at the Indira Gandhi Institute of Development Research, Mumbai, and an honorary visiting professor of economics at the Indian Statistical Institute, Kolkata. He is a renowned name in the fields of studies of voting game theory and measurement of inequality, poverty, and polarization, and has published widely, both books and in reputed journals. He authored *A Course on Cooperative Game Theory* with Manipushpak Mitra and Palash Sarkar, which was published by the Press in 2015.

**Manipushpak Mitra** is professor of economics at the Indian Statistical Institute, Kolkata. His research interests are game theory, individual and collective choice, industrial organization, and mechanism design.

**Suresh Mutuswami** is professor of economics at the School of Business, University of Leicester. His research interests are social choice and microeconomic theory.

# Social Aggregations and Distributional Ethics

Satya R. Chakravarty  
Manipushpak Mitra  
Suresh Mutuswami



Cambridge University Press & Assessment  
978-1-108-83204-5 — Social Aggregations and Distributional Ethics  
Satya R. Chakravarty, Manipushpak Mitra, Suresh Mutuswami  
Frontmatter  
[More Information](#)

---



**CAMBRIDGE**  
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781108832045](http://www.cambridge.org/9781108832045)

© Satya R. Chakravarty, Manipushpak Mitra, and Suresh Mutuswami 2022

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press & Assessment.

First published 2022

*A catalogue record for this publication is available from the British Library*

ISBN 978-1-108-83204-5 Hardback

ISBN 978-1-108-92763-5 Paperback

Cambridge University Press & Assessment has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

CONTENTS

*Preface* ix

**1. Introduction 1**

    References . . . . . 12

**2. Individual and Social Orderings 17**

    2.1 Introduction . . . . . 17

    2.2 Relations . . . . . 17

    2.3 Preference Relations and Choice Sets . . . . . 19

    2.4 Social Orderings, Quasi-orderings, and Weak  
        Quasi-orderings . . . . . 22

    2.5 Exercises . . . . . 27

    2.6 Bibliographical Notes . . . . . 28

    References . . . . . 28

**3. May’s Theorem 29**

    3.1 Introduction . . . . . 29

    3.2 The Framework . . . . . 30

    3.3 May’s Theorem . . . . . 32

    3.4 Robustness of the Axioms . . . . . 35

    3.5 Exercises . . . . . 37

    3.6 Bibliographical Notes . . . . . 38

    References . . . . . 38

**4. Arrow’s Theorem with Individual Preferences 39**

    4.1 Introduction . . . . . 39

    4.2 The Framework . . . . . 40

    4.3 The Arrow Impossibility Theorem . . . . . 41

    4.4 Two Proofs of Arrow’s Theorem . . . . . 45

    4.5 Exercises . . . . . 53

4.6	Bibliographical Notes . . . . .	54
	References . . . . .	54
<b>5.</b>	<b>Relaxing Arrow’s Axioms</b>	<b>57</b>
5.1	Introduction . . . . .	57
5.2	Relaxing Weak Pareto . . . . .	58
5.3	Relaxing Transitivity of (Social) Binary Relations . . . . .	62
5.4	Domain Restriction: Single-Peaked Preferences . . . . .	70
5.5	Exercises . . . . .	73
5.6	Bibliographical Notes . . . . .	74
	References . . . . .	74
<b>6.</b>	<b>Arrow’s Theorem with Utilities</b>	<b>77</b>
6.1	Introduction . . . . .	77
6.2	The Framework and Assumptions . . . . .	78
6.3	Measurability and Comparability . . . . .	81
6.4	Arrow’s Theorem . . . . .	85
6.5	Positional Dictatorships . . . . .	89
6.6	Leximin . . . . .	93
6.7	Utilitarianism . . . . .	95
6.8	Exercises . . . . .	97
6.9	Bibliographical Notes . . . . .	98
	References . . . . .	99
<b>7.</b>	<b>Harsanyi’s Social Aggregation Theorem</b>	<b>101</b>
7.1	Introduction . . . . .	101
7.2	The model . . . . .	102
7.3	Appendix . . . . .	116
7.4	Illustrative Examples . . . . .	116
7.5	Exercises . . . . .	118
7.6	Bibliographical Notes and Discussions . . . . .	119
	References . . . . .	122
<b>8.</b>	<b>Distributional Ethics: Single Dimensional Approaches</b>	<b>127</b>
8.1	Introduction . . . . .	127
8.2	Basics and Preliminaries . . . . .	128
8.3	Common Features . . . . .	133
8.4	The Direct Approach . . . . .	137
8.5	The Inclusive Measure of Well-Being Approach . . . . .	140

8.6	Direct Descriptive Inequality Indices and Reduced Form Welfare Functions . . . . .	143
8.7	Measuring Inequality within the Harsanyi Framework . . . . .	146
8.8	Comparability between Achievement and Shortfall Inequality: An Expository Analysis . . . . .	149
8.9	Equality of Opportunity: An Illustrative Discussion . . . . .	154
8.10	Inequality and Welfare with an Ordinal Dimension of Well-Being . . . . .	159
8.11	Inequality as an Ordinal Notion . . . . .	163
8.12	Fairness in Network Resource Allocation: An Analytical Exposition . . . . .	165
8.13	Exercises . . . . .	169
8.14	Bibliographical Notes . . . . .	171
	References . . . . .	172
<b>9.</b>	<b>Distributional Ethics: Multidimensional Approaches</b>	<b>183</b>
9.1	Motivations . . . . .	183
9.2	Basics and Preliminaries . . . . .	185
9.3	Common Features . . . . .	194
9.4	The Direct Approach . . . . .	197
9.5	The Inclusive Measure of Well-Being Approach . . . . .	200
9.6	Direct Descriptive Multidimensional Inequality Indices and Reduced Form Welfare Functions . . . . .	205
9.7	Inequality under Uncertainty: A Brief Discussion . . . . .	207
9.8	Exercises . . . . .	209
9.9	Bibliographical Notes . . . . .	210
	References . . . . .	211
<b>10.</b>	<b>Social Choice Functions</b>	<b>219</b>
10.1	Introduction . . . . .	219
10.2	The Framework and the Gibbard–Satterthwaite Theorem . . . . .	220
10.3	Two Proofs of the Gibbard–Satterthwaite Theorem . . . . .	223
10.4	Single-Peaked Preferences . . . . .	232
10.5	Exercises . . . . .	236
10.6	Bibliographical Notes . . . . .	237
	References . . . . .	238

viii	<i>Contents</i>
<b>11.</b>	<b>Strategyproofness on Quasi-linear Domains 241</b>
11.1	Introduction . . . . . 241
11.2	The Pure Public Goods Problem . . . . . 242
11.3	Allocation of a Single Indivisible Object . . . . . 253
11.4	Relaxing Outcome Efficiency: Affine Maximizers . . . . . 254
11.5	Exercises . . . . . 257
11.6	Bibliographical Notes . . . . . 258
	References . . . . . 259
<i>Index</i>	261

---

## PREFACE

Social aggregation theory deals with the problem of amalgamation of the values assigned by different individuals to alternative social or economic states in a society into values for the entire society. A social state provides a description of materials that are related to the well-being of a population in different ways. One fundamental question that arises at the outset is how an individual can rank two alternative states in a well-defined manner. The problem of social aggregation theory can then be regarded as one of clubbing individual rankings into a social ranking in a meaningful way.

Chapter 1 presents an introductory outline of the materials analyzed in the remaining chapters. Chapter 2 of the monograph formally defines individual and social rankings of alternative states of affairs. Chapter 3 provides a rigorous discussion on May's remarkable theorem on "social choice functions," which represents group or collective decision rules, limited to two alternatives. In Chapter 4 we analyze Arrow's social welfare function, a mapping from the set of all possible profiles of individual orderings of social states to the set of all possible orderings. We discuss this with some elaboration in view of the fact that Arrow's model forms the basis of almost the entire social choice theory. Chapter 5 investigates to what extent the "dictatorship" result can be avoided when some of Arrow's axioms are relaxed. In Arrow's framework, when individual preferences are represented by utility functions, utilities are of the ordinal and non-comparable types. Arrow's theorem with utility functions constitutes a part of Chapter 6 of the book. We use geometric technique to prove this fundamental result. In the recent past, the non-comparability assumption has been relaxed to partial and full comparability assumptions. A discussion on alternative notions of measurability and comparability of utility functions is also presented in Chapter 6. Simple numerical examples have been provided to illustrate the ideas. Possibilities of social welfare functions are expanded as a



consequence of interpersonal comparability. Proofs of most theorems in the literature are sophisticated mathematically, which may not be easygoing for non-specialist readers. In view of this, we use simple graphical proofs for many such results.

Harsanyi's social aggregation theorems indicate how individual utility functions can be aggregated into a social utility function under the assumption that individual and social preferences satisfy the expected utility hypotheses. There is a clear line of demarcation between the Arrowian approach and the Harsanyi approach. This is because in the former and its variants, decision-making by different individuals takes place in an environment characterized by certainty. On the contrary, uncertainty has an impact on the behavior of different individuals in the latter. We include a discussion on Harsanyi's theorems in Chapter 7.

Chapters 8 and 9 present real life applications of social welfare functions by analyzing welfare-theoretic approaches to the measurement of both single and multidimensional inequalities. An inequality index determines the size of welfare loss resulting from the existence of inequality. There will be extensive discussions on policy implications of the underlying indices.

Social choice functions are addressed with greater detail in Chapters 10–11. While Chapter 10 mainly concentrates on the well-known Gibbard–Satterthwaite theorem, Chapter 11 deals with “strategyproof” decision rules.

Chakravarty and Mitra have used different chapters of the book to offer a course on social choice and welfare economics at the level of Master of Science in Quantitative Economics, a graduate program offered at the Indian Statistical Institute, Kolkata. Mitra has also used different chapters of the book as part of the Game Theory Course at the level of Master of Science in Economics, a graduate program offered at the University of Calcutta. Some parts of Chapters 2, 4, 5, 10, and 11 have been taken from the online notes of Debasis Mishra and Arunava Sen.<sup>1</sup> We thank Debasis Mishra and Arunava Sen for allowing us to use their notes. We also thank Sreoshi Banerjee, Parikshit De, and Arindam Paul for their suggestions and constructive feedback.

<sup>1</sup> For Debasis Mishra's notes, see <https://www.isid.ac.in/~dmishra/game.html> (accessed on August 17, 2022) and for Arunava Sen's notes, see <https://www.isid.ac.in/~asen/teach.html> (accessed on August 17, 2020).

A major advantage of this book is that it covers four distinct but not unrelated areas of social choice theory and welfare economics, namely, nonstrategic choice, Harsanyi's aggregation theorems, distributional ethics, and strategic choice. The book will therefore serve as a useful resource for students or researchers interested in this area of research. Since the materials presented in the book are distinct but not unrelated, for a researcher interested in one or more of the topics covered, it will work as a device used to connect his research areas to some other closely associated issues. A concrete example is the analysis of fairness in network resource allocation. This may be regarded as an added advantage of our monograph.

There are several innovative features of the book that are worth noting. The methodology of the book is analytical and geometric. The technical terms and mathematical operations employed to discuss the results are explained in a non-technical language and intuitive explanations of the mathematical results are given. Wide coverage of the topics and their analytical, articulate, and authoritative presentation make the book theoretically and methodologically quite contemporary and inclusive. Often analytical examples are used to illustrate the results. Integration of theory and practice helps students understand the theoretical issues first and then see their practical relevance.

We express our sincere gratitude to the students who encouraged us to teach this course. We are grateful to them for the joys and immense benefit we have derived through interactive teaching.